Accessibility of Referents in Discourse Semantics

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Accessibilité des référents en sémantique du discours

THÈSE

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Résumé

Les anaphores recouvrent un phénomène linguistique omniprésent, dans lequel l'interprétation d'une expression, appelée anaphore, dépend de celle d'une autre, appelé l'antécédent. Cette thèse étudie la sémantique d'un type particulier d'anaphore : les anaphores pronominales où l'anaphore et l'antécédent sont des syntagmes nominaux singuliers. Plus précisément, la thèse traite de l'accessibilité des référents de discours à l'aide d'un système formel de la sémantique dynamique. Nos préoccupations principales sont les facteurs qui déterminent les "antécédents potentiels" d'un syntagme nominal, à savoir, les conditions dans lesquelles un syntagme nominal peut agir comme antécédent d'une expression anaphorique donnée.

Grâce au travail de pionniers du siècle précédent comme Tarski et Montague, il a été montré que le langage naturel, en particulier l'anglais, peut être interprété comme un langage formel. Toutefois, la grammaire de Montague (MG) est conçu pour calculer la sémantique de phrases isolées. Mais nous sommes également intéressé par le discours qui est plus qu'une collection aléatoire de phrases sans rapport. Empiriquement, MG ne résout pas une série de phénomènes discursifs, comme les anaphores inter-phrastiques et les *Donkey Sentences*.

Depuis les années 1980, un certain nombre de théories sémantiques ont été établies pour la sémantique du discours, par exemple, la *Discourse Representation Theory* (DRT), *File Change Semantics* (FCS), et *Dynamic Predicate Logic* (DPL). Ces théories sont rassemblées dans la sémantique dynamique, car elles proposent un nouveau point de vue sur le sens: le sens de l'expression est identifié par son potentiel de modification du contexte plutôt que de ses conditions de vérité (comme pour MG). Cependant, les théories dynamiques classiques ne sont pas parfaitement satisfaisantes. Par exemple, la DRT s'appuie sur un niveau indispensable de structure de représentation, où le principe de compositionnalité de Frege et Montague n'est pas respectée. Pour DPL, sa syntaxe est celle de la logique des prédicats standard, qui est une sémantique non-classiques. De plus, à la fois la DRT et DPL souffrent du problème de *destructive assignment*.

Plus récemment, De Groote propose un autre cadre dynamique appelé type théorique Dynamic Logic (TTDL), qui introduit les bases théoriques des travaux de cette thèse. Ce cadre s'inscrit dans la tradition montagovienne et respecte le principe de compositionnalité. Il n'utilise que des outils mathématiques et logiques bien établies, telles que le λ-calcul et la théorie des types. Dans TTDL, les notions de contexte gauche et droit sont introduits afin de rendre compte de la dynamique du discours: le contexte gauche est constitué d'une liste de variables accessibles et le contexte droit est sa continuation. L'accessibilité d'un référent de discours dans TTDL est alors sa présence dans le contexte gauche.

Malgré les précieuses avancées apportées par les théories classiques de la sémantique du discours, il persiste un certain nombre d'exceptions non résolues, par exemple, les anaphores sous double négation et la gestion des modalités. Ce travail de thèse propose une adaptation de TTDL pour chacun de ces deux cas. Brièvement, le problème de la double négation est d'encapsuler dans un tuple à la fois les représentations positive et négative d'une expression. La négation est alors vue comme une opération qui commute les positions des deux représentations. Ainsi, la présence d'une deuxième négation rétablira les positions comme si aucune négation n'avait jamais eu lieu. De cette manière, une double négation peut être éliminé et l'accessibilité aux référents souhaitée est possible. Quant à l'anaphore sous modalité, nous proposons d'enrichir le contexte gauche de TTDL avec la notion de base modale, introduite par Kratzer. Ainsi le modèle de monde possible est ajoutée à la représentation sémantique. Enfin, nous montrons comment les différentes adaptations peuvent coexister.

Mots-clés : Logique, Anaphore, Montague, λ-calcul, Sémantique Dynamique, Discours, Pronom, Accessibilité, Référent, Modalité.

Abstract

Anaphora is a ubiquitous linguistic phenomenon whereby the interpretation of one expression, called the anaphor, depends on that of another, called the antecedent. This thesis studies the semantics of one particular sort of anaphora: pronominal anaphora, where both anaphor and antecedent are singular noun phrases. More specifically, the thesis deals with the accessibility of discourse referents using a formal system of dynamic semantics. Our central concerns are the factors which determine the "antecedent potential" of a noun phrase, namely, the conditions under which a noun phrase may act as antecedent of a particular anaphoric expression.

Due to the pioneering work of Tarski and Montague in the last century, it has been shown that natural language, in particular English, can be interpreted as a formal language. However, Montague Grammar (MG) is designed to account for the semantics of isolated sentences. But we are also interested in discourse which is more than a random collection of unrelated sentences. MG is empirically problematic for a series of discourse phenomena, such as the inter-sentential anaphora and the donkey anaphora.

Since the 1980s, a number of semantic theories have been established for the semantics of discourses, e.g., Discourse Representation Theory (DRT), File Change Semantics (FCS), and Dynamic Predicate Logic (DPL). These theories are subsumed under dynamic semantics because they propose a novel point of view: the meaning of an expression is identified with its potential to change the context, rather than its truth conditions (as in MG). However, the classical dynamic theories are not completely satisfactory. For instance, DRT relies on an indispensable level of representational structure, hence the Fregean and Montagovian tradition of compositionality is not restored. As for DPL, although its syntax is the one of standard predicate logic, which is a non-classical semantics. Further more, both DRT and DPL suffer from the so-called destructive assignment problem.

More recently, De Groote proposes another dynamic framework called Type Theoretic Dynamic Logic (TTDL), which lays the theoretical foundation of this thesis. This framework follows the Montagovian tradition and is completely compositional. It only makes use of well-established mathematical and logical tools, such as λ-calculus and theory of types. In TTDL, the notion of left and right context are introduced in order to achieve dynamics: the left context consists of a list of accessible variables for future reference, and the right context is its continuation. The lift-span of a discourse referent in TTDL is boiled down to its existence in the left context.

Despite the valuable insights yielded by the classical theories of discourse semantics, there is a wide range of exceptional phenomena that they fail to address, e.g., anaphora under double negation and modality. Concentrating on these two exceptions, this thesis provides a corresponding adaptation of TTDL for each case. Briefly speaking, for the problem of double negation, we propose to encapsulate both the affirmative representation and the negative representation of an expression in its semantics. Negation is treated as

an operation which switches the positions of the two representations. Thus a second negation will switch the positions again as if no negation had ever occurred. In this way, a double negation can be eliminated and the desired referent accessibility is modeled. As for anaphora under modality, we propose to enrich the TTDL left context with the notion of modal base, which is proposed by Kratzer. The possible world model is integrated in the semantic representation as well. Moreover, we show how the different adaptations could work in an unified framework.

Keywords: Logic, Anaphora, Montague, λ-calculus, Dynamic Semantics, Discourse, Pronoun, Accessibility, Referent, Modality.

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Le panorama

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0.1 Sémantique formelle des langues naturelles

Le terme de langage naturel, aussi appelé langage humain, ou langage ordinaire, apparaît dans différentes disciplines de recherche comme la philosophie, la linguistique et la logique. On l'entend comme un concept générique qui dénote des langages parlés ou écrits par des humains. À proprement parler, l'étude scientifique des langues est la linguistique. Une vision contemporaine la décompose en six sous-domaines principaux, chacun étant un sujet de recherche, Fromkin (2000). La phonétique et la phonologie étudient les sons et les systèmes abstraits de sons ; la morphologie étudie la structure des mots ; la syntaxe s'intéresse à la structure des syntagmes et des phrases ; la sémantique observe le sens ; et la pragmatique interprète dans un environnement de communication global. Cette thèse se concentre sur la sémantique formelle du langage naturel, à savoir, l'analyse du sens des expressions linguistiques par des systèmes formels, en particulier la logique.

Au milieu du siècle précédent, Alfred Tarski a étudié la sémantique des langages formels en définissant la notion de vérité Tarski (1944, 1956)[1]. Cependant, l'auteur ne s'est pas montré optimiste envers la formalisation de la sémantique des langues naturelles. À la fin de la première section de Tarski (1956), il fait la remarque suivante :

[1] Tarski (1956) a été traduit de l'allemand et publié en 1936. Les travaux originaux l'ont été en polonais en 1933.

> *... the very possibility of a consistent use of the expression 'true sentence' which is in harmony with the laws of logic and the spirit of everyday language seems to be very questionable, and consequently the same doubt attaches to the possibility of constructing a correct definition of this expression*
>
> ...la possibilité même de l'utilisation conforme de l'expression 'phrase vraie', qui serait en harmonie avec les lois de la logique et l'esprit de la langue de tous les jours semble être très discutable, et par conséquent, le même doute persiste quant à la possibilité de construire une définition correcte de cette expression

> *... I now abandon the attempt to solve our problem for the language of everyday life and restrict myself henceforth entirely to formalized languages.* Tarski (1956)
>
> ... j'abandonne maintenant la tentative de résoudre notre problème de la langue de la vie quotidienne et me limite désormais entièrement aux langages formalisés. Tarski (1956)

Dans les années 1970, en utilisant des outils mathématiques (la logique des prédicats d'ordre supérieur, le λ-calcul, la théorie des types, la logique intensionnelle, etc.), Richard Montague établit une sémantique des langages naturelles dans une perspective *model-theoretic* Montague (1970a,b, 1973). Cette série de travaux est connue comme la grammaire de Montague (MG) qui propose d'interpréter le langage naturel, en particulier l'anglais, comme un langue formel.

Plus précisément, dans Montague (1973), l'auteur propose une interprétation du langage naturel en deux étapes. Tout d'abord, les expressions linguistiques sont exprimées dans un langage formel, par exemple la logique des prédicats d'ordre supérieur. Pour cela, chaque constituant est représenté par un λ-terme qui définit sa contribution sémantique. À partir de la structure grammaticale de la phrase, la combinaison des différentes entrées lexicales construit une expression logique globale, grâce à la β-réduction. Le lien entre la structure grammaticale et la structure logique est ainsi conservé, et permet de donner une représentation du sens de l'énoncé. Ensuite, les formules logiques obtenues à partir des étapes précédentes reçoivent une interprétation dans un modèle, comme tout autre système formel, qui fournit une interprétation des expressions linguistiques en termes de conditions de vérité.

0.2 Cohésion and Anaphore

À première vue, une phrase se compose d'un ensemble de mots. Mais c'est évidemment bien plus que cela. Un ensemble aléatoire de mots qui respectent les règles de la grammaire ne donne pas forcément une phrase acceptable. Chomsky a proposé un exemple devenu célèbre :

(1) Colorless green ideas sleep furiously. Chomsky (1957)

Grammaticalement, la phrase (1) est correcte. Mais du point de vue sémantique, elle n'a pas vraiment de sens : la combinaison des constituants de (1) (ie, *colorless*, *green*, *ideas*, *sleep* et *furiously*) ne construit pas un tout sémantiquement cohérent, notamment parce que le vocabulaire n'est pas relié. De manière analogue, un discours est plus qu'un ensemble aléatoire de phrases, par exemple:

(2) a. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. A law firm in which Mr Sarkozy owns shares was also searched, reports say. (émission de BBC News Europe du 3 juillet 2012)
b. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. Tens of thousands have turned out in the streets of the Spanish capital Madrid to welcome the national football team after their victory at Euro 2012. (mélange d'émissions de BBC News Europe du 3 juillet 2012)

Chacun des deux discours de (2) est constitué de deux phrases. Les phrases qui les composent sont parfaitement compréhensibles pour elles-mêmes. Cependant, comme on peut le remarquer, (2-a) est un texte cohérent alors que (2-b) est juste un alignement arbitraire de deux phrases (c'est en effet un mélange de deux articles indépendants). Les deux phrases de (2-a) sont centrées autour du même thème et des indices peuvent être mis en avant (répétition du nom propre *Sarkozy*, relations lexicales entre les expressions telles que *the police*, *law firm*, *search*, etc.). Alors que dans (2-b), ces relations n'apparaissent pas. Cela en fait un texte non compréhensible, à savoir qu'il ne parvient pas à former un "ensemble unifié" en termes de Halliday and Hasan (1976).

Dans Halliday and Hasan (1976), les auteurs caractérisent la connectivité d'un texte cohérent en termes de groupes de mécanismes linguistiques dits de cohésion, y compris les références, la substitution, l'ellipse, la conjonction, et la cohésion lexicale. Ces dispositifs sont présents pour lier les énoncés entre eux et par cela former la connectivité. Ce processus construit alors le texte. Dans la suite de cette thèse, nous allons utiliser indifféremment les deux termes "discours" et "texte", pour désigner un ensemble de phrases connectées. De manière analogue, nous utiliserons "phrase" et "énoncé" pour désigner les constituants de base d'un discours.

La classification des dispositifs cohérents fournit des heuristiques utiles pour des recherches ultérieures, en particulier pour l'analyse de texte. Ces dispositifs ne sont pas mutuellement exclusifs, plutôt, ils se chevauchent et ils s'étendent. Nous ne reviendrons pas sur ces détails, les lecteurs intéressés peuvent se référer à l'ouvrage original Halliday and Hasan (1976). Ici, nous nous intéressons à un sous-domaine particulier de la référence : les anaphores.

Depuis le milieu du 20e siècle, l'étude de l'anaphore a suscité l'intérêt des chercheurs de différentes branches, en particulier celles liées à la linguistique. D'une manière générale, l'anaphore est entendue comme la relation entre deux expressions linguistiques, où l'interprétation d'un élément, appelé l'anaphore, est déterminée par l'interprétation d'un autre, appelé l'antécédent. Nous dirons qu'il y a un lien anaphorique entre l'antécédent et l'anaphore. Ce qui signifie qu'une phrase contenant une anaphore ne peut pas être interprétée pour elle-même, mais doit être plongée dans un contexte. Par exemple:

(3) a. John walks in. He smiles.
b. Bill walks in. He smiles.

Les deux discours ci-dessus partagent la même seconde phrase, qui contient le pronom *he*. Son interprétation est clairement dépendante du contexte : dans (3-a), c'est John qui sourit, tandis que dans (3-b), la personne qui sourit est Bill. Dans ces exemples, les noms propres *John* et *Bill* sont nommés les antécédents et *he* est l'anaphore.

L'anaphore peut relever de différentes catégories syntaxiques. On trouve des anaphores sur des syntagmes nominaux (NP), des syntagmes verbaux (VP), des adjectifs, etc. Pour une étude complète sur la taxonomie des anaphores, le lecteur pourra se référer à Hirst (1981). Nous nous restreignons à un type spécifique d'anaphore : les anaphores pronominales, et plus encore sur les cas où l'anaphore et l'antécédent sont tous les deux des syntagmes nominaux singuliers, comme dans l'exemple (3).

Du point de vue syntaxique, l'un des cadres les plus influents sur l'analyse des anaphores est la théorie du gouvernement et du liage (Gouvernement and Binding (GB)) proposé par Chomsky (1981, 1986b). Pour les syntacticiens chomskiens, l'anaphore est un phénomène de grammaire. Ainsi, elle est exprimée en termes purement syntaxiques, tels que *local domain*, *domain*, etc. Trois principes ont été introduits pour justifier la distribution de l'anaphore. Par exemple, un pronom (*he* ou *him*) ne doit pas trouver son antécédent dans le domaine local, au contraire des pronoms réflexifs. Ces propriétés sont illustrées dans les exemples suivants, où nous avons utilisons le symbole " $*$ " devant un indice pour indiquer que la relation anaphorique sur cet indice n'est pas acceptable :

(4) a. Russell$_i$ admired him$_{*i/j}$.
b. Russell$_i$ admired himself$_{i/*j}$. Huang (2006)

Selon la théorie GB, les deux NP de (4-a) (*Russell* et *him*) ne peuvent pas être anaphoriquement liés, sinon la phrase n'est pas interprétable. Quant à (4-b), le nom propre *Russell* doit être l'antécédent de *himself*. Ces prévisions correspondent bien à l'intuition que nous apporte ces exemples.

A contrario de la théorie GB, le point de vue sémantique de l'anaphore vise à en préciser son interprétation. Il est généralement acquis que les anaphores peuvent être classées du point de vue sémantique en au moins deux types : anaphores référentielles et anaphores liées Bach and Partee (1980); Evans (1980). Cette distinction est illustrée par les exemples suivants, où les deux anaphores ne diffèrent que par l'antécédent:

(5) a. John$_i$ loves his$_i$ mother.
b. Every man$_i$ loves his$_i$ mother. Evans (1980)

L'anaphore dans (5-a) est référentielle en ce que l'expression anaphorique *his* se réfère à l'individu particulier John. Celle de (5-b) est appelée anaphore liée parce que l'anaphore *his* est interprétée par analogie avec la variable liée en logique des prédicats traditionnelle à laquelle il fait référence : *his* est alors lié par le quantificateur universel de *every man*. Les représentations sémantiques de ces deux phrases reflètent bien cela (où (respectivement) **John** est une constante d'individu, **love** est un prédicat à deux arguments, **mother_of** est une fonction qui prend un individu qui retourne un autre individu) :

$$\mathbf{love\ john}\ (\mathbf{mother\_of\ john})$$

$$\forall x.(\mathbf{man}\ x \rightarrow \mathbf{love}\ x\ (\mathbf{mother\_of}\ x))$$

0.3 Accessibilité et sémantique dynamique

Il serait erroné de croire que toutes les paires de NP peuvent former des relations antécédent-anaphore. Ainsi, en plus de l'interprétation des anaphores, il faut considérer si un NP peut servir d'antécédent pour une expression anaphorique particulière. Ce problème est nommé accessibilité de l'antécédent.

La théorie GB définit plusieurs contraintes sur le plan syntaxique, qui ont été établies en fonction de relations structurelles entre l'antécédent et l'anaphore. Cependant, malgré les idées fécondes de la théorie GB, elle ne prend pas en compte toutes les caractéristiques de l'anaphore. En particulier, son étude reste limitée au niveau phrastique, comme dans les exemples (4) et (5) (phénomène que l'on retrouve son le terme d'anaphore intra-phrastique dans la littérature). Pourtant, Halliday and Hasan (1976) montre bien qu'elles sont utilisées pour rassembler des phrases en un texte, ce qui en fait des éléments fondateurs du discours. Cependant, il est important de noter que la plupart de ces interactions se produisent au-delà de la phrase, comme dans l'exemple (3), ou le suivant :

(6) $\underline{\text{A man}}_i$ walks in the park. $\underline{\text{He}}_i$ whistles.

La classification sémantique abordée précédemment ne suffit pas toujours. Par exemple, l'anaphore dans l'exemple (6) ne peut pas être référentielle parce que les deux NP(*a man* et *he*, ne se réfèrent pas à une personne en particulier. Et d'autre part, l'anaphora ne peut être liée à un quantificateur. Suivant Russell (1905), nous traitons les antécédents indéfinis comme *a man* comme des quantifications existentielles. Cependant, la portée du quantificateur ne peut pas être étendue au-delà de la phrase. Un autre exemple notoire d'anaphores complexes est les *Donkey Sentences* :

(7) Every farmer who owns $\underline{\text{a donkey}}_i$ beats $\underline{\text{it}}_i$.

Encore aujourd'hui, l'interprétation sémantique exacte de l'exemple (7) est un sujet de débat. Indépendamment de son interprétation, nous pouvons voir que, bien que l'indéfini *a donkey* est dans la portée d'une quantification universelle (c'est-à-dire, *every farmer*), il est acceptable de le relier anaphoriquement comme référent au pronom *it*.

En conséquence, afin de surmonter les problèmes empiriques issus des analyses syntaxique et sémantique, les chercheurs ont étudié l'anaphore au niveau du discours. C'est dans cette perspective que s'inscrivent ces travaux de thèse. Certaines questions sont évidemment corrélées : comment rendre compte de l'accessibilité des antécédents dans l'anaphore inter-phrastique ? Quelle est la différence entre anaphore inter-phrastique et intra-phrastique ? Peuvent-elles être prises en compte dans une solution unifiée ?

À la fin des années 1960, Karttunen a proposé une façon intuitive et générale de décrire les anaphores, en particulier les anaphores intra-phrastiques Karttunen (1969). En introduisant la notion de référent discours, il classe d'une manière uniforme les anaphores selon deux classes sémantiques (référentielle et liée). Essentiellement, un référent de discours est une entité fonctionnant comme une variable. Lors de l'analyse d'un discours, les NP indéfinis introduissent un nouveau référent de discours, au contraire des anaphores qui n'en ont pas la capacité. Les anaphores doivent donc chercher un référent de discours précédemment introduit et qui sera donc interprété comme son antécédent. Par exemple dans (6), le NP *a man* introduit un référent de discours et l'anaphore *he* est ainsi identifiée avec le même référent. Cependant, il n'est plus possible de poursuivre (6) avec des phrases telles que *he smokes*, où le pronom doit être anaphoriquement relié au même référent de *a man*. Ce qui donne une explication pour les anaphores non traitées aux niveaux syntaxique et sémantique.

Karttunen a remarqué que le référent de discours a une durée de vie, à savoir un référent ne peut pas toujours être accessible pour résoudre une expressions anaphoriques. Par exemple:

(8) Bill doesn't have a car$_i$. *It$_i$ is black. Karttunen (1969)

(9) You must write a letter$_i$ to your parents. *They are expecting the letter$_i$. Karttunen (1969)

À partir des exemples ci-dessus, Karttunen conclut que la durée de vie d'un référent du discours est généralement déterminée par la portée de l'opérateur logique dans lequel il est introduit. Plus précisément, si un NP indéfini apparaît dans la portée d'un opérateur, la durée de vie du référent est identique à celle de cet opérateur. Dans l'exemple (8), l'indéfini est dans la portée d'une négation, de sorte que le référent de discours de *a car* n'est pas accessible pour le pronom *it*. De manière analogue, dans l'exemple (9), l'auxiliaire modal *doit* prend une portée plus large que l'indéfini *a letter*, l'expression anaphorique *the letter* ne peut donc pas à être résolue avec *a letter*.

Les observations de Karttunen fournissent une description précieuse de l'accessibilité de l'antécédent du point de vue sémantique. Cette thèse s'inscrit clairement dans cette lignée. Bien que Karttunen n'ait pas établi une théorie sémantique formelle, son travail a lancé un nouveau mouvement pour les théories sémantiques et ce depuis les années 1980, appelé *la sémantique dynamique*. Les principaux cadres formels établis depuis lors sont la Discourse Representation Theory (DRT) Kamp (1981), File Change Semantics (FCS) Heim (1982), et Dynamic Predicate Logic (DPL) Groenendijk and Stokhof (1991). Leurs modes de fonctionnement sont divers, bien qu'ils se basent tous sur le principe des référents de discours et tentent de décrire l'anaphore en ces termes. La question de l'accessibilité de l'antécédent est systématiquement caractérisée par les interactions entre les différents opérateurs logiques.

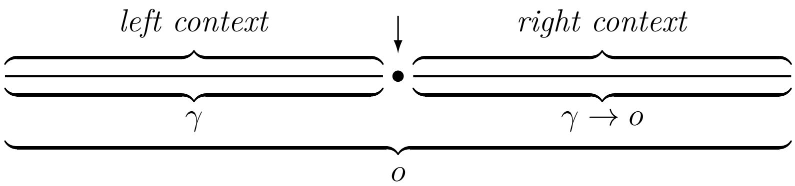
Le fait de chercher à interpréter au niveau du discours et non plus seulement au niveau de la phrase introduit un nouveau point de vue sur le sens. Selon la sémantique dynamique, le sens d'une expression est identifié par son potentiel de modification du contexte, plutôt que par ses conditions de vérité (comme dans la logique classique, telle que dans les MG). Le contexte est alors utilisé pour désigner les affectations de référents de discours (variables) (un contexte est nécessaire pour interpréter une phrase) et, en retour, le résultat de l'interprétation est un contexte mis à jour.

Cependant, les théories dynamiques classiques ne sont pas entièrement satisfaisantes. Par exemple, la DRT repose sur un niveau indispensable de structure de représentation, où la tradition de Frege et Montague de compositionnalité n'est pas respectée[2]. Quant à DPL, bien que sa syntaxe soit celle de la logique des prédicats standard, une sémantique non-classique est nécessaire. De plus, à la fois DRT et DPL souffrent du problème appelé *destructive assignment*.

Plus récemment, de Groote a proposé un autre cadre dynamique, que nous appelons Type Theoretic Dynamic Logic (TTDL) de Groote (2006). Il établit le fondement théorique de cette thèse. TTDL est principalement motivé par deux aspects. Tout d'abord, il vise à étudier la sémantique des phrases et du discours dans un cadre uniforme et compositionnel. Deuxièmement, il tente de résoudre le problème de *destructive assignment* de la DRT et de DPL. Il se fonde sur des outils logiques et mathématiques bien établis, tels que le λ-calcul et la théorie des types. Pour TTDL, la notion de con-

[2]La notion de compositionnalité a été intégrée avec succès dans certaines versions ultérieures de la DRT.

texte gauche et droit est proposée afin de modéliser la dynamique : le contexte gauche est constitué d'une liste de variables accessibles pour consultation ultérieure, le contexte droit est la continuation du point d'évaluation. Ici, la période de vie d'un référent de discours est celle de sa présence dans un contexte gauche. Quant à l'interprétation d'une phrase, elle est dépendante à la fois de l'interprétation de son contexte gauche et de son contexte droit. Outre les deux types atomiques classiques en théorie des types Church (1940), à savoir ι (le type des individus) et o (le type des propositions), un troisième type γ est introduit pour désigner le type des contextes gauches. Puis le contexte droit, qui est vu comme une continuation de la phrase, est une fonction de contexte gauche vers une valeur de vérité : son type est $\gamma \to o$. Les informations sur les types sont reprises dans la figure suivante :



Cette proposition est différente des autres théories dynamiques classiques comme la DRT et DPL, où seul le discours précédent (le contexte de gauche) est pris en compte. Comme d'autres théories dynamiques standards, TTDL rend également les prédications correctes sur des exemples tels que (6), (7), (8) et (9).

0.4 Problèmes à résoudre

Les observations de Karttunen sur l'accessibilité des antécédents ont été bien modélisé dans les théories dynamiques que nous avons mentionnés ci-dessus, par exemple, DRT, DPL et TTDL. Ces théories sont conçues de manière à ce qu'un référent de discours introduit dans la portée d'un opérateur logique reste accessible seulement dans le cadre de cet opérateur. Toutefois, dans l'intervalle, Karttunen a également souligné quelques exceptions à sa propre conclusion sur lesquelles nous revenons.

0.4.1 Double Négation

Le premier problème de cette thèse portera sur les interactions entre anaphore et double négation. Par exemple:

(10) a. John did not fail to find an answer$_i$. The answer$_i$ was even right.
b. John did not remember not to bring an umbrella$_i$, although we had no room for it$_i$. Karttunen (1969)

Dans (10-a), l'expression définie *the answer* est anaphoriquement liée à l'indéfini précédent *an answer*. Et il en est de même pour *it* et *an umbrealla* dans (10-b). Ainsi, même si un référent de discours est bloqué par une négation, il semble qu'une seconde négation permette de rendre le référent à nouveau accessible. Dans toutes les théories dynamiques précédentes, la négation est considérée comme un opérateur qui bloque les référents de discours de manière définitive. Par conséquent, une double négation bloquera l'accès aux référents de discours deux fois, plutôt que d'annuler le blocage. Les théories dynamiques standards ne pourront jamais rendre compte de ces liens anaphoriques tels que dans (10). Nous proposerons donc une solution où la double négation permettra de rendre accessible les référents de discours sous la portée de la première négation. Un problème similaire est la disjonction, qui peut être illustrée par le fameux exemple de la salle de bain :

(11) Either there's no bathroom$_i$ in the house, or it$_i$'s in a funny place. Roberts (1989)

Comme la partie droite de la disjonction porte une négation, les référents de discours dans sa portée ne sont plus accessibles. L'exemple nous montre bien que la seconde partie de la disjonction est quand même capable d'y faire référence. Comment définir la relation entre les exemples (11) et (10), pourquoi ne pas les traiter comme de manière analogue ? Supposons que p et q sont des propositions, selon la loi de De Morgan, nous pouvons réécrire $\neg p \vee q$ en $\neg(\neg(\neg p) \wedge \neg q)$ qui peut se réduire selon les doubles négations en $\neg(\neg(\neg p) \wedge \neg q)$ peut être encore réduit à $\neg(p \wedge \neg q)$. Ainsi, sous l'hypothèse de simplification de la négation, l'exemple (11) peut être paraphrasé en " Ce n'est pas le cas qu'il y a une salle de bains, et elle est dans un drôle d'endroit ". Cette fois, l'anaphore de (11) peut être résolue.

0.4.2 Subordination modale

Une autre exception qui nous intéresse est la subordination modale. Selon Karttunen, les verbes modaux sont traités de la même manière que les autres opérateurs logiques : ils bloquent l'accessibilité des référents dans leur portée, voir l'exemple (9). Cependant, ce n'est pas toujours le cas :

(12) If John bought a book$_i$, he'll be home reading it$_i$ by now. It$_i$'ll be a murder mystery. Roberts (1989)

(13) A thief$_i$ might break into the house. He$_i$ would take the silver. Roberts (1989)

À nouveau, les théories classiques de la sémantique du discours ne prédisent pas correctement les anaphores de (12) et (13), notamment parce que les potentiels NP antécédents apparaissent sous la portée d'opérateurs modaux. Bien que des modalités soient impliquées dans ces exemples, les expressions anaphoriques peuvent parfaitement être liées à leurs antécédents. Les travaux en cours tentent de donner une interprétation des discours contenant des modaux comme dans (12) et (13).

0.5 Propositions

0.5.1 DN-TTDL

Un premier cadre formel s'est proposé de gérer la question de la double négation et a été avancé dans Krahmer and Muskens (1995). Il étend la DRT traditionnelle en associant à chaque DRS deux extensions, une positive et une négative. Cette modification complique la sémantique du système formel. Pour éviter cela, nous proposons de traiter le même phénomène par une extension de TTDL, que l'on appelle *double negation*-TTDL (DN-TTDL). Cette propriété distingue DN-TTDL de la DRT, DPL, et TTDL, où la représentation sémantique ne contient que le cas positif. De plus, dans ces cadres, la négation est une opération irréversible.

L'idée de base de DN-TTDL est que lorsque nous traduisons une expression du langage naturel en formule logique, nous encapsulons deux propositions dans une paire ordonnée. Parmi les deux représentations, l'une correspond à la représentation affirmative de l'expression, l'autre correspond à la représentation négative. L'objectif est de conserver les variables introduites sous la portée de la négation et de pouvoir les ré-introduire en cas de seconde négation en modifiant l'ordre des éléments de la paire. Ainsi le système est capable "d'annuler" les effets d'une négation. Cette proposition permet de gérer les cas de négations multiples.

En outre, nous étudions le lien formel entre TTDL et DN-TTDL. Nous montrons que tous les exemples reconnus correctement par TTDL, le sont par DN-TTDL.

0.5.2 M-TTDL

Pour résoudre le seconde problème, à savoir la résolution d'anaphore dans la portée d'un auxiliaire modal, nous introduisons une autre adaptation de TTDL, appelée modal-TTDL (M-TTDL). Selon la théorie de Kratzer sur la modalité du langage naturel Kratzer (1977, 1981), un arrière-plan conversationnel est utilisé comme fonction des mondes possibles vers un ensemble de propositions nécessairement vraies (dans un monde donné). Ils modélisent l'information connue par tous, ou d'ensemble d'hypothèses vraies, pour la suite des énoncés modaux subordonnés. Notre stratégie est d'enrichir le contexte de TTDL avec la notion de points communs, en utilisant le principe de la base modale.

Comme DN-TTDL, le détail formel de M-TTDL est similaire à celui de TTDL : les signatures de différents cadres sont modifiées, tandis que la manière dont propositions, contextes gauches, contextes droits, etc., sont interprétés est presque le même. Avec des entrées lexicales spécifiques, M-TTDL se révèle être capable de gérer un certain nombre de cas complexes de subordinations modales. De plus, le lien formel entre M-TTDL et TTDL est également étudié. Un cadre intermédiaire : propositional-TTDL (P-TTDL), qui est similaire à GL, Lebedeva (2012), est mis en place pour relier M-TTDL avec TTDL. Avec cette connexion, les exemples qui sont reconnus par TTDL le sont par M-TTDL.

A la fin de la thèse, nous proposons un cadre intégré : double négation et modalité - TTDL (DNM-TTDL), qui est capable de traiter en même temps les deux exceptions. Le système ainsi obtenu est plus puissant et il montre la grande flexibilité de TTDL pour la gestion de phénomènes de discours complexes. Le schéma de la figure 1 reprend les relations entre les systèmes présentés dans cette thèse.

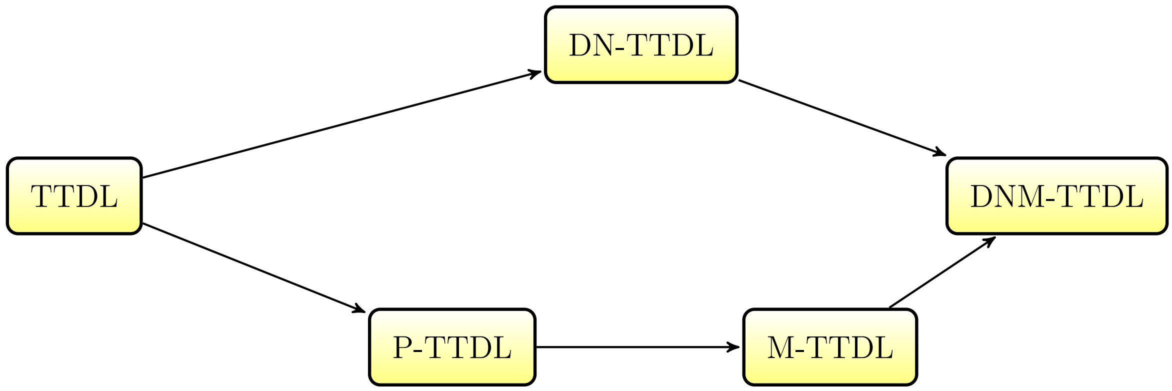


Fig. 1 Relations entre les différentes extensions de TTDL

0.6 Conclusion

L'anaphore est un mécanisme essentiel de la langue naturelle. Il désigne le phénomène linguistique par lequel l'interprétation d'une expression, appelée anaphore, dépend de celle d'un autre élément, appelée antécédent. Pour comprendre le sens d'un énoncé contenant une expression anaphorique, il faut résoudre l'anaphore, à savoir relier correctement l'anaphore avec son antécédent.

D'une manière générale, cette thèse a étudié la sémantique d'un type spécifique d'anaphore : les anaphores pronominales inter-phrastiques, où les deux éléments, antécédent et anaphore, sont des NP singuliers, apparaissant dans des phrases différentes. Et plus précisément encore, nous nous sommes intéressés à un phénomène particulier de l'anaphore, soit l'accessibilité de l'antécédent. Suivant la tradition de la sémantique dynamique, cette thèse a proposé d'interpréter les expressions linguistiques dans le cadre de la sémantique dynamique. L'interprétation d'une phrase génère un contexte qui est mis à jour par la phrase, et par lequel les expressions anaphoriques peuvent être résolues, tandis que les anaphores qui ne doivent pas trouver leur antécédent ne le trouvent pas.

En résumé, la présente thèse n'établit pas une nouvelle théorie dynamique de la sémantique du discours, mais au contraire, cherche à adapter un cadre formel, TTDL, en proposant des extensions permettant un traitement approprié pour les cas mal pris en compte. Pour cela, nous avons travaillé à rendre compte de deux phénomènes particuliers : l'accessibilité sous la portée de la double négation et de la subordination modale. Pour ces deux cas, nous proposons une extension du formelle de TTDL. En particulier, DN-TTDL a été introduite pour traiter de la double négation, et M-TTDL a été proposée pour la subordination modale. En outre, les deux extensions ci-dessus ont été intégrées avec succès pour former un système unifié performant, appelé DNM-TTDL.

Chapter 1

Introduction

Contents

Language is the most massive and inclusive art we know, a mountainous and anonymous work of unconscious generations. Sapir (1921)

1.1 Notations

At the very beginning, we would like to clarify the notations that are employed in this thesis. We will first see notations in linguistic examples, then talk about notations in the main body.

- In linguistic examples:
  - The anaphor and its antecedent will be marked with the same subscript (index);
  - The symbol “∗” is the marker of infelicity, it is used to indicate either a syntactic, or a semantic, or a pragmatic infelicity: when “∗” appears in front of a discourse, it means that the whole discourse is infelicitous; otherwise, when “∗” is in front of a particular sentence in a discourse, it means that only the marked sentence is infelicitous;
  - The symbol “?” is used to indicate that the acceptability of an example is controversial;
  - Underline mark is used to highlight a particular segment of an example;
  - Most linguistic examples in this thesis come from the literature, the original references will be explicitly indicated at the end of the examples. Sometimes, a single example entry may contain several sub-examples. In this case, the

particular reference will be appended to every corresponding sub-example if they are from different works; otherwise, if they are cited from the same work, a single reference is directly appended at the end of the last sub-example.

- In the main body of the thesis:
  - The **bold font** is used to highlight a particular part of the text, for instance, a terminology to be defined, or a keyword deserving certain attention;
  - The *italic font* is used to mark quotations from references and examples.

1.2 Natural Language Formal Semantics

The term natural language, which is sometimes called human language, or ordinary language, occurs constantly in research fields such as philosophy, linguistics and logics. It is a generic concept denoting all languages spoken or written by human beings. The scientific study of language is called linguistics. Contemporarily, it includes six major sub-domains, each of which has its own emphasis Fromkin (2000). Phonetics/Phonology studies the sounds and the abstract sound system; morphology studies the structure of words; syntax studies the structure of phrases and sentences; semantics studies the meaning; pragmatics studies the general communicative environment. This thesis is mainly concerned with formal semantics of natural language, namely, to analyze the meaning of linguistic expressions with formal systems, in particular, logics.

Around the middle of the last century, Alfred Tarski investigated the semantics of formal languages by defining the notion of truth Tarski (1944, 1956)[1]. However, he was not very optimistic towards formalizing the semantics for natural language. At the conclusion of section 1 in Tarski (1956), the author remarked that:

> *... the very possibility of a consistent use of the expression 'true sentence' which is in harmony with the laws of logic and the spirit of everyday language seems to be very questionable, and consequently the same doubt attaches to the possibility of constructing a correct definition of this expression*
> *... I now abandon the attempt to solve our problem for the language of everyday life and restrict myself henceforth entirely to formalized languages.* Tarski (1956)

Later on in the 1970s, using the mathematic tools of that time, e.g., higher-order predicate logic, λ-calculus, type theory, intensional logic, etc., Richard Montague established a model-theoretic semantics for natural language Montague (1970a,b, 1973). This series of work is known as the Montague Grammar (MG). It provides the possibility to interpret natural language, in particular English, as a formal language.

Specifically, in Montague (1973), the author proposed to interpret natural language with a two-step method. Firstly, linguistic expressions are translated into a formal language, e.g., higher-order predicate logic in MG. Each sentence constituent is represented by a λ-term, which specifies its semantic contribution. The combinations of various lexical entries, which conform to the grammatical structure of the sentence, result in other logical expressions through β-reduction. Then the link between grammatical structure and

[1] Tarski (1956) has been translated from the German version, which was published in 1936. The original work was published in Polish in 1933.

logical structure is captured, and the translation process is considered to be meaning-preserving. Then secondly, the logical formulas obtained from the previous step will receive a model-theoretic interpretation with the semantics of the formal system. This interpretation ultimately provides the meaning of the corresponding linguistic expressions in terms of truth conditions.

1.3 Cohesion and Anaphora

Superficially, a sentence consists of a set of words. But it should be more than that: a random set of expressions which conform to the rules of grammar do not always yield a meaningful sentence. Take the famous example from Chomsky:

(1) Colorless green ideas sleep furiously. Chomsky (1957)

Grammatically, sentence (1) is completely correct. But from the semantic perspective, it does not really make sense: the combination of the constituents of (1) (i.e., *colorless*, *green*, *ideas*, *sleep*, and *furiously*) does not mean anything, the vocabulary is not connected. Analogously, a discourse is more than a random set of sentences, for instance:

(2) a. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. A law firm in which Mr Sarkozy owns shares was also searched, reports say. (episode from BBC News Europe on 3 July 2012)

b. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. Tens of thousands have turned out in the streets of the Spanish capital Madrid to welcome the national football team after their victory at Euro 2012. (mixed episode from BBC News Europe on 3 July 2012)

Each of the two discourses in (2) is made up of two sentences. The component sentences are perfectly understandable by themselves. However, as one might have noticed, (2-a) is an integrate text while (2-b) is just an alignment of two arbitrary sentences (it is indeed a mixture of episodes from two unrelated articles). The two sentences of (2-a) are centered around the same topic, clues can be drawn from the repetition of the proper name *Sarkozy*, the lexical relations between expressions such as *police*, *law firm*, *search* and so on. Whilst similar connectedness is missing in (2-b). This prevents it to be an appropriate text, namely it fails to form a "unified whole" in terms of Halliday and Hasan (1976).

In Halliday and Hasan (1976), the authors characterize the connectedness of a coherent text in terms of a group of linguistic mechanisms called cohesive devices, including reference, substitution, ellipsis, conjunction, and lexical cohesion. These devices link utterances in a meaningful way, thus to create the connectedness and make the text. For the rest of this thesis, we shall use the two terms: discourse and text, interchangeably, to denote a set of meaningful and connected sentences. Another pair of interchangeable terms: sentence and utterance, will denote the basic constituent of a discourse.

The classification of cohesive devices provides useful heuristics to subsequent researches, especially for the analysis of text beyond the sentence level. These devices are not mutually exclusive, rather, they overlap to some certain extend. We will not dive into the details, interested readers may refer back to the original book Halliday and

Hasan (1976). In this thesis, we are interested in one particular sub-field of reference: anaphora.

Since the middle of the 20th century, the study of anaphora has attracted interest of researchers from various research branches, in particular those related with linguistics. Generally speaking, anaphora is understood as the relationship between two linguistic expressions, whereby the interpretation of one, called the anaphor, is somewhat determined by the interpretation of the other, called the antecedent. And we say that there is an anaphoric link between the antecedent and the anaphor. That is to say, a sentence containing anaphor can not be comprehended on its own, rather, a context is required for its interpretation. For instance:

(3) a. John walks in. He smiles.
 b. Bill walks in. He smiles.

The above two discourses share the same second part, which contains the pronoun *he*. The interpretation of that sentence is obviously context-dependent: in (3-a), it is John who smiles, while in (3-b), the person who smiles is Bill. And in this example, proper name *John* and *Bill* are antecedents, *he* is anaphor.

In terms of syntactic category, anaphora can be divided into various categories, e.g., noun phrase (NP) anaphora, verb phrase (VP) anaphora, adjective anaphora, etc. For a comprehensive survey on the taxonomy of anaphora, please refer to Hirst (1981). In this thesis, we will focus on one specific sort of anaphora: pronominal anaphora. Particularly, both the anaphor and the antecedent are singular NPs, and the anaphor is a pronoun, such as the ones in example (3).

The research on anaphora spans various branches of linguistics. From the syntactic perspective, one of the most influential frameworks is the Government and Binding Theory (GB Theory) proposed by Chomsky (1981, 1986b). For Chomskian syntacticians, anaphora is a grammar phenomenon. So it can be accounted for in purely syntactic terms, such as local domain, command, etc. Three principles were introduced to justify the distribution of anaphor. For instance, a pronoun (e.g., *he*, *him*) should not have an antecedent in its local domain, while a reflexive should. Let's have a look at the following example, where we put the symbol "$*$" in front of a particular index to indicate the infelicity of that anaphoric relation:

(4) a. $\underline{\text{Russell}}_i$ admired $\underline{\text{him}}_{*i/j}$.
 b. $\underline{\text{Russell}}_i$ admired $\underline{\text{himself}}_{i/*j}$. Huang (2006)

According to the GB Theory, the two NPs in (4-a): *Russell* and *him*, can not be anaphorically linked. Otherwise, the sentence would be infelicitous. As for (4-b), the proper name *Russell* must act as the antecedent of *himself*. These predictions correctly correspond to our intuition on examples such as (4).

Different from the GB Theory, research oriented from the semantic perspective aimed to specify the interpretation of anaphora, namely the semantic relationship between the anaphor and its antecedent. It has been commonly acknowledged that anaphora can be semantically classified at least into the following two types: referential anaphora and bound anaphora Bach and Partee (1980); Evans (1980); Partee (1978). This distinction can be best illustrated with the following examples, where the two anaphoras only differ in the antecedent:

(5) a. $\underline{\text{John}}_i$ loves $\underline{\text{his}}_i$ mother.

b. Every man_i loves his_i mother. Evans (1980)

The anaphora in (5-a) is referential because the anaphoric expression *his* refers to the particular individual John. The one in (5-b) is called bound anaphora because the anaphor *his* can be construed in analogy with the bound variable in classical predicate logic: *his* is bound by the universally quantified antecedent *every man*. This can be reflected by the semantic representations of the two sentences, which are provided as follows, respectively (where **john** is an individual constant, **love** is a two-place predicate, **mother_of** is a function which takes an individual and returns another individual):

$$\textbf{love john } (\textbf{mother\_of john})$$

$$\forall x.(\textbf{man } x \to \textbf{love } x \ (\textbf{mother\_of } x))$$

1.4 Accessibility and Dynamic Semantics

It is not the case that any pair of NPs can form an antecedent-anaphor relation. Hence besides the interpretation of anaphora, there is an additional aspect of the phenomenon that should be taken into account. One central concern of a theory of anaphora is to specify when a NP is possible to serve as antecedent for a particular anaphoric expression. This problem is characterized as the accessibility of the antecedent.

The GB Theory provides various constraints on the syntactic level, which were established on the structural relation between the antecedent and the anaphor. However, despite the fruitful insights from the GB Theory, it does not reflect all the characteristics of anaphora. In particular, the study of anaphora in the GB Theory has been restricted at the sentential level, typically the ones in example (4) and (5). They are called intra-sentential anaphora in the literatures. However, anaphora is a discourse phenomena in nature: according to Halliday and Hasan (1976), it is a cohesive device which glues utterances to form a text. In fact, most anaphoras occur beyond a sentence, e.g., the ones in example (3), and the following well-known example:

(6) A man_i walks in the park. He_i whistles.

Appealing to the semantic viewpoint, plenty of anaphoras fail to fit the semantic classification. For instance, on the one hand, the anaphora in example (6) can not be referential because the two NPs: *a man* and *he*, do not refer to any particular individual. On the other hand, the anaphora can not be bound, either. Following Russell (1905), we treat the indefinite antecedent *a man* as an existential quantification. The scope of the quantifier is not allowed to extend beyond the sentence boundary. Another notoriously problematic anaphora is the so-called donkey sentence:

(7) Every farmer who owns a donkey_i beats it_i.

Even until nowadays, the exact semantics of example (7) is still a matter of debate. Regardless of its interpretation, we may see that although the indefinite *a donkey* is located within the scope of a universal quantification (i.e., *every farmer*), it is fairly acceptable to anaphorically link its referent with the subsequent pronoun *it*.

As a result, in order to overcome the empirical problems arising from the standard syntactic and semantic analysis, researchers have been motivated to study anaphora from the discourse perspective, and this is what this thesis is concerned with. Some correlated questions are: how to capture the accessibility of antecedent in inter-sentential anaphora? What is the difference between intra-sentential and inter-sentential anaphora? Can they be accounted for with a unified solution?

At the end of the 1960s, Karttunen proposed an intuitive and general way to describe anaphora, in particular those that span across two or more sentences Karttunen (1969). By introducing the notion of discourse referent, he described the two semantic classes of anaphora (i.e., referential and bound) in a uniform way. Essentially, a discourse referent is a variable-like entity. When processing a discourse, an indefinite NP establishes a new discourse referent, while an anaphoric expression does not. Rather, an anaphor retrieves a corresponding referent from its antecedent. It is through the common referent that the two NPs are anaphorically linked together. Take example (6) for instance, the NP *a man* introduces a discourse referent, the anaphor *he* is thus identified with the same referent. We can further continue (6) with sentences such as *he smokes*, where the pronoun will also be directed to the same referent from *a man*. This gives an explanation for the felicitous anaphoras that are not properly treated by traditional syntactic and semantic analysis.

Karttunen noticed that a discourse referent has a life-span, namely a referent may not always be possible to antecede subsequent anaphoric expressions. For instance:

(8) Bill doesn't have a car$_i$. *It$_i$ is black. Karttunen (1969)

(9) You must write a letter$_i$ to your parents. *They are expecting the letter$_i$. Karttunen (1969)

Based on the above examples, Karttunen concludes that the life-span of a discourse referent is generally determined by the the scope of the logical operator within which it is introduced. More specifically, if an indefinite NP occurs in the scope of some operator, the life-span of its referent is identical to the scope of that operator. In example (8), the indefinite is within the scope of a negation, so the discourse referent from *a car* is not accessible to the subsequent pronoun *it*. Analogously, in example (9), the modal auxiliary *must* takes a wider scope over the indefinite *a letter*, hence the anaphoric expression *the letter* fails to be resolved with *a letter*.

Karttunen's observations provide a valuable account on the accessibility of antecedent from the semantic perspective. The current thesis will follow this line. Although Karttunen did not establish a formal semantic theory, his work motivated a new stream of semantic theories since the 1980s, called dynamic semantics. Representative work includes Discourse Representation Theory (DRT) Kamp (1981), File Change Semantics (FCS) Heim (1982), and Dynamic Predicate Logic (DPL) Groenendijk and Stokhof (1991). These frameworks have different appearances, but they capture the idea of discourse referent and try to describe anaphora in terms of it. The accessibility of antecedent is thus characterized by their interactions with various logical operators.

The shift of attention from sentence semantics to discourse semantics brings about a novel point of view towards meaning. According to dynamic semantics, the meaning of an expression is identified with its potential to change the context, rather than its truth conditions (as in classical logical semantics such as MG). The term context is used to denote assignments for discourse referents (variables). With the dynamic viewpoint, a context is required to interpret a sentence, in return, the result of interpretation is an update context.

However, the classical dynamic theories are not completely satisfactory. For instance, DRT relies on an indispensable level of representational structure, hence the Fregean and Montagovian tradition of compositionality is not restored[2]. As for DPL, although its syntax is the one of standard predicate logic, a non-classical semantics is employed. Further more, both DRT and DPL suffer from the so-called destructive assignment problem.

More recently, De Groote proposed another dynamic framework, which we call Type Theoretic Dynamic Logic (TTDL) de Groote (2006). This framework lays the theoretical foundation of the present thesis. TTDL is mainly motivated from two aspects. Firstly, it aims to study the semantics of sentence, discourse under a uniform and compositional framework. Secondly, it tries to solve the destructive assignment problem, which occurs in DRT and DPL. It only makes use of basic and well-established mathematical and logical tools, such as λ-calculus and theory of types. In TTDL, the notion of left and right context are proposed in order to achieve dynamics: the left context consists of a list of accessible variables for subsequent reference, the right context is its continuation. The lift-span of a discourse referent in TTDL is boiled down to its presence in the left context. As for the interpretation of sentence, it is conducted with respect to both the left and right contexts, and the semantics is abstracted over the two contexts. This is different from other classical dynamic theories such as DRT and DPL, where only the preceding discourse (the left context) is taken into account. Like other standard dynamic theories, TTDL also makes the correct predications on examples such as (6), (7), (8) and (9).

1.5 Problems to Tackle

The observations of Karttunen on the accessibiliy of antecedent have been well-modeled in the dynamic theories we have mentioned above, e.g., DRT, DPL and TTDL. Namely, these theories are designed in a way such that a discourse referent introduced in the scope of a logical operator is only accessible within the scope of that operator. However in the meantime, Karttunen has also pointed out some exceptions to his own conclusion.

The first problem this thesis will deal with concerns the interplay between anaphora and double negation. For instance:

(10) a. John did not fail to find <u>an answer</u>$_i$. <u>The answer</u>$_i$ was even right.
b. John did not remember not to bring <u>an umbrella</u>$_i$, although we had no room for <u>it</u>$_i$. Karttunen (1969)

In (10-a), the definite expression *the answer* is anaphorically linked to the preceding indefinite *an answer*, it is the same case for *it* and *an umbreалla* in (10-b). Hence although a discourse referent is blocked by a single negation, it seems that a second negation re-allows the referent to be accessible. In all the mentioned dynamic theories, negation is treated as an operator that blocks discourse referents within its scope once and forever. Accordingly, a double negation will block the referents twice, rather than canceling each other out. Standard dynamic theories will run into trouble when handling the anaphoric links in (10). A remedy where double negation can be removed is expected. A problem of the same sort involves disjunction, which can be illustrated with the bathroom example:

[2]The notion of compositionality has been successfully integrated in some later versions of DRT.

(11) Either there's no bathroom$_i$ in the house, or it$_i$'s in a funny place. Roberts (1989) (motivated by Barbara Partee)

The first disjunct is itself a negation, so no referent may survive outside its scope. While the anaphoric pronoun in the second disjunct can somehow refer back to the antecedent referent. Then what is the relation between example (11) and (10), why do we say they are similar problems? Assume p and q are propositions, according to the De Morgan's law, we may rewrite $\neg p \vee q$ as $\neg(\neg(\neg p) \wedge \neg q)$. With the law of double negations, $\neg(\neg(\neg p) \wedge \neg q)$ may be further reduced to $\neg(p \wedge \neg q)$. Thus, if double negation could be eliminated, example (11) can be paraphrased as "it is not the case that there is a bathroom, and it's not in a funny place". This time, the felicitous anaphora in (11) automatically gains an account.

Another exception we are interested in is the so-called modal subordination. According to Karttunen, modal verbs are treated in a similar way as other logical operators: they block referents within their scopes, see example (9). However, this is not always the case:

(12) If John bought a book$_i$, he'll be home reading it$_i$ by now. It$_i$'ll be a murder mystery. Roberts (1989)

(13) A thief$_i$ might break into the house. He$_i$ would take the silver. Roberts (1989)

Again, standard theories of discourse semantics will predict that the anaphoras in (12) and (13) are infelicitous, because the potential antecedent NPs occur in the scope of some modality. However, it seems that if modality is also involved in subsequent sentences, anaphoric expressions may perfectly be linked with their antecedents. The current work tries to give an interpretation for modalized discourses such as (12) and (13), where the extraordinary anaphoras are accounted for.

1.6 Thesis Outline

To sum up, this thesis has its roots in MG and dynamic semantics. It focuses on the pronominal anaphoras which go beyond the sentential level. The purpose is to extend the coverage of a specific dynamic framework: TTDL, on those anaphoras (i.e., double negation and modal subordination) which are naturally problematic for standard dynamic theories such as DRT and DPL. The organization for the rest of the thesis is generally as follows.

Chapter 2 aims to present the necessary linguistic background. We will investigate anaphora in detail from the linguistic perspective. Correlated terminologies will be clarified. In addition, various sorts of taxonomy, as well as existing analysis of anaphora in various research fields, will be discussed.

Chapter 3 is intended to provide the mathematical preliminaries that subsequent chapters will make reference to. Four classical formal systems, which are widely used to formalize natural language semantics, will be introduced, i.e., Propositional Logic (PL), First-Order Logic (FOL), Modal Propositional Logic (MPL), and the Simply Typed λ-Calculus. We shall end chapter 3 with a toy illustration of Montague Grammar (MG). The two above chapters supplied many of the essential notions on which the whole thesis could be understood.

Chapter 4 deals with dynamic semantics, which emerges for the semantics of discourses, rather than isolated sentences. Different from traditional (truth-conditional) semantics such as MG, the meaning of an expression is identified with its potential to change the context in dynamic semantics. Standardly, the notion of context denotes a set of assignment functions, with respect to which anaphors receive their interpretation. For instance, different contexts (sets of assignments) may have different potential in resolving anaphora. And this potential can be modified when some subsequent utterances are added. We will first review two standard dynamic theories: Discourse Representation Theory (DRT) Kamp (1981) and Dynamic Predicate Logic (DPL) Groenendijk and Stokhof (1991). For each framework, the syntax and the semantics will be presented in a succinct yet precise way. After that, we will look into a more recently proposed dynamic framework: Type Theoretic Dynamic Logic (TTDL) de Groote (2006), which forms the theoretical backbone of most of the work in the current research. This framework has been designed to describe the same set of semantic phenomena as other dynamic theories, e.g., inter-sentential anaphora, donkey sentence. However different from other dynamic theories, it was established on classical mathematical and logical tools. Hence it is completely compositional and does not suffer from the destructive assignment problem.

In chapter 5, we will dive deeper into the notion of discourse referent, which was first introduced by Karttunen to account for NP anaphora in discourse Karttunen (1969). Following Karttunen, the dynamic theories presented in chapter 4, namely DRT, DPL, and TTDL, share a common analysis on anaphor: anaphoric pronouns are uniformly treated as variable-like entities. Hence from the dynamic point of view, the semantic distinction between kinds of anaphoras (i.e., referential and bound) is of little importance: all anaphors correspond to bound variables in context. In addition, we will show that the three dynamic theories correctly model most accessibility constraints generalized by Karttunen, such as (single) negation, implication, disjunction, etc. However, all of them run into trouble when encountering anaphora under certain context environments, in particular, double negation and modality. These two exceptions are the problems that the thesis aims to tackle.

Chapter 6 focuses on the first problem, namely anaphora under double negation. It starts with an existing work on the same issue: Double Negation DRT (DN-DRT) Krahmer and Muskens (1995). This framework is an extension of the standard DRT. It treats negation as a DRS rather than a DRS condition. In order to restore the law of double negation, the notion of positive extension and negative extension have been proposed. This allows negation to be interpreted as a flip-flop operation. Thus, DN-DRT successfully handles double negation by complicating the semantics of the framework. In the second part of this chapter, we will adapt TTDL to double negation, yielding a new system called Double Negation TTDL (DN-TTDL). Different from DN-DRT, DN-TTDL avoids a more complicated semantics by restricting the computations on the syntactic level. We propose to encapsulate both the positive and negative representation of an expression in a pair, and to retrieve the appropriate representation with respect to the polarity of the sentence. The formal link between TTDL and DN-TTDL will also be investigated at the end of the chapter.

Chapter 7 envisages the second exception, namely anaphora under modalized context. We will first present Kratzer's theory on natural language modality Kratzer (1977, 1981). The notion of conversation background, in particular, the modal base usage, will be extensively discussed. Then two existing proposals on modal subordination shall be briefly presented Asher and Pogodalla (2011a); Roberts (1989). After that, we will bring forward another adaption of TTDL, namely Modal TTDL (M-TTDL), in order to describe the semantics of anaphora across modality. The relation between M-TTDL and TTDL will be formally established. We finally propose an integrated framework: Double Negation Modal TTDL (DNM-TTDL), which is capable to address both exceptions, i.e., double negation and modal subordination, at the same time.

Finally, chapter 8 summarizes our findings and draws some general conclusions. Suggestions on future research will also be made.

Chapter 2

Linguistic Preliminaries

Contents

> *Anaphora is the device of making in discourse an abbreviated reference to some entity (or entities) in the expectation that the perceiver of the discourse will be able to disabbreviate the reference and thereby determine the identity of the entity.* Hirst (1981)

This chapter aims to provide the necessary linguistic background of this thesis. We focus extensively on the phenomenon of anaphora by providing its detailed definition, taxonomy, and the way in which it has been treated in the literatures. Then we will restrain ourselves to a certain group of data which will be further investigated in this thesis.

2.1 What is Anaphora?

From the etymological point of view, the word *anaphora* originates from its Greek ancestor *ἀνάφορον*, which has the meaning of "carrying back", "carrying up", "offering" Liddell et al. (1940), or "picking-up", "updating" Seuren (2009). Along the history, the term anaphora has been used in various fields, for instance, liturgy, rhetorics, and linguistics. In this thesis, we are interested is the linguistic sense, which shall be elaborated immediately below in this section.

Generally speaking, linguistic anaphora denotes a relation between two linguistic elements, which are semantically related. Before going to its formal details, let's first have a look at some prototypical examples of anaphora. For instance:

(14) $\underline{\text{Max}}_i$ claims $\underline{\text{he}}_i$ wasn't told about it. Huddleston et al. (2002)

(15) $\underline{\text{The idea was preposterous}}_i$, but no one dared say $\underline{\text{so}}_i$. Huddleston et al. (2002)

In example (14), the pronoun *he* can be considered as a shorthand for *Max*, it is used to avoid the repetition of an other occurrence of *Max*. Since the two expressions *he* and *Max* actually denote the same individual, by substituting *he* with *Max*, we can rephrase (14) as "Max claims Max wasn't told about it", which expresses the same meaning. Similarly, the word *so* in (15) functions in the same way as *he* in (14): *so* is "carried backwards" to the whole clause *the idea was preposterous*.

Conventionally, sentences like such, (14) and (15), are considered as anaphora-involved. But how is anaphora precisely defined in linguistics? We will examine the phenomenon in detail in the following subsection.

2.1.1 Classical Definitions

Even though the phenomenon of anaphora has been studied intensively, its definition varies for different researchers. Looking back into the literatures, we can find two main categories of definition.

- **Co-referential Point of View**

  The first group of definitions consider anaphora as a **co-referential** phenomenon. A representative one of this sort is as follows:

  > *The term anaphora is used most commonly in theoretical linguistics to denote any case where two nominal expressions are assigned the same referential value or range.* Reinhart (1999)

- **Context-dependent Point of View**

  The second group of definitions consider anaphora as a **context-dependent** phenomenon. By way of illustration, let's consider the following samples:

  > *Anaphora is the phenomenon whereby an expression, which is called a proform (e.g., 'he' [pronoun], 'so' [proadjective]), is interpreted in light of another expression in its immediate linguistic context, which is called the proform's antecedent.* King (2006)

  > *In the first place, it can be used for reference to a relation between two linguistic elements, in which the interpretation of one (called an anaphor) is in some way determined by the interpretation of the other (called an antecedent).* Huang (2006)

These two threads of definitions are not mutually exclusive, many commonly acknowledged anaphoras fall into both categories. In fact, each thread merely provides a partial characterization of anaphora. What we want to show below is that, it is really a difficult task to assign an accurate and precise definition on anaphora. In what follows we will take a closer look at the two groups of definitions one by another.

Above all, the first group of definitions are inadequate: co-reference is neither a necessary nor a sufficient condition of anaphora. To explicate that, we have to explain several correlated notions such as referring, referent, and co-reference.

Definition 2.1.1. An expression is **referring** if it is used to single out an object or a group of objects. The object, or the group of objects, is called the **referent** of the expression. The relation between a referring expression and its referent is called **reference**.

For instance, proper names are prototypical examples of referring expressions, e.g., *Max* in example (14). Sometimes, a whole clause can also refer, such as the highlighted fragment *the idea was preposterous* in example (15). However, quantified noun phrases (QNPs), such as *every man*, *no one*, *few men*, and *most men*, are generally not considered as referring expressions. They are not used to pick out any particular object or group of objects. Rather, they denote sets of set of objects. Besides, predicative noun phrases, namely the noun phrases that follow a copula (e.g., *be*, *become*, *get*), are not referring either:

(16) Wim Kok is <u>the prime minister of the Netherlands</u>. Krahmer and Piwek (2000)

As in example (16), the under-scored expression *the prime minister of the Netherlands* does not refer to any specific individual. Instead, it is used to ascribe some specific property (being the prime minister of the Netherlands). With the above knowledge, we can thus define the notion of co-reference as follows:

Definition 2.1.2. Let E_1 and E_2 be two linguistic expressions. We say E_1 **co-refers** with E_2, or equivalently, E_1 and E_1 are **co-referring**, iff

1. both E_1 and E_2 are referring;
2. the referents of E_1 and E_2 are identical.

The relationship between E_1 and E_2 is called **co-reference**.

It is true that many anaphoras are concerned with co-reference, such as the above example (14) and (15), where *he* refers to the individual denoted by *Max*, *so* refers to the proposition expressed by the clause *the idea was preposterous*. However, these two relations do not necessarily correlate. We will show this from two aspects. Take the following example for instance:

(17) The people who work for <u>him</u> love <u>Al</u>. Conroy et al. (2009)

In example (17), both highlighted expressions: *him* and *Al*, are referring. Assume that the speaker is pointing at somebody when he utters (17), and the person being pointed at happens to be Al (but the speaker might not know that). Then by chance, *him* and *Al* refer to the same individual. According to definition 2.1.2, the two NPs are co-referring. However, the relation between the two expressions is usually not considered as anaphora.

Similarly, one may imagine another situation, where there are several different documents, and the proper name *Al* happens to occur in all of them. If these documents are talking about the same person, then all the occurrences of *Al* refer to the same individual, namely they are co-referring. Again, this phenomenon, which is often referred to as cross-document co-reference in the literatures Poesio et al. (2011), should not be considered as anaphora. In fact, relations such as the one in (17) and the cross-document co-reference are subsumed under a more general category: **accidental co-reference** Carlson (2002). As a result, co-reference does not imply anaphora.

Then does the reverse hold? Namely does anaphora results in co-reference? The answer is negative as well. There are many anaphoras which do not involve co-reference at all, for instance:

(18) I was at a wedding$_i$ last week.

a. The bride$_i$ was pregnant.

b. The mock turtle soup$_i$ was a dream. Geurts (2009)

As we can see, (18-a) and (18-b) are possible continuations of (18). The expression *the bride* in (18-a), and *the mock turtle soup* in (18-b), are both anaphorically linked to the indefinite *a wedding* in the first sentence. However, the referent of neither expression is given in the prior context. This is different from example (14) and (15), where the referents of *he* and *so* have been explicitly introduced before their occurrences. Anaphoras such as the ones in example (18) are called **bridging anaphora** Haviland and Clark (1974), because the anaphoric links require some context inference. We will see more examples of this type in section 2.3.1. Moreover, it is often even the case that anaphora is not concerned with referring expressions at all, for instance:

(19) Every man$_i$ thinks that he$_i$ deserves a raise. Carlson (2002)

(20) No one$_i$ wanted to admit that he$_i$ might be wrong. Partee (2008)

As explained above, expressions such as *every man*, *no one* are QNPs, which are not referring expressions. Hence, it is obvious that *every man* and *he* in (19), *no one* and *he* in (20) can not co-refer, despite that they are in anaphoric relations. So it is reasonable enough to abandon the first group of definitions, at least not to adopt it completely: it disqualifies in characterizing anaphora in both ways.

Now let's turn to the second group of definitions. It seems that they provide a more appropriate characterization on anaphora, since anaphora indeed ought to be context-dependent. However, context-dependence and anaphora are not interchangeable concepts either, because not every context-dependent phenomenon is anaphoric. For instance, resolving different senses of a word, namely word sense disambiguation (WSD), is a task which requires context-dependence Van Deemter (1992). Nevertheless, this task should not be considered as anaphora. Take the following sentence for example:

(21) John needed some cash so he went to a bank. Krahmer and Piwek (2000)

In example (21), due to the occurrence of the noun *cash* in the earlier context, the word *bank* is more likely to be interpreted in the sense of a financial establishment, rather than a land alongside water. Hence, despite the fact that the interpretation of *bank* in (21) is context-dependent, in particular, it depends on *cash*, there is no anaphora involved in (21).

As we have shown, strictly speaking, neither of the above groups of definitions provides a satisfactory description of anaphora. However, the second group is comparatively less problematic. Since a formal definition on anaphora is very difficult, and it is not the focus of our work, we will thus adopt the definition from Barbara in this thesis:

Definition 2.1.3. Anaphora as a phenomenon refers to the relationship between a "referentially dependent" expression (the anaphoric expression, or anaphor) and a "referentially independent" expression that serves as its antecedent and from which the anaphoric expression gets its reference (or other semantic value). Partee (2008)

In the next subsection, we will discuss some more terminologies which are often involved in anaphora.

2.1.2 Terminologies on Anaphora

As presented in the previous subsection, anaphora, as far as it is concerned by us, is a general linguistic phenomenon where the interpretation of an expression depends on another in some certain ways. Now we shall give the definitions of two fundamental concepts related to anaphora, namely anaphor and antecedent.

Definition 2.1.4. In anaphora, the expression, whose interpretation depends on others, is called **anaphor**; the information supplier, namely the expression on which the anaphor depends for the interpretation is called **antecedent**.

Accordingly, let's look back at example (14) and (15), *he* and *so* are anaphors, *Max* and *the idea was preposterous* are their antecedents, respectively. From the literal point of view, antecedent is the expression whose occurrence precedes that of anaphor. However, it does not seem to be necessary, for instance:

(22) a. When $\underline{\text{he}}_i$ saw the damage, $\underline{\text{the headmaster}}_i$ called in the police.
b. The repeated attacks on $\underline{\text{him}}_i$ had made $\underline{\text{Max}}_i$ quite paranoid. Huddleston et al. (2002)

According to definition 2.1.4, pronouns *he* and *him* in (22) are anaphors, because their interpretations depend on the definite expressions *the headmaster* and *Max*, respectively. However, different from previous anaphoric examples, such as (14), (15), (18), (19), and (20), both antecedents (i.e., *the headmaster* and *Max*) occur after their anaphors in example (22). In fact, contrasting to anaphora, relations like such are more commonly referred to as **cataphora**[1].

In real practice, cataphora is used much less frequent compared to anaphora. The reasons are as follows. Firstly, the construction of cataphora is subject to many structural constraints Quirk et al. (1985), e.g., the anaphor should be located within a subordinate clause, as in example (22-a), the anaphor ought to occupy a subordinate position within a larger NP, as in example (22-b). Further more, cataphora is only employed for some special purposes, e.g., to add rhetorical effects, as in the following examples:

(23) a. $\underline{\text{He}}_i$'s the biggest slob I know. $\underline{\text{He}}_i$'s really stupid. $\underline{\text{He}}_i$'s so cruel. $\underline{\text{He}}_i$'s my boyfriend $\underline{\text{Nick}}_i$. Wikipedia (2014)
b. $\underline{\text{It}}_i$'s a complete mystery to me: $\underline{\text{Why did he turn down such a marvellous offer}}_i$? Huddleston et al. (2002)

[1]Sometimes, anaphora and cataphora are respectively called **retrospective** and **anticipatory** anaphora, or anaphora and **backward** anaphora Huddleston et al. (2002).

Same as in anaphora, the dependent expression and the information supplier in cataphora are still called anaphor and antecedent respectively, despite a reversed order with respect to their appearances in the context. Because of the close semantic resemblance of the anaphora and cataphora, and also because one dominantly outnumbers the other in practice, there is a certain trend in the field of semantics to generalize both phenomena uniformly as anaphora. Since we will not consider cataphora in this thesis, the generalization does not seem to be crucial for us. In what follows, we stick to the etymological meaning of anaphora, which refers to cases where antecedents appear earlier.

In fact, both anaphora and cataphora are subsumed under a more general concept: **endophora**, where the anaphor and its antecedent are both within the linguistic material. In contrast to that, **exophora** is a phenomenon whereby the interpretation of an expression depends on the extra-linguistic context (the visual context in terms of Poesio et al. (2011)), e.g., gesture, direction of gaze, prosody, etc. For instance, **deixis** is a prototypical example of exophora:

(24) He went to Spain <u>last week</u>. Huddleston et al. (2002)

(25) a. <u>He</u>'s up early.
b. I'm glad <u>he</u>'s left. Evans (1980)

As one may see, there is no explicit antecedent presented in (24) or (25). In example (24), the interpretation of the expression *last week* depends on the time when the utterance takes place, which we can not draw from linguistic material. It is the same case for example (25), where the referent of the highlighted pronoun *he* must be derived from some information which are not recorded in the text, such as the direction that the speaker points to or stares at. Practically, many of the linguistic forms used to realize anaphora, particularly pronouns, may also be used for deixis.

Besides the above deictic examples, there is another particular phenomenon which is counted as exophora. This phenomenon, which is called **homophora**, refers to the relation where the antecedent is derived from cultural knowledge or commonsense knowledge, for instance:

(26) a. <u>The President of the U.S.</u> visited France last week.
b. <u>The moon</u> orbits around <u>the earth</u>.

As we can see, the highlighted expression in example (26-a), namely *the President of the U.S.*, is interpreted with respect to the reader's knowledge on the politics in the U.S., it does not depend on the linguistic information. Analogously, the two expressions in (26-b), i.e., *the earth* and *the moon*, unambiguously refer to the planet we are living on, and its satellite planet, respectively, as long as the reader has some knowledge about the Solar System.

To summarize, the following figure **??** generally describes the hierarchy of the set of phenomena akin to anaphora as introduced above.

Among all the phenomena in diagram **??**, anaphora will be the focus of this thesis. As a result, we shall leave out cataphora and exophora henceforth, even though they are important in both theoretical Huddleston et al. (2002) and computational linguistics Kelleher et al. (2005). For the rest of this chapter, we will conduct a more detailed investigation on anaphora.

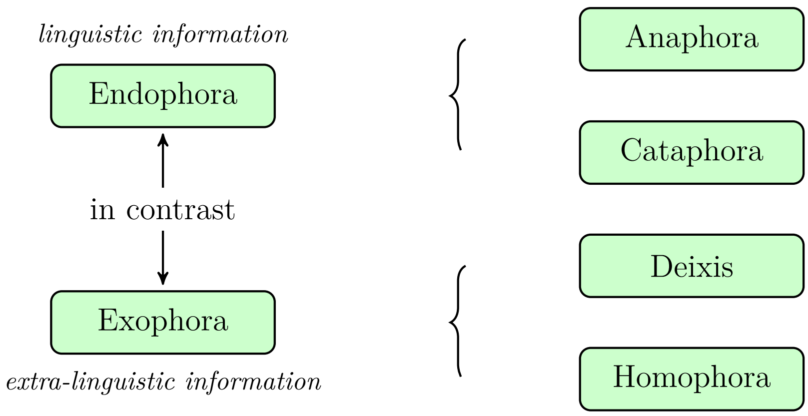


Fig. 2.1 Various Types of References

2.2 Taxonomy of Anaphora

After presenting some preliminaries of anaphora, we will have a look at its taxonomy. The goal of this section is to present various ways that anaphoras are classified. Each classification captures one property of anaphora from a particular perspective.

2.2.1 Syntactic Category of Anaphor

One of the most intuitive classifications of anaphora is based on the syntactic category of the involved expressions. An anaphor can be of almost any grammatical type, ranging from noun to noun phrase (NP), from verb phrase (VP) to adjective, as well as various sorts of ellipses. In this subsection, we will present the classification of anaphora with respect to the grammatical category.

Noun Anaphora

Some specific nouns, in particular *one* and *other*, can serve as anaphor. The anaphoras which involve the former are often called ***one* anaphora** in the literatures[2].

(27) I asked for a green shirt$_i$, but he gave me a white one$_i$. Huddleston et al. (2002)

(28) These boxes$_i$ are more suitable than the others$_i$. Huddleston et al. (2002)

It is often the case that the anaphor and the antecedent are of the same syntactic category. This is revealed in the above examples: the antecedents of *one* in (27) and *others* in (28) are *shirt* and *boxes*, respectively, which are also bare nouns.

NP Anaphora

NP is probably the most prominent syntactic category which is involved in anaphoric relations. However, it is not the case that all NPs are suitable to function as anaphor. In the course of anaphora, we are especially interested in two types of NPs: pronoun and definite NP. We shall look at them one by another.

[2]For a comphrehensive survey on *one* anaphora, please refer to Gardiner (2003).

Pronoun In English, pronouns constitutes a closed word class and can be subdivided into several categories, such as personal (e.g. *I*, *you*, *he*, *they*), reflexive (e.g. *ourselves*, *himself*, *herself*), reciprocal (e.g. *each other*, *one another*), possessive (e.g. *my*, *your*, *his*, *her*), demonstrative (e.g. *this*, *that*, *these*, *those*), interrogative and relative (e.g., *which*, *who*, *whose*), etc.

As a particular case of the pro-form[3], a **pronoun** is a word which is used in place of a noun or a NP. Hence the interpretation of a pronoun does not derive from itself, rather, it depends on the nominal that the pronoun substitutes. As a result, the most preliminary functionality of pronoun is to serve as anaphor. We have already seen several examples where personal pronouns are used as anaphor, such as (14), (19) and (20). In the following, we provide some anaphoric examples where other types of pronouns are involved:

(29) a. Ann$_i$ blamed herself$_i$ for the accident. (Reflexive)
b. They$_i$ are required to consult with each other$_i$/one another$_i$. (Reciprocal)
c. Everyone$_i$ had cast his$_i$ vote. (Possessive)
d. I raised some money$_i$ by hocking the good clothes I had left, but when that$_i$ was gone I didn't have a cent. (Demonstrative)
e. She wrote personally to those$_i$ whose$_i$ proposals had been accepted. Huddleston et al. (2002) (Relative)

The third-person neuter pronoun *it* is a little bit special. Besides referring to human babies, animals or inanimate objects, it is often used to refer to a whole sentence or a propositional. For instance:

(30) John insulted the ambassador$_i$. It$_i$ happened at noon. Gundel et al. (2005)

Sometimes, the pronoun *it* is also involved in some special usages, where it occurs only for constructional reason. We call the pronoun in these cases **pleonastic**, for instance:

(31) a. It is fortunate that Nadia will never read this thesis.
b. It is half past two. Hirst (1981)

Definite NP In general, **definite NP** or **definite description**, refers to those NPs which start with a definite article or a demonstrative, for instance, *the man*, *the President*, *that book*. Besides, proper names are typically considered as a subcategory of definite NP as well. A definite NP can often be used as anaphor, as in the following examples:

(32) Mary saw a movie$_i$ last week. The movie$_i$ was not very interesting. Abbott (2006)

The definite NP *the movie* in (32) refers back to the object which is introduced by its antecedent *a movie* in the preceding sentence. A definite NP can be as plain as

[3]For more discussions on pro-form, please refer to Huddleston et al. (2002).

the man, *the movie*. In addition, it may contain descriptive content as well, which is expressed by additional words (usually adjectives) in the construction. In this case, the definite NP is called an **epithet**, which can be used as anaphor as well:

(33) $\underline{\text{John}_i}$ was playing. $\underline{\text{The tall boy}_i}$ was happy. Elworthy (1992)

(34) $\underline{\text{Ross}_i}$ used his Bankcard so much, $\underline{\text{the poor guy}_i}$ had to declare bankruptcy. Hirst (1981)

In example (33) and (34), the definite NPs containing descriptive content: *the tall boy* and *the poor guy*, refer to the individuals introduced by the proper name *John* and *Ross*, respectively. As we can see, different from plain definite NPs, epithets can ascribe additional properties (e.g., being tall, being poor) to the referent.

VP Anaphora

VP of some particular forms such as *do so*, *do it* and *do this/that* can function as anaphor. They are also called **proaction**. For instance:

(35) a. She $\underline{\text{agreed to help}_i}$, but she $\underline{\text{did so}_i}$ reluctantly.
b. If we are going to $\underline{\text{live together}_i}$, we may as well $\underline{\text{do it}_i}$ properly.
c. There are times when I'd just like to $\underline{\text{go down to the library and get some books}_i}$, but often you can't $\underline{\text{do that}_i}$ on the spur of the moment. Huddleston et al. (2002)

Sometimes, the bare verb *do* can serve as anaphor by itself, in particular when the antecedent is an intransitive verb. Analogous to other pro-forms, it is called **proverb**.

(36) a. Daryel $\underline{\text{thinks}_i}$ like I $\underline{\text{do}_i}$.
b. When Ross $\underline{\text{orders sweet and sour fried short soup}_i}$, Nadia $\underline{\text{does}_i}$ too. Hirst (1981)

In example (35-a) and (35-b), the VP anaphors refer to the same actions or events expressed by their antecedents. However, it is more often the case that VP anaphora is concerned with different actions or events in the same form, as in example (35-c) and (36). In these cases, the actions or events introduced by the antecedent and the anaphor are not identical, they distinguish from each other with respect to at least one of the following aspects: agent, location, etc.

Adjective Anaphora

It is not so common that adjectives act as anaphor, except for the particular one *such* Postal (1969), which is also a specific type of the pro-form, called a **proadjective**:

(37) a. Some $\underline{\text{careless}_i}$ driver backed into our car. $\underline{\text{Such}_i}$ people make me mad. Partee (2008)
b. I was looking for a $\underline{\text{purple}_i}$ wombat, but I couldn't find $\underline{\text{such}_i}$ a wombat. Hirst (1981)

As shown in the two discourses of (37), the anaphor *such* is interpreted based on the antecedent adjective *careless* and *purple*, respectively.

Adverb or Prepositional Phrase Anaphora

Sometimes, temporal and locative reference are also considered as anaphora, which are achieved through particular adverbs (e.g. *then*, *there*) or prepositional phrases (e.g., *at that time*):

(38) a. In the mid-sixties$_i$, free love was rampant across campus. It was then$_i$ that Sue turned to Scientology.

b. In the mid-sixties$_i$, free love was rampant across campus. At that time$_i$, however, bisexuality had not come into vogue. Hirst (1981)

Hence both *then* in (38-a) and *at that time* in (38-b) refer to the time denoted by the preceding prepositional phrase *in the mid-sixties*.

Ellipsis as Anaphor

An **ellipsis** is a structural null which takes the place of an intended linguistic expression. It is an ultimate example of context-dependence because ellipsis is the linguistic entity which carries the least information one can possibly imagine. In other words, due to the minimal lexical information it contains (in fact, it does not contain anything at all), the interpretation of an ellipsis completely comes from the context.

Based on the syntactic category of the elided expression, an ellipsis can be of various types as well, among which VP ellipsis is the most common and well-studied one. In what follows, we provide a comprehensive list of examples which are concerned with ellipsis, where the symbol "∅" is used to indicate the occurrence of an ellipsis.

(39) a. John's brother$_i$ is an anti-war campaigner, and Bill's $\emptyset_i$ is an anti-globalization activist. Huang (2006) (Nominal Ellipsis)

b. *A*: Have you finished your assignment$_i$ yet?
B: I haven't even started $\emptyset_i$. Huddleston et al. (2002) (NP Ellipsis)

c. I couldn't hear what he was saying$_i$, but fortunately Kim could $\emptyset_i$. Huddleston et al. (2002) (VP Ellipsis)

d. She will help me$_i$, won't she $\emptyset_i$? Huddleston et al. (2002) (VP Ellipsis)

e. I liked it$_i$, but Kim didn't $\emptyset_i$. Huddleston et al. (2002) (VP Ellipsis)

f. I asked Max to tidy up his room$_i$, but he refused $\emptyset_i$. Huddleston et al. (2002) (Complement Ellipsis)

Note that the above list does not exhaust the range of all ellipsis-involved anaphoras. Some researchers have proposed a more fine-grained typology for ellipsis. For instance, Webber (1979) makes a distinction among VP deletion, null complement anaphora, sluicing, gapping, stripping. However, we will not go into further details since this is outside the scope of this thesis.

2.2.2 Type of Identity

The type of identity is another perspective with respect to which we can classify anaphora. A standard theory in this fashion is attributed to Graeme Hirst, who distinguishes between two types of anaphora: **identity of reference anaphora** (**IRA**) and **identity of sense anaphora** (**ISA**) Hirst (1981). The former indicates the anaphoric relations where both the anaphor and its antecedent denote the same referent (namely co-reference); the latter indicates the anaphoric relations where the anaphor is interpreted with respect to the descriptive content of the antecedent, rather than its referent. For instance, let's examine the following pair of examples:

(40) a. The President (of the United States)$_i$ walked off the plane. He$_i$ waved to the crowd.
b. The President$_i$ is elected every four years. He$_i$ has been from a southern state ten times. Carlson (2002)

According to the above discussion, the anaphora in (40-a) is an IRA, while the one in (40-b) is an ISA. Assume both highlighted NPs in (40-a) refer to the current president of the United States, namely Barack Obama, then the two NPs: the definite expression *The President* and the pronoun *he*, are co-referring (see definition 2.1.2). By substituting them with the proper name *Barack Obama*, we can thus paraphrase (40-a) as *Barack Obama walked off the plane. Barack Obama waved to the crowd.* However, a similar replacement does not work for (40-b). That is because the pronoun *he* in (40-b) is interpreted with respect to the concept of presidency (which is the descriptive content of its antecedent *the President*), rather than any particular individual.

In addition, the famous paycheck anaphora and the above *one* anaphora (e.g., example (27) in section 2.2.1) are also typical examples of ISA:

(41) The man who gave his paycheck$_i$ to his wife was wiser than the man who gave it$_i$ to his mistress. Karttunen (1969)

(42) Kelly is seeking a unicorn$_i$ and Millie is seeking one$_i$ too. Luperfoy (1991)

In the paycheck example (41), the anaphoric relation between the pronoun *it* and the preceding NP *his paycheck* is not co-reference. The most natural interpretation one may obtain is that *it* refers to a different paycheck from the one denoted by the antecedent. Hence, the anaphoric link between the two NPs does not reside in the identity of referents, rather, it is the common property of the referents that plays an important role. It is an analogous case for the anaphora in example (42). The anaphor *one* and its antecedent *a unicorn* denote two different unicorns.

Moreover, one may notice that in both of the two examples, the anaphor can be regarded as a literal repetition of its antecedent. That is to say, by literally substituting the anaphor with the antecedent, we do not change the meaning of the original sentence. Because of this feature, anaphors like such are sometimes called **pronoun of laziness** Geach (1962).

Empirically, it is not an easy task to distinguish between IRA from ISA, because an anaphora is often ambiguous with respect to these two categories. For instance:

(43) Ross likes his hair$_i$ short, but Daryel likes it$_i$ long. Hirst (1981)

In Example (43), the pronoun *it* can be either an IRA or an ISA. In the former case, *it* refers to Ross's hair; while in the latter, *it* refers to the hair of Daryel.

The IRA and ISA distinction is often exclusively limited to NP anaphora, as it is the case in Hirst (1981). That is because both IRA and ISA are concerned with referring expressions, and referentiality is seldom discussed for expressions other than NPs. However, as we have shown in the section 2.2.1, a similar contrast, which extends the IRA and ISA difference between NPs, can be drawn among VP or verb anaphora as well, please consult example (35) and (36) for more details.

2.2.3 Anaphor-Antecedent Position

Anaphora can also be classified according to the relative position of the anaphor and its antecedent. This position may refer either to the linear precedence between the two in the text, or the distribution across the discourse (with respect to sentence boundary).

Linear Precedence

As we have seen in section 2.1, the antecedent does not necessarily need to precede the anaphor. In situations where the anaphor comes earlier, we more often speak of cataphora rather than anaphora. Both cases are subsumed under a more general phenomenon: endophora, as shown in figure **??**. The current work will analyze anaphora from the semantic perspective, while the linear precedence is largely a syntactic issue. Consequently, as we have already mentioned, we will exclusively focus on the strict sense of anaphora, where anaphor appears after its antecedent. Other context-dependent phenomena, such as cataphora and exophora, will be left apart.

Distribution with respect to Sentence Boundary

In some previous examples, such as (14), (15), (19) and (20), the anaphor and its antecedent occur within the same sentence (either simplex or complex). Relation like such is called **intra-sentential anaphora**. In contrast, there are examples where the anaphor and its antecedent are distributed across sentence boundary, they are hence called **inter-sentential** or **discourse anaphora**. Examples of this type include (32), (33), (34), and (37), where definite NPs and adjectives are involved. Pronoun, which is a paradigmatic anaphor, can of course serve in inter-sentential anaphora:

(44) $\underline{\text{Leonard}}_i$ is a famous conductor. $\underline{\text{He}}_i$ writes operas. Carlson (2002)

(45) $\underline{\text{Few professors}}_i$ came to the party. $\underline{\text{They}}_i$ had a good time. King (2013)

The main focus of this thesis is dynamic semantics, which studies the semantics of discourse rather than isolated sentences. Hence we are mostly interested in inter-sentential anaphora. We will come back to inter-sentential anaphora and investigate more data of this sort in chapter 4.

As a summary, anaphora is a complex linguistic phenomenon involving *structural, cognitive and pragmatic factors that interact with each other* Huang (2000). In this thesis, we will confine our interest mainly to pronominal anaphora, where both the antecedent and the anaphor are singular NPs[4], and the anaphor is pronoun. In addition, we will mainly restrict our attention to anaphoras that span across multiple sentences thereinafter.

2.3 Anaphora on Different Linguistic Levels

This section provides a brief survey of previous analysis on anaphora. We start from investigating the classical semantic relations between an anaphor and its antecedent. Then we will introduce an influential syntactic theory on anaphora.

[4]For a comprehensive study of plural anaphora, please refer to Nouwen (2003b).

2.3.1 Semantics: Anaphor-Antecedent Relations

The main objective of a semantic theory on anaphora is to assign it an appropriate interpretation. As can be inferred from definition 2.1.4, in an anaphoric relation, the interpretation of the anaphor depends on that of the antecedent. In order to study the semantics of anaphora, we first have to understand the semantic relation between the two ingredients. However, this does not seem to be an easy question, because an anaphor can stand in several kinds of relations to its antecedent.

In the semantic tradition, there are mainly two ways that an anaphor is interpreted: either as a co-referring expression, or as a bound variable. This point of view is widely agreed in the literature. For instance, co-indexing, which has been adopted in this thesis as an anaphora indicator, is distinguished among various usages:

> *Let's summarize the places where something like coindexing is used in the literature:*
>
> 1. *The same pronoun appears in several places in a sentence:*
> <u>*He*</u>$_i$ *said* <u>*he*</u>$_i$ *was OK.*
> 2. *A pronoun appears together with a referring NP:*
> <u>*John*</u>$_i$ *said that* <u>*he*</u>$_i$ *was OK.*
> 3. *A pronoun appears together with a quantificational NP:*
> <u>*No woman*</u>$_i$ *doubts that* <u>*she*</u>$_i$ *is OK.*
> 4. *A pronoun occurs in a relative clause:*
> *... the woman* <u>*who*</u>$_i$ *said that* <u>*she*</u>$_i$ *had found the answer.*
> 5. *A reflexive or other obligatorily bound pronoun appears in a sentence:*
> <u>*John*</u>$_i$ *loves* <u>*himself*</u>$_i$*.*
> <u>*Oscar*</u>$_i$ *is out of* <u>*his*</u>$_i$ *head.*
>
> *It is really only in situation 1 (in some sentences), and 2 that it seems appropriate to talk about coreference. In every other case ... coindexing a pronoun with some other expression is a shorthand way of saying that the pronoun in question is being interpreted as a bound-variable...* Bach and Partee (1980)

For the rest of this subsection, we will sequentially discuss each of two classical interpretations in more detail.

Referential Interpretation

As indicated by the first group of definitions on anaphora in section 2.1, co-reference is a paradigmatic relation between an anaphor and its antecedent. In the case of co-reference, the anaphor obtains a referential interpretation: it simply refers to whatever the antecedent does. We call this **co-referential anaphora**. Various previous examples, such as (14), (15), (32), (33), and (34), fall within this category.

In section 2.1.1, we have seen some example where the anaphor is interpreted referentially, while it is not co-referring with its antecedent, e.g., (18). Instead of being identical, the referents from the two NPs are semantically or pragmatically related. As we mentioned, cases like such are called bridging anaphora[5]. The antecedent in bridging anaphora is often called **antecedent trigger** Cornish (1999). This is because the

[5]It is also sometimes called associative anaphora in the literatures Hawkins (1978).

information based on which the anaphor is interpreted is not supplied directly by the antecedent. Instead, the antecedent triggers a tacit inferential process, where the information can be deduced. In Clark (1975), the author provides a comprehensive survey on bridging anaphora, and classifies it into various sub-types, such as set membership, necessary part, inducible part, etc. In what follows, we provide some corresponding examples:

(46) a. I met two people$_i$ yesterday. The woman$_i$ told me a story. (set membership)
b. I looked into the room$_i$. The ceiling$_i$ was very high. (necessary part)
c. I walked into the room$_i$. The chandeliers$_i$ sparkled brightly. (inducible part) Krahmer and Piwek (2000)

As can be implied from their names, the relations in the above examples are relatively easy to understand. For instance, in (46-a), the definite NP *the woman* refers to an individual that is an element of the referent (a set of individuals) introduced by the antecedent NP: *two people*. In (46-b), the anaphor *the ceiling* denotes an indispensable component of the room object established by its antecedent: *the room*. In (46-c), *the chandeliers* is associated with *the room* based on the commonsense knowledge: it is likely that there are chandeliers in a room. Generally speaking, bridging is a more complex relation than co-reference. The former is additionally concerned with certain amounts of lexical and pragmatic information, thus it is more variable and more difficult to handle.

In this thesis, we subsume both co-referential anaphora and bridging anaphora into a more general type: **referential anaphora**, in which the anaphor is interpreted as a referring expression. Since bridging anaphora falls outside the domain of our work, future examples will not be concerned with it any more.

Bound-Variable Interpretation

Anaphora would be a much easier phenomenon if all of them were co-references. However, it does not seem to be the case (see section 2.1.1). Let's re-examine two earlier examples: (19) and (20), repeated as follows:

(19) Every man$_i$ thinks that he$_i$ deserves a raise. Carlson (2002)

(20) No one$_i$ wanted to admit that he$_i$ might be wrong. Partee (2008)

As explained before, in example (19), although the pronouns *he* is anaphorically linked to the antecedent *every man*, it does not refer to any specific entity. It is the same case for (20). Suppose anaphoras in (19) and (20) are co-references, then by literally substituting the anaphors with their corresponding co-referring antecedents, the meanings of the sentences should be pertained. That is because co-referring NPs always denote the same referent. However, we will end up with the following paraphrases after carrying out the operation described above:

(47) Every man thinks that every man deserves a raise.

(48) No one wanted to admit that no one might be wrong.

Although (47) and (48) are grammatically correct, they have a very different meaning from (19) and (20). In fact, as we mentioned earlier in section 2.1.1, the two NPs: *every man* and *no one*, are QNPs, and they are not referring expressions. Hence by no means do the pronouns in (19) and (20) co-refer with their antecedents: the antecedents do not even have the potential to refer.

So what could the semantic relation for anaphoras in (19) and (20) be? Let's take a closer look at the two examples. Sentence (19) roughly means that for every man x, x thinks that x deserves a raise. Analogously, sentence (20) means that there is no person x such that x wants to admit that x may be wrong. The pronouns in (19) and (20) are interpreted with respect to some quantified expressions, they behave like variables in classical predicate logic. Because these variables are under the scope of some quantifications (from the antecedent NPs), anaphoras of this sort are thus termed **bound anaphora**. Different from referential anaphora, the anaphoric expression in bound anaphora does not have a particular referent. Rather, it obtains the semantics through variable assignment. This will be discussed in more detail in section 3.1.2.

In bound anaphora, the antecedent does not necessarily need to be a QNP, as it is the case in (19) and (20). Take the following example for instance:

(49) $\underline{\text{John}_i}$ loves $\underline{\text{his}_i}$ wife. Partee (2008)

According to the index information in example (49), the antecedent of the possessive pronoun *his* is the proper name *John*[6], which is a typical referring expression. Then the anaphora of (49) is ambiguous between the two semantic interpretations. Namely, it is unclear whether *his* refers to John; or the other possibility: *his* is a variable bound by *John*. This distinction does not make much sense for (49) because both interpretations render an identical semantics for the whole sentence. For instance, assume John's wife is Mary, then what (49) means is simply John loves Mary, no matter which semantic interpretation is applied to *his*. However, a noticeable difference would be revealed if we complicate (49) as follows:

(50) John loves his wife and so does Bill. Partee (2008)

The first part of example (50) is just (49), it has only one possible semantics. However, the whole sentence is now ambiguous in two ways. Again, assume John's wife is Mary, Bill's wife is Susan, then (50) can express either of the following meanings:

1. John loves Mary, and Bill loves Mary, too;

2. John loves Mary, and Bill loves Susan, too.

The two interpretations are called the strict identity reading and the sloppy identity reading, respectively Ross (1967). To capture the ambiguity, we can distinguish the two semantic interpretations (referential and bound-variable) of the anaphoric pronoun: the referential interpretation brings about the strict identity reading, the bound-variable interpretation gives rise to the sloppy identity reading. As a result, example (50) gives a fair motivation for the two semantic interpretations that we have been discussing in this subsection.

In the field of natural language semantics, there are also many theories which attempt to reduce the two categories to bound anaphora, most notably Geach (1962); Groenendijk and Stokhof (1991); Kamp (1981); Karttunen (1969). These theories are mostly interested in NPs, so examples such as (50) are not concerned with. Then the uniform interpretation

[6] In this case, we do not consider the situation where *his* is anaphorically linked to some other NPs in the preceding context, which is another possibility to resolve the anaphora.

of anaphor seems to be adequate. In the current thesis, we will follow this tradition and assume that anaphors are interpreted uniformly as variable-like entities. More discussion on this topic will be conducted in chapter 4.

2.3.2 Syntax: Binding Theory

A central concern of a linguistic theory of anaphora is how to determine the antecedent for an anaphor. One of the most influential work to this problem is proposed by Chomsky. This is what we are going to discuss shortly.

As it has been observed by linguists, there is a different distribution between pronouns such as *he*, *she*, *him*, and reflexives such as *himself*, *herself*, for instance:

(51) Zelda$_i$ bores herself$_{i/*j}$. Büring (2005)

(52) Bob$_i$ was nominated by him$_{*i/j}$. Carlson (2002)

(53) She$_{*i/j}$ hoped that Mary$_i$ would win the contest. Carlson (2002)

In example (51), the reflexive pronoun *herself* can only be resolved by taking the subject proper name *Zelda* as its antecedent. However in example (52), the subject *Bob* can never serve as antecedent for the pronoun *him*. Otherwise, the sentence will be infelicitous. In example (53), the pronoun *she* precedes the proper name *Mary*, and they can not be anaphorically associated.

In order to account for the different distributions of anaphors in the above examples a series of syntactic works emerges. The most representative is the Government and Binding Theory (GB Theory), or simply the Binding Theory (BT), which is proposed and further developed by Chomsky and his associates Chomsky (1981, 1986b, 1995); Reinhart (1983, 1984, 1976). Generally speaking, the GB Theory considers anaphora as a syntactic phenomenon in nature. By proposing various structural constraints and conditions, the GB Theory provides a syntax for nominal anaphora, in particular, the intra-sentential, or sentence-internal anaphora. Hence, two nominal expressions should bear an appropriate syntactic relation if they are to be anaphorically linked.

In the GB Theory, NPs are classified into several different groups: **R-expressions**, which denote full NPs, typically including proper names and definite NPs (e.g., *John*, *the man*, etc.), **plain pronouns** (e.g., *he*, *she*, *you*, etc.), **reflexive pronouns** (e.g., *himself*, *herself*, *yourself*, etc.) and **reciprocal pronouns** (e.g., *each other*, *one another*).

Please note that in Chomskian generative syntax, the term “anaphor” has been used in a much narrower sense compared to the way we have presented it above. According to Chomsky, anaphor exclusively refers to reflexive and reciprocal pronouns. However, as shown in definition 2.1.4, we consider it as an expression whose interpretation depends on others. As a result, not only reflexives and reciprocals, but also plain pronouns, even some R-expressions, are counted as anaphor in our usage. In order to keep the presentation uniform, we use the term **R-pronoun** Reinhart (1983) in place of Chomsky's anaphor, which subsumes reflexives and reciprocals. Then the hierarchy of NPs in the GB Theory can be summarized in figure 2.2[7]:

In addition to the classification on NPs, Chomsky introduces some purely structural notions in the GB Theory, such as local domain/locality, command, etc. The local domain for a NP denotes the syntactic region where the binding of the NP will be affected. The formal definition of local domain depends on governing category Chomsky (1981), which

[7]Figure 2.2 is a slightly modified variant of a diagram in Asudeh and Dalrymple (2006).

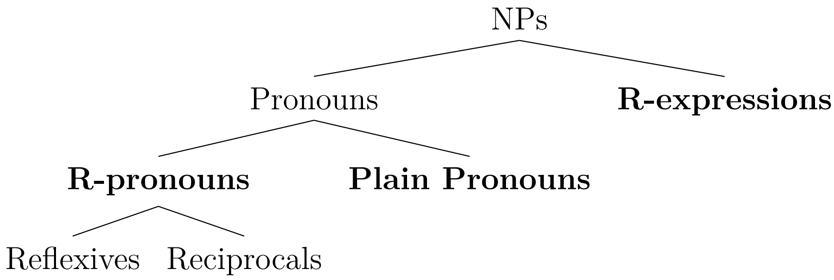


Fig. 2.2 Classification of NPs in BT

is a fairly complex notion. Since we do not want to dive into the technical details, we may roughly consider local domain as the "(minimal) clause", which is the minimal domain containing the head of the clause (usually the verb) and all its arguments. Besides local domain, the notion of command is also a key ingredient in the formation of the GB Theory. It refers to the structural domination or superiority between two nodes in a syntactic tree. One of the most common version is **c-command** Reinhart (1976), which is short for constituent-command. It is defined as follows:

Definition 2.3.1. Assume there is a constituent structure tree, where A and B are two nodes. We say A **c-commands** B iff

1. neither A nor B dominates one another;
2. the branching node most immediately dominating A also dominates B.

We will illustrate the c-command relation with the tree structure in figure 2.3:

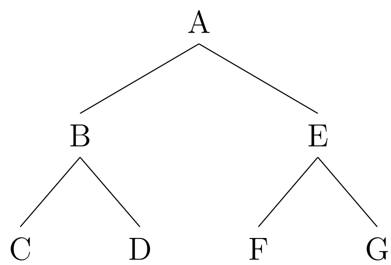


Fig. 2.3 C-Command Illustration

First of all, it is obvious that we have the following dominating relations among the nodes: A dominates B, C, D, E, F, G; B dominates C, D; E dominates F, G; C, D, F, G do not dominate any node. Then following definition 2.3.1, it is straightforward to infer the following c-command relations:

- A does not c-command any node because A dominates all the nodes;
- B c-commands E, F, G;
- E c-commands B, C, D;
- C c-commands D, D also c-commands C;

- F c-commands G, G also c-commands F.

As we can see, c-command is a relation mutually existing between sibling nodes. Based on above discussion, we can further define the notion of (syntactic) binding, which is another crucial concept in the GB Theory.

Definition 2.3.2. Assume there is a constituent structure tree, where A and B are two nodes. We say A (**syntactically**) **binds** B, or equivalently, B is (**syntactically**) **bound** by A, iff

1. A c-commands B;
2. A and B are co-indexed.

In addition, A is called the **binder** of B. If a node is not bound, we say it is **free**.

A remark on the terminology: we prefixed the above definition with "syntactic" because it is distinguished from the notion semantic binding, namely the variable-binding in classical predicate logic. In the rest of this subsection, since we are mostly focusing on the syntactic theory, the term binding will refer to syntactic binding by default, unless it is specifically indicated.

Finally, with all the above concepts, e.g., local domain, command, and binding, Chomsky proposes the following binding conditions Chomsky (1981):

Condition A An R-pronoun must be bound in its local domain;

Condition B A plain pronoun must be free in its local domain;

Condition C An R-expression must be free.

In the literatures, the above three conditions are often associated with the condition on R-pronouns, the condition on plain pronouns, and the condition on R-expressions, respectively. Hence each type of nominals, in particular the three highlighted ones in figure 2.2, is distributed under the guidance of its own binding condition.

As one may have already noticed, the three conditions can be respectively used to account for the examples at the beginning of this subsection. For instance, according to condition A, the R-pronoun *herself* in (51) ought to be bound in its local domain, where *Zelda* is the only available potential binder; in (52), the plain pronoun *him* should be free in its local domain, hence if we take *Bob* as its antecedent, condition B is violated; finally in (53), since the R-expression *Mary* is c-commanded by *she*, it can not be co-indexed with *she* any more because that will violate condition C.

There are, however, serious criticisms that the GB Theory suffers from, at the very least for the version that we presented above. At the heart of the GB Theory (the three conditions), the most important concepts are local domain and command. It has been assumed that all nominals, in particular R-pronouns and plain pronouns, share the same notion of local domain. Thus according to condition A and B, R-pronouns and plain pronouns should stand in complementary distribution. That is to say, R-pronouns ought to be bound exactly in the domain where plain pronouns should not be bound, and vice versa. Although this prediction seems to qualify most examples, such as (51) and (52), it has been challenged by counter-examples as follows, where R-pronouns and plain pronouns are in fact not mutually exclusive:

(54) a. $\underline{\text{They}_i}$ saw $\underline{\text{each other}_i}$'s pictures.
b. $\underline{\text{They}_i}$ saw $\underline{\text{their}_i}$ pictures. Huang (1983)

(55) a. $\underline{\text{They}_i}$ saw pictures of $\underline{\text{each other}_i}$.
b. $\underline{\text{They}_i}$ saw pictures of $\underline{\text{them}_i}$. Huang (1983)

Obviously, the R-pronoun *each other* and the plain pronoun *them* can occur at the same position in both (54) and (55). In order to amend the GB Theory to cover these examples, a more sophisticated definition of local domain has been proposed Chomsky (1986b). In this thesis, we will not go into details.

Besides local domain, the notion of command also needs further refinements. Above, we have only introduced the c-command relation. However, the GB Theory is unable to account for a number of examples with this setup, in particular when it is transplanted to languages where constituent ordering is more flexible, such as Chinese, Japanese, German, etc. For more examples in these languages, please refer to Büring (2005); Huang (2006). Because of that, subsequent developments of the GB Theory are heavily concerned with other types of command relations. For instance, there are tree-based **m-command** Chomsky (1986a), feature structure-based **f-command** Dalrymple (2001) and **o-command** Pollard and Sag (1994)[8], as well as thematic role-based **Θ-command** Jackendoff (1972). Again, we will not proceed any further because it goes beyond the scope of the thesis.

Up until now, all the examples we have seen in this subsection, namely (51), (52), (53), (54) and (55), are co-referential anaphora based on the semantic classifications (see subsection 2.3.1). Does it mean that the GB Theory only works for referring NPs? In fact, the GB Theory has a much wider coverage, it can be applied to anaphoras where antecedents are QNPs as well. If we count QNPs as R-expressions (QNPs are full NPs, although they are not referring expressions), then the three binding conditions still hold. For instance, in example (19) and (20), the plain pronoun *he* must be free in its local domain as predicted by condition B. Thus the prediction made by the GB Theory is correct because *he* is bound by its antecedent outside its local domain in (19) and (20). Analogously, the following example (56) is infelicitous because the R-expression *each of the tenors* is bound by *he*, which is an obvious violation of condition C.

(56) *$\underline{\text{He}_i}$ exploits the secretary that $\underline{\text{each of the tenors}_i}$ hired. Büring (2005)

In the above discussion, we have distinguished between the syntactic binding and the semantic binding. As we can see, in examples such as (19) and (20), the anaphor is not only syntactically but also semantically bound. Then how do these two bindings correlate, do they always coincide with each other? It has been observed semantic binding implies syntactic binding, but not the other way round Heim and Kratzer (1998). For more discussions on this topic, please refer to Büring (2005).

2.3.3 Computational Linguistics: Anaphora Resolution

Besides the theoretical work, there has also been extensive research on anaphora in the field of artificial intelligence and computational linguistics.

For computational linguists, the study of anaphora mainly refers to the so-called **anaphora resolution**, that is, the process of automatically identifying the most probable

[8]The term "f-command" and "o-command" are abbreviations for "functional-command" and "obliqueness-command", respectively.

antecedent for an anaphor. Another closely related task is **co-reference resolution**, which aims to automatically retrieve parts of a text that denote the same referent. We have explained the difference between anaphora and co-reference in section 2.1.1 and 2.3.1, but for most computational linguists, these two terms are more or less interchangeable.

Early approaches to anaphora resolution heavily rely on linguistic knowledge. Most of them involve a two-stage procedure. The first step is to filter out a set of NPs, which are potential antecedents for a given anaphor. This process is often accomplished by various linguistic constraints, such as morphological agreements (e.g., person, number, gender), syntactic constraints (e.g., conditions in the GB Theory). The second step is concerned with picking up the most preferable one(s) from the potential set. It is carried out based on factors such as commonsense knowledge (lexical semantics), syntactic structure (e.g., parallel position of a pronoun and its antecedent), salience (e.g., distance between a pronoun and its antecedent), etc.

Nowadays in real implementations, those early knowledge-based approaches are generally replaced by corpus-based systems. The reason is twofold. On the one hand, the purely linguistic approaches require an extensive amount of linguistic rules which are labor-intensive and time-consuming. On the other hand, they are not sufficiently adaptive as well: most linguistic rules are highly language-dependent. In fact, due to the rapid development of computational technologies and the availability of electronic corpus, anaphora resolution, like other natural language processing tasks, is experiencing the data-driven era. Machine learning techniques, which are implemented upon the annotated corpus, start to change the field in many aspects.

We will stop discussing the issue of anaphora resolution, because this thesis investigates anaphora from the formal semantic perspective. For comprehensive surveys on the state of the art of anaphora resolution, please refer to Mitkov (1999); Poesio et al. (2011).

Up until now, we have examined anaphora from various aspects. As a conclusion of this chapter, the goal of this thesis is twofold. On the one hand, we will investigate the conditions under which a NP may act as the antecedent for an anaphor, namely the accessibility constraints. On the other hand, we will provide appropriate semantic representations to texts containing anaphoras.

Chapter 3

Mathematical Preliminaries

Contents

> *There is in my opinion no important theoretical difference between natural languages and the artificial languages of logicians; indeed I consider it possible to comprehend the syntax and semantics of both kinds of languages with a single natural and mathematically precise theory.* Montague (1970b)

One of the fundamental properties of natural language is that it is able to convey **meaning**. But due to the complexity of natural language, to characterize what we mean by “meaning” is not an easy task. This is where formal language such as **logic** intervenes, to which we appeal for a precise mathematical model capturing the relation between natural language expressions and their meanings. But what is logic and why we choose logic to analyze natural language?

Logic, which is often seen as the science of reasoning, was introduced more than two thousand years ago (e.g., Aristotle’s theory of syllogism). It has played an important role in argumentation theory, namely, how to draw conclusions based upon a set of premises. Like natural language, logic also closely correlates with meaning: in order to construct valid arguments, logic has to firstly figure out the meanings of expressions that are involved, otherwise, the arguments will simply be nonsensical. As a result, it is the analysis of meaning that brings natural language and logic together.

Along the history, although developments in logic and natural language have been carried out independently for a long period[1], there is a growing recognition that the two

[1] For a historical survey on the relationship between logic, philosophy and linguistics, please refer to Gamut (1991a).

fields are closely connected and should be studied side by side. In particular, since the beginning of the 20th century, work on the relationship between logic and linguistics has started to gather pace. At first, notions such as **model**, **truth**, **entailment** have been extensively developed in mathematical logic Frege (1879); Frege et al. (1966); Tarski (1944, 1956). Later on, a corresponding set of notions were proposed in linguistics as well Davidson (1965, 1967).

> *There is no need to suppress, of course, the obvious connection between a definition of truth of the kind Tarski has shown how to construct, and the concept of meaning. It is this: the definition works by giving necessary and sufficient conditions for the truth of every sentence, and to give truth conditions is a way of giving the meaning of a sentence. To know the semantic concept of truth for a language is to know what it is for a sentence - any sentence - to be true, and this amounts, in one good sense we can give to the phrase, to understanding the language.* Davidson (1967)

Despite Davidson's contribution to some particular linguistic phenomena, no detailed and formal theory in natural language semantics was established until the 1970s, when Montague proposed a series of works Montague (1970a,b, 1973). These frameworks, named after him, are known as **Montague Grammar** (**MG**)[2]. Going one step further than Chomsky, who suggests that the syntax of natural language could be treated analogously to that of formal language, Montague claimed that the semantics could also be analyzed in the same way (please refer to the quotation at the beginning of this chapter).

Following Davidson and Montague, natural language sentences, in particular declarative sentences, are related to states of affairs in the world by means of the concept of truth. For instance, the sentence *John loves Mary* is **true** if and only if (iff) John loves Mary in the world, otherwise it is **false**. Then, to know the meaning of a sentence is to understand what the situation in the world would suffice the truth of the sentence. In other words, specifying the meaning of a sentence comes down to giving its **truth conditions**[3], viz., the circumstances under which the sentence is true. In other words. Adopting this point of view, the truth-conditional meaning of sentence *John loves Mary* is the situation where John loves Mary.

In order to depict the truth conditions of natural language sentences, MG translates them into logical languages, in particular, intensional logic. In such a way, the interpretations of logical formulas are expected to reflect the interpretations of natural language expressions. Since the semantics of logical language has been formally and precisely defined, natural language semantics is thus reduced to a much easier task. Because logical language, as well as the notion of truth and model, are heavily dependent in MG and a number of similar approaches, they are also referred to as **logical semantics**, **truth-conditional semantics** and **model-theoretic semantics**.

In MG, natural language behaves like logic such that each expression is linked to its meaning in a systematic way. This linking relation is achieved through the **Principle of Compositionality**, which it is generally attributed to Frege:

> *The meaning of a complex expression is determined by the meanings of its constituent expressions and the way they are syntactically combined.*

[2]The work in this thesis is heavily influenced by MG. However, we will not present the whole theory of MG here, for more reference, see Dowty et al. (1981); Montague (1974).

[3]This view has though been contested by various researchers Dummett (1975); Soames (1992).

Due to the above principle, elementary linguistic expressions are associated with the semantic representations of their own; in addition, each syntactic rule, which indicates how a composite expression is constructed from its components, is accompanied with a semantic rule, which indicates how the meaning of the composite expression is formulated by meanings of the components. One immediate consequence of the close correspondence between syntax and semantics is that it can easily account the semantic productivity: with a finite number of (recursive) syntactic rules, people can recognize a potentially infinite number of grammatical sentences; correspondingly, a finite number of semantic rules, which are in parallel with the syntactic rules, ensure the understanding of an infinite number of unseen sentences.

The objective of this chapter is to provide some fundamental mathematical preliminaries, which have been used extensively in formal semantics of natural language. In particular, three logical languages: propositional logic, first-order logic and modal propositional logic, together with a formal system: simply typed λ-calculus, will be presented. In the following context, all frameworks will be described from both the syntactic and the semantic aspects, though with a emphasis on the latter. As we said, we will introduce logic from a linguistic perspective. To this end, some natural language examples, which the formal system can well handle, will be included at the end of the exposition for each system.

3.1 Logical Languages

In this section we will present three logical languages: propositional logic, first-order logic and modal propositional logic. All systems will be exhibited in the same way: we first describe the syntactic rules and semantic interpretations, based on which we shall introduce some key notions such as satisfiability and validity. Propositional logic, the topic of section 3.1.1, concerns the meanings of expressions with respect to a set of logical connectives, e.g., conjunction, negation; first-order logic, the topic of section 3.1.2, extends the former system with the notion of quantification; in section 3.1.3, we focus on modal propositional logic, which incorporates the notion of possible world in propositional logic, it can serve to account for the phenomenon of modality in natural language semantics.

Our presentation will be as brief as possible. For a more detailed introduction on the three systems, please refer to Fitting (1996); Kleene (1952); Smullyan (1968) from a pure logical perspective, or Gamut (1991a) from a multidisciplinary (logic, philosophy and linguistic) perspective.

3.1.1 Propositional Logic

Propositional Logic (PL) is one of the simplest logical systems. As its name implies, it is a formal system of propositions. Roughly speaking, a proposition is the description of a situation, or a state of affairs in the world. In natural languages, a declarative sentence, or an assertion, is assumed to express a proposition. A proposition will be either true or false, depending on the world, or the circumstances in which it occurs.

The goal of this subsection is to lay out the syntax of PL. To begin with, we present the vocabulary of PL as follows, which determines the basic expressions that the logical system contains.

Definition 3.1.1. The alphabet for propositional logic (PL) consists of the following symbols:

1. Propositional variables: p, q, r, ...;

2. Logical connectives: $\neg$ (negation), $\wedge$ (conjunction).

Notation 3.1.1. We use $\mathcal{A}$ to denote the countable set of propositional variables. Lowercase letters p, q, r will denote propositional variables.

Below is the syntax of PL, which prescribes the valid forms of expressions in the language. The definition comprises a number of explicit rules indicating how composite expressions are established through the combination of other component expressions.

Definition 3.1.2. The set of PL formulas $\mathbb{F}$ is inductively defined as follows:

1. $p \in \mathbb{F}$, whenever $p \in \mathcal{A}$;

2. $(\neg\phi) \in \mathbb{F}$, whenever $\phi \in \mathbb{F}$;

3. $(\phi \wedge \psi) \in \mathbb{F}$, whenever $\phi, \psi \in \mathbb{F}$.

Formulas constructed from rule 1 are called **atomic formulas**, those constructed from rule 2 and 3 are called **complex formulas**.

Notation 3.1.2. As used in definition 3.1.2, lowercase greek letters ϕ, ψ, ρ will denote propositions/formulas.

In the following context, we will not write all the parentheses in a formula. First of all, we will leave off the outermost parentheses, for instance,

- $\neg\phi$ denotes $(\neg\phi)$;

- $\phi \wedge \psi$ denotes $(\phi \wedge \psi)$.

Further more, other omissions, which occur within complex formulas, will not bring about ambiguity because there is a conventional order of precedence among logical operators: $\neg$ has a higher precedence than $\wedge$. Generally speaking, arguments taken by symbol with a higher precedence will be grabbed within a pair of parentheses by default before being taken by symbol with a lower precedence. For instance:

- $\neg\phi \wedge \psi$ denotes $(\neg\phi) \wedge \psi$;

- $\phi \wedge \neg\psi$ denotes $\phi \wedge (\neg\psi)$.

Finally, when more than two conjuncts occur, the binary operator $\wedge$ is to associate to the right. For instance:

- $\phi \wedge \psi \wedge \rho$ denotes $(\phi \wedge (\psi \wedge \rho))$.

Some other conventional logical connectives, such as $\rightarrow$ (implication) and $\vee$ (disjunction), can be defined respectively through De Morgan's Laws with the previous symbols:

$$\phi \rightarrow \psi \triangleq \neg(\phi \wedge \neg\psi) \tag{3.1}$$

$$\phi \vee \psi \triangleq \neg(\neg\phi \wedge \neg\psi) \tag{3.2}$$

Above we have only presented the syntax of PL, basically the formulas are "meaningless" up until now. As we mentioned, the meaning of a proposition is subject to the circumstances under which it occurs. Before formally presenting the semantics, namely the interpretation of PL formulas, we shall introduce the concept of interpretation function.

Definition 3.1.3. A **truth value** is either 1 or 0. An **interpretation function** is a mapping such that it assigns a truth value to every propositional variable, namely $I : \mathcal{A} \rightarrow \{0, 1\}$.

With the interpretation function, we can define the interpretation of formulas in PL as follows.

Definition 3.1.4. Let $\phi \in \mathbb{F}$ be a formula, I an interpretation function. The interpretation of ϕ with respect to I, in notation $[\![\phi]\!]^I_{PL}$, is defined inductively as follows:

1. $[\![p]\!]^I_{PL} = I(p)$, if $p \in \mathcal{A}$;
2. $[\![\neg\phi]\!]^I_{PL} = 1 - [\![\phi]\!]^I_{PL}$;
3. $[\![\phi \wedge \psi]\!]^I_{PL} = [\![\phi]\!]^I_{PL} * [\![\psi]\!]^I_{PL}$, where symbol "$*$" denotes the multiplication function.

As shown above, the interpretation of a formula ϕ with respect to an interpretation function I is a truth value. It is determined solely by I if ϕ is atomic; it is determined by the interpretations of its constituent parts, together with their mode of combination (the specific logical connective involved) if ϕ is complex. Logical connectives can thus be viewed as functions on truth values.

Based on the above definition on interpretation, we can introduce a list of correlated concepts, such as truth, satisfiability and validity.

Definition 3.1.5. Let $\phi \in \mathbb{F}$ be a formula, I an interpretation function. We say that ϕ is **true** given I, or equivalently, I **satisfies** ϕ, or equivalently, I is a **model** of ϕ, in notation $I \models_{PL} \phi$, iff $[\![\phi]\!]^I_{PL} = 1$.

Definition 3.1.6. ϕ is **satisfiable** if there is some interpretation function I such that $I \models_{PL} \phi$ (otherwise it is **unsatisfiable**); ϕ is **valid** if for any interpretation function I, $I \models_{PL} \phi$ (otherwise it is **invalid**). A valid formula is called a **tautology**; an unsatisfiable formula is called a **contradiction**.

Now let's compute the semantics of the derived logical connectives based on the ones of the primitives. We will first examine $\rightarrow$ (implication), which is defined in formula 3.1, then go to $\vee$ (disjunction), which is defined in 3.2. Assume ϕ and ψ are two formulas, I an interpretation function, then:

$$\begin{aligned}
[\![\phi \to \psi]\!]^I_{PL} &= [\![\neg(\phi \wedge \neg\psi)]\!]^I_{PL} \\
&= 1 - [\![\phi \wedge \neg\psi]\!]^I_{PL} \\
&= 1 - ([\![\phi]\!]^I_{PL} * [\![\neg\psi]\!]^I_{PL}) \\
&= 1 - ([\![\phi]\!]^I_{PL} * (1 - [\![\psi]\!]^I_{PL}))
\end{aligned} \tag{3.3}$$

As we can see, an implication $\phi \to \psi$ is true given I iff $1 - ([\![\phi]\!]^I_{PL} * (1 - [\![\psi]\!]^I_{PL}))$ is 1, viz. iff $([\![\phi]\!]^I_{PL} * (1 - [\![\psi]\!]^I_{PL}))$ is 0. Since we know that the value of a multiplication is 0 iff either its argument is 0, accordingly, $\phi \to \psi$ is true given I iff either $[\![\phi]\!]^I_{PL} = 0$ or $[\![\psi]\!]^I_{PL} = 1$, namely, either the antecedent ϕ is false given I, or the consequence ψ is true given I. As to disjunction:

$$\begin{aligned}
[\![\phi \vee \psi]\!]^I_{PL} &= [\![\neg(\neg\phi \wedge \neg\psi)]\!]^I_{PL} \\
&= 1 - [\![\neg\phi \wedge \neg\psi]\!]^I_{PL} \\
&= 1 - ([\![\neg\phi]\!]^I_{PL} * [\![\neg\psi]\!]^I_{PL}) \\
&= 1 - ((1 - [\![\phi]\!]^I_{PL}) * (1 - [\![\psi]\!]^I_{PL}))
\end{aligned} \tag{3.4}$$

As we can see, a disjunction $\phi \vee \psi$ is true given I iff $1 - ((1 - [\![\phi]\!]^I_{PL}) * (1 - [\![\psi]\!]^I_{PL}))$ is 1, viz. iff $(1 - [\![\phi]\!]^I_{PL}) * (1 - [\![\psi]\!]^I_{PL})$ is 0. Analogously, the computation is reduced to a multiplication. Hence $\phi \vee \psi$ is true given I iff either $[\![\phi]\!]^I_{PL} = 1$ or $[\![\psi]\!]^I_{PL} = 1$, namely, either the first disjunct ϕ is true given I, or the second disjunct ψ is true given I.

Finally, let's have a look at an illustration. For the following sentences:

(57) a. John loves Mary.
b. Mary loves John.
c. John loves Mary and Mary does not love John.

Assume the proposition expressed by (57-a) is p, the proposition expressed by (57-b) is q. Then the proposition expressed by (57-c) is compositionally constructed as $p \wedge \neg q$. Let I be an interpretation function such that $I(p) = 1$, $I(q) = 0$. The interpretations of the three formulas, as defined above in 3.1.4, are respectively:

$$[\![p]\!]^I_{PL} = I(p) = 1$$

$$[\![q]\!]^I_{PL} = I(q) = 0$$

$$\begin{aligned}
[\![p \wedge \neg q]\!]^I_{PL} &= [\![p]\!]^I_{PL} * [\![\neg q]\!]^I_{PL} \\
&= [\![p]\!]^I_{PL} * (1 - [\![q]\!]^I_{PL}) \\
&= 1 * (1 - 0) \\
&= 1
\end{aligned}$$

Hence, the interpretation function I which we provided, is a model of p and $p \wedge \neg q$, but not of q. In other words, both (57-a) and (57-c) correctly describe the states of affairs in model I, while (57-b) does not. Also as we can see, to determine the interpretation of a composite formula, such as $p \wedge \neg q$, we only need to pay attention to the interpretations of its constituents and the way they are combined, this precisely reflects the principle of compositionality.

As we have shown above, PL is a useful tool for investigating meanings of natural language sentences, such as the ones in example (57). However, sometimes we may want to see inside sentences and take advantage of relationships among constituent expressions. For instance, the following sentences have a clear predicate-argument(s) structure that we can make use of:

(58) a. Casper is bigger than John.
b. John is bigger than Peter.
c. Casper is bigger than Peter. Gamut (1991a)

To interpret (58) in PL, one will have to symbolize the proposition expressed by each sentence as an independent propositional variable, e.g., p, q, and r, whose truth values will be assigned by an interpretation function. However, since we know that the "bigger than" relation is transitive, with (58-a) and (58-b) as premises, it is natural to come up with (58-c). This inference can not be achieved in PL because p, q, and r are treated as unrelated atomic formulas, by no means do p and q together imply r.

Exploiting beyond the sentential level for example (58), proper names *Casper*, *John*, and *Peter* refer to some individuals, we use **casper**, **john**, and **peter** to represent them, respectively; the expression *is bigger than* is a predicate which refers to some particular property (the "bigger than" relation) that two individuals may or may not bear to each other, we use **bigger_than** to represent it. Then under this new schema, sentences in (58) can be translated into the following formulas:

- **bigger_than casper john**
- **bigger_than john peter**
- **bigger_than casper peter**

By adding the transitive property to the predicate symbol **bigger_than**, the implication from (58-a) and (58-b) to (58-c) will be rather straightforward. As a result, to account for relations among sentences, a deeper analysis is called for.

In addition, it is often the case that we encounter quantifying expressions in natural language. Let's have a look at the following example:

(59) Every man loves Mary.

Again, PL is not able to capture the meaning of the above sentence. It does not seem sufficient even when we go deeper into the sentence structure. As we explained before, the proper name $Mary$ refers to an individual. However, the subject NP *every man* can not be dealt with in the way: there does not exist any particular individual who is called *every man*. In fact, it picks up every individual who is a man. Similar expressions include *some man*, *no man*, etc. In order to account for sentences containing these quantifying expressions, we are in a position to extend the logical system with a machinery ranging over individuals.

Consequently, the above problems motivate a more powerful form of logic: first-order predicate logic, or simply first-order logic, which we will be introducing in the next subsection.

3.1.2 First-Order Logic

As we can see from the previous subsection, in dealing with natural language semantics, PL resides on the sentential level, which means PL can only describe sentences as a whole. In order to extend the capability of PL and go deeper into the components of a sentence, we appeal to a more powerful formal system: First-Order Logic (FOL).

As a type of predicate logic, FOL is able to account for natural language sentences with explicit predicate-argument(s) structures, where the predicate is treated as properties, its arguments as entities (individuals or objects). In addition, another prominent feature which distinguishes FOL from PL is that the former involves quantification, this allows a proposition to be generalized over sets of individuals and can be used to deal with examples such as the above (59). These two properties largely extends the expressive power of the logical language in the previous subsection.

In what follows we will introduce the formal details of FOL. The syntax of FOL can be viewed as an extension of the one of PL. In general, the vocabulary contains a countable set of constant and predicate symbols, an infinite set of variables, some logical connectives and quantifiers. Logical connectives are rather standard; constants and predicate symbols are like propositional variables in PL; variables and quantifiers are a novelty in FOL, they aim to account for the concept of individual and quantification, respectively.

Please note that the framework in this thesis is a restricted version of the classical FOL. For the sake of simplicity, we do not include function symbols, because we will not be concerned with functions in our linguistic examples.

Definition 3.1.7. The alphabet for first-order logic (FOL) consists of the following symbols:

1. Constant symbols: $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, ..., $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, ...;
2. Variables: x, y, z, ..., x_1, x_2, x_3, ...;
3. Predicate symbols: $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$, ...;
4. Logical connectives: $\wedge$ (conjunction), $\neg$ (negation);
5. Quantifiers: $\exists$ (existential).

Notation 3.1.3. We use $\mathcal{C}$ to denote the countable set of constant symbols, $\mathcal{X}$ to denote the set of variables, $\mathcal{P}$ to denote the countable set of predicate symbols. Lowercase bold letters $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ will denote constant symbols; lowercase letters x, y, z will denote variables; uppercase bold letters $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$ will denote predicate symbols.

Each predicate symbol goes with a fixed natural number n called the **arity**, which denotes the number of arguments it takes. Hence, given a $\mathbf{P}$, it can be called a n-place predicate. Among all elements of the alphabet, the constant symbols, variables, and predicate symbols are **non-logical symbols**, the logical connectives and quantifiers are **logical symbols**.

The expressions of FOL, which are composed of symbols from the alphabet defined above, are divided into terms and formulas.

Definition 3.1.8. The set of FOL terms $\mathcal{T}$ is defined as the union of $\mathcal{C}$ and $\mathcal{X}$, namely $\mathcal{T} = \mathcal{C} \cup \mathcal{X}$.

The following syntactic rules explain what is meant to be a formula in FOL.

Definition 3.1.9. The set of FOL formulas $\mathbb{F}$ is inductively defined as follows:

1. $\mathbf{P}t_1, ..., t_n \in \mathbb{F}$, whenever $\mathbf{P} \in \mathcal{P}$, $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
2. $(\neg\phi) \in \mathbb{F}$, whenever $\phi \in \mathbb{F}$;
3. $(\phi \wedge \psi) \in \mathbb{F}$, whenever $\phi, \psi \in \mathbb{F}$;
4. $(\exists x.\phi) \in \mathbb{F}$, whenever $x \in \mathcal{X}$, $\phi \in \mathbb{F}$.

Formulas constructed from rule 1 are called **atomic formulas**, those constructed from rule 2 and 3 are called **complex formulas**, those constructed from rule 4 are called **existentially quantified formulas**.

Notation 3.1.4. As used in definition 3.1.9, lowercase greek letters ϕ, ψ, ρ will denote formulas.

Same as in PL, we can leave off unnecessary parentheses in FOL formulas. All the conventions in PL, which have been explained above, are inherited in FOL. In addition, since one more logical operator (quantifier $\exists$) is involved in FOL, we need to update the precedence as follows: $\exists$ has the highest precedence, then comes $\neg$, finally is $\wedge$. This is shown in figure 3.1.

Precedence:	*lower* →		*higher*
Logical Connectives:	$\wedge$	$\neg$	$\exists$

Fig. 3.1 Precedence Among Logical Symbols in FOL

And same as $\neg$ and $\wedge$, the parentheses between several continuous quantifiers will be omitted with respect to the "association to the right" convention. The following examples illustrate how the abbreviation rules work:

- $\exists x.\phi \wedge \neg\psi$ denotes $(\exists x.\phi) \wedge (\neg\psi)$;
- $\exists x \exists y \exists z.\phi$ denotes $(\exists x.(\exists y.(\exists z.\phi)))$.

Some other conventional logical connectives, such as $\rightarrow$ (implication), $\vee$ (disjunction), and $\forall$ (universal quantifier), can be defined through De Morgan's Laws with the previous symbols. For $\rightarrow$ and $\vee$, please refer to formula 3.1 and 3.2, the definition of $\forall$ is provided below:

$$\forall x.\phi \triangleq \neg\exists x.\neg\phi \tag{3.5}$$

As we said before, different from in PL, quantifiers are involved in FOL. The scope of a quantifier is defined as follows.

Definition 3.1.10. Let $x \in \mathcal{X}$ be a variable, $\phi \in \mathbb{F}$ a formula. For a quantified formula of the following form: $\exists x.\phi$, we say that ϕ is the **scope** of this particular occurrence of quantifier $\exists x$, or equivalently, ϕ or any part of ϕ lies in the scope of this particular occurrence of quantifier $\exists x$.

Basically, a quantifier binds a variable ranging over a domain of formula. Thus variables which are quantified by a quantifier and occur in its scope are bound. However, not all variables in the formula are bound. Accordingly, we introduce the notion of free variable.

Definition 3.1.11. The set of **free variables** of a term $t \in \mathcal{T}$, in notation $FV(t)$, is defined as follows:

1. $FV(\mathbf{a}) = \emptyset$;
2. $FV(x) = \{x\}$.

Definition 3.1.12. The set of **free variables** of a formula $\phi \in \mathbb{F}$, in notation $FV(\phi)$, is inductively defined as follows:

1. $FV(\mathbf{P}t_1...t_n) = FV(t_1) \cup ... \cup FV(t_n)$, where $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
2. $FV(\neg\phi) = FV(\phi)$;
3. $FV(\phi \wedge \psi) = FV(\phi) \cup FV(\psi)$;
4. $FV(\exists x.\phi) = FV(\phi) - \{x\}$.

For a formula ϕ, if $FV(\phi)$ is an empty set, namely $FV(\phi) = \emptyset$, we say ϕ is a **closed formula**.

We will now discuss the interpretation of FOL formulas, namely FOL semantics. In PL, the model is basically the interpretation function, which assigns truth values to atomic propositions. However in FOL, since the analysis is more fine-grained, notions such as individual and property are involved, the model for evaluation ought to be more complex as well.

Definition 3.1.13. A **model** M is a pair $\langle D, I\rangle$, where

1. D is a non-empty set called the **domain**, whose elements are called **individuals**;
2. I is an **interpretation function** whose domain is $\mathcal{C} \cup \mathcal{P}$, such that:

   i. $I(\mathbf{a}) \in D$, where $\mathbf{a} \in \mathcal{C}$;

   ii. $I(\mathbf{P}) \begin{cases} \in \{0,1\} & \text{if } n = 0, \\ \subseteq D^n & \text{otherwise.} \end{cases}$ where $\mathbf{P} \in \mathcal{P}$, n is the arity of $\mathbf{P}$.

Definition 3.1.14. Let $M = \langle D, I\rangle$ be a model, an **assignment function** is a mapping $f : \mathcal{X} \to D$.

Notation 3.1.5. We use $\mathcal{G}$ to denote the set of assignment functions. As used in 3.1.14, lowercase letters f, g, h will denote assignment functions.

Hence, the role of an assignment function (or simply assignment, or valuation) is to associate each variable with an individual in the domain.

Definition 3.1.15. Let g and h be assignment functions, $X \subseteq \mathcal{X}$ a set of variables. Notation $h[X]g$ is used to denote that h differs from g at most with respect to the values it assigns to elements of X, namely:

$$h[X]g \text{ iff } \forall x \in \mathcal{X} : (x \notin X) \to (h(x) = g(x))$$

The above notation as in definition 3.1.15 can be used in a concatenated style to describe relations among multiple assignment functions. Let g, h, and f be assignment functions, X and Y sets of variables. If $h[X]g$ and $f[Y]h$ then we can infer $f[X \cup Y]g$. The reason is straightforward, if h agrees with g possibly except for the values it assigns to elements of X, at the same time, f agrees with h possibly except for the values it assigns to elements of Y, then f must agree with g on all variables that are not in X and Y, while possibly, for variables in X and Y, they might differ. We provide a concrete example as follows. For instance, assume we have $h[\{x, y\}]g$, $f[\{y, z\}]h$. Then it is possible that $h(x) \neq g(x)$, $h(y) \neq g(y)$, while for any rest variable, such as z, h and g always agree, so h(z) = g(z); similarly, it is possible that $f(y) \neq h(y)$, $f(z) \neq h(z)$, while for any rest variable, such as x, f and h agree, so $f(x) = h(x)$. Because $h(z) = g(z)$, so it is possible that $f(z) \neq g(z)$; because $f(x) = h(x)$, so it is possible that $f(x) \neq g(x)$; as for variable y, all three assignments possibly differ from one another, namely $f(y) \neq h(y) \neq g(y)$. As a result, f agrees with g possibly except for the variables in $\{x, y, z\}$, which is the union $\{x, y\} \cup \{y, z\}$.

With the notion of model and assignment function, we can induce the interpretation (for both terms and formulas), as well as the notion of satisfiability in FOL as follows.

Definition 3.1.16. Let $M = \langle D, I \rangle$ be a model, $g \in \mathcal{G}$ an assignment function, $t \in \mathcal{T}$ a term. The interpretation of t in M with respect to g, in notation $[\![t]\!]^{M,g}_{FOL}$, is defined as follows:

1. $[\![\mathbf{a}]\!]^{M,g}_{FOL} = I(\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}$;
2. $[\![x]\!]^{M,g}_{FOL} = g(x)$, if $x \in \mathcal{X}$.

Definition 3.1.17. Let $M = \langle D, I \rangle$ be a model, $\phi \in \mathbb{F}$ a formula. The interpretation of ϕ in M, in notation $[\![\phi]\!]^{M}_{FOL}$, is defined inductively as follows:

1. $[\![\mathbf{P}t_1 ... t_n]\!]^{M}_{FOL} = \begin{cases} \emptyset \text{ or } \mathcal{G} & \text{if } n = 0, \\ \{g \mid \langle [\![t_1]\!]^{M,g}_{FOL}, ..., [\![t_n]\!]^{M,g}_{FOL} \rangle \in I(\mathbf{P})\} & \text{otherwise.} \end{cases}$
   where $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
2. $[\![\neg\phi]\!]^{M}_{FOL} = \mathcal{G} - [\![\phi]\!]^{M}_{FOL}$, where $\mathcal{G}$ is the set of all assignment functions;
3. $[\![\phi \wedge \psi]\!]^{M}_{FOL} = [\![\phi]\!]^{M}_{FOL} \cap [\![\psi]\!]^{M}_{FOL}$;
4. $[\![\exists x.\phi]\!]^{M}_{FOL} = \{g \mid \exists h : h[\{x\}]g \text{ and } h \in [\![\phi]\!]^{M}_{FOL}\}$.

In rule 4, the symbol ∃ on the right hand side of the definition is an abbreviation for the phrase "there is a". It is different from the quantifier ∃ on the left hand side, whose meaning is to be defined. And as shown above, the interpretation of a formula in FOL is a set of assignment functions, namely those which verify it under a model.

Below, we define the notion of truth, satisfiability and validity in FOL, which are similar as in PL.

Definition 3.1.18. Let M be a model, $g \in \mathcal{G}$ an assignment function, and $\phi \in \mathbb{F}$ a formula. We say that ϕ is **true** in M with respect to g, or equivalently, M **satisfies** ϕ with respect to g, or equivalently, g **verifies** ϕ in M, in notation $M, g \models_{FOL} \phi$, iff $g \in [\![\phi]\!]^{M}_{FOL}$.

Definition 3.1.19. Let M be a model, $g \in \mathcal{G}$ an assignment function, and $\phi \in \mathbb{F}$ a formula. We say that ϕ is **satisfiable** in M if there is some assignment function g such that $M, g \models_{FOL} \phi$ (otherwise it is **unsatisfiable**); ϕ is **valid** in M if for any interpretation function g, $M, g \models_{FOL} \phi$ (otherwise it is **invalid**). A valid formula is called a **tautology**; an unsatisfiable formula is called a **contradiction**.

In section 3.1.1, we have demonstrated the semantics of $\to$ (implication) and $\vee$ (disjunction), see formula 3.1 and 3.2, based on the meanings of the primitive logical constants. A similar computation can be carried out in FOL as well, we shall give the results directly, instead of repeating the same process here.

$$[\![\phi \to \psi]\!]^M_{FOL} = (\mathcal{G} - [\![\phi]\!]^M_{FOL}) \cup [\![\psi]\!]^M_{FOL} \tag{3.6}$$

$$[\![\phi \vee \psi]\!]^M_{FOL} = [\![\phi]\!]^M_{FOL} \cup [\![\psi]\!]^M_{FOL} \tag{3.7}$$

In what follows, we would like to spell out the semantics of the new operator: $\forall$ (universal quantifier), which is defined in formula 3.5. Assume x is a variable, ϕ is a formula, then:

$$\begin{aligned}
[\![\forall x.\phi]\!]^M_{FOL} &= [\![\neg\exists x.\neg\phi]\!]^M_{FOL} \\
&= \mathcal{G} - [\![\exists x.\neg\phi]\!]^M_{FOL} \\
&= \mathcal{G} - \{g \mid \exists h : h[\{x\}]g \text{ and } h \in [\![\neg\phi]\!]^M_{FOL}\} \\
&= \{g \mid \neg(\exists h : h[\{x\}]g \text{ and } h \in [\![\neg\phi]\!]^M_{FOL})\} \\
&= \{g \mid \forall h : \text{ if } h[\{x\}]g \text{ then } h \notin [\![\neg\phi]\!]^M_{FOL}\} \\
&= \{g \mid \forall h : \text{ if } h[\{x\}]g \text{ then } h \notin (\mathcal{G} - [\![\phi]\!]^M_{FOL})\} \\
&= \{g \mid \forall h : \text{ if } h[\{x\}]g \text{ then } h \in [\![\phi]\!]^M_{FOL}\}
\end{aligned} \tag{3.8}$$

As shown from the above computation, a universally quantified formula $\forall x.\phi$ is true iff for every individual in the domain (every possible assignment h is applied to variable x), it makes the formula ϕ true.

Now let's turn to examples. Firstly, the FOL translation of example (58-a) has been provided above: **bigger_than casper john**, we abbreviate the formula as ϕ. Let $M = \langle D, I\rangle$ be a model, the semantic interpretation of ϕ is as follows:

$$\begin{aligned}
[\![\phi]\!]^M_{FOL} &= \{g \mid \langle [\![\mathbf{casper}]\!]^{M,g}_{FOL}, [\![\mathbf{john}]\!]^{M,g}_{FOL}\rangle \in I(\mathbf{bigger_than})\} \\
&= \{g \mid \langle I(\mathbf{casper}), I(\mathbf{john})\rangle \in I(\mathbf{bigger_than})\}
\end{aligned}$$

Since there are no variables occuring in ϕ, its interpretation does not depend on the particular performance of assignment functions. Hence, if the two individuals denoted by *Casper* and *John* bear the "bigger than" relation, $[\![\phi]\!]^M_{FOL}$ will be the complete set of assignments, namely ϕ is valid; otherwise, $[\![\phi]\!]^M_{FOL}$ will be the empty set, namely ϕ is unsatisfiable.

As for example (59), based on the FOL system we have just introduced, it is relatively easy to translate example (59) into the following logical form:

$$\forall x.(\mathbf{man}\ x \to \mathbf{love}\ x\ \mathbf{mary}) \tag{3.9}$$

The above formula 3.9 literally means for any individual, which is associated to the variable x, if it has the property of being a man, then it is in the love relationship with the constant **mary**. Based on above deduced semantics for implication and universally quantifier, we can formally examine the interpretation of the 3.9 step by step as follows. Let $M = \langle D, I \rangle$ be a model, we abbreviate the whole formula 3.9 as ϕ. Then:

$$\begin{aligned}
[\![\phi]\!]^M_{FOL} &= \{g \mid \forall h: \text{ if } h[\{x\}]g \text{ then } h \in [\![\textbf{man } x \rightarrow \textbf{love } x \textbf{ mary}]\!]^M_{FOL}\} \\
&= \{g \mid \forall h: \text{ if } h[\{x\}]g \text{ then } h \in ((\mathcal{G} - [\![\textbf{man } x)]\!]^M_{FOL} \cup [\![\textbf{love } x \textbf{ mary}]\!]^M_{FOL})\} \\
&= \{g \mid \forall h: \text{ if } h[\{x\}]g \text{ then } h \notin [\![\textbf{man } x]\!]^M_{FOL} \text{ or } h \in [\![\textbf{love } x \textbf{ mary}]\!]^M_{FOL}\} \\
&= \{g \mid \forall h: \text{ if } h[\{x\}]g \text{ then } h \notin \{f \mid f(x) \in I(\textbf{man})\} \text{ or} \\
&\qquad\qquad h \in \{f \mid \langle f(x), I(\textbf{mary})\rangle \in I(\textbf{love})\}\} \\
&= \{g \mid \forall h: \text{ if } h[\{x\}]g \text{ then } h(x) \notin I(\textbf{man}) \text{ or } \langle h(x), I(\textbf{mary})\rangle \in I(\textbf{love})\} \\
&= \{g \mid \forall d \in D: d \notin I(\textbf{man}) \text{ or } \langle d, I(\textbf{mary})\rangle \in I(\textbf{love})\}
\end{aligned}$$

As defined in 3.1.18, the satisfiability of ϕ in M with respect to g is deduced as follows:

$$\begin{gathered}
M, g \models_{FOL} \phi \text{ iff} \\
g \in \{g \mid \forall d \in D: d \notin I(\textbf{man}) \text{ or } \langle d, I(\textbf{mary})\rangle \in I(\textbf{love})\} \text{ iff} \\
\text{for any } d \in D \text{ either } d \notin I(\textbf{man}) \text{ or } \langle d, I(\textbf{mary})\rangle \in I(\textbf{love}).
\end{gathered}$$

That is to say, g verifies ϕ in M iff for any element d in domain D, either d is not a man individual, or d and the constant **mary** are in love relation. Obviously, this correctly reflects the semantics of (59).

Finally let's also have a quick look at the following example:

(60) He loves Mary.

One possible way of translating it into the FOL language is as follows, where pronoun *he* is treated as a variable:

$$\textbf{love } x \textbf{ mary} \tag{3.10}$$

As we can see, different from previous FOL translations, such as the ones for example (58-a) and (59), the above formula 3.10 contains a free occurrence of variable x. Then what is its interpretation? We abbreviate 3.10 as ϕ, then:

$$\begin{aligned}
[\![\phi]\!]^M_{FOL} &= \{g \mid \langle [\![x]\!]^M_{FOL}, [\![\textbf{mary}]\!]^M_{FOL}\rangle \in [\![\textbf{love}]\!]^M_{FOL}\} \\
&= \{g \mid \langle g(x), I(\textbf{mary})\rangle \in I(\textbf{love})\}
\end{aligned}$$

The semantic interpretation of ϕ is a set of assignment functions. Each element in the set maps variable x to an individual, and the love relation holds between the mapped individual and the individual denoted by constant **mary**. In fact, this can be viewed as a function from individuals to truth values. Because of that, in FOL, a closed formula, such as 3.9, is called a **sentence**; while a formula which contains free variable(s), such as 3.10, is called a **propositional function** Gamut (1991a).

Although PL and FOL, as we presented above, are helpful in semantic analysis, natural language is much richer than what they can express. For instance, we can talk about modality in natural language, which is a category of linguistic meaning concerned with possibility and necessity. Expressions that are commonly involved include *it is possible that*, *it is necessary that*, *might*, *must*, etc. For instance:

(61) a. Sandy might be home.
b. Sandy must be home. von Fintel (2006)

Different from previous examples we have seen, such as (58), (59), the meaning of sentences in (61) does not depend on the actual state of affairs. For instance, what (61-a) says is that there is a possibility that Sandy is at home, what (61-b) says is that Sandy is at home in all possibilities, they are not concerned with whether Sandy is actually at home or not. Then how can we formally encode the semantics expressed by sentences such as the ones in (61). In order to account for that, we will have to extend the standard logic, namely PL or FOL, with the device of **possible worlds**. This gives rise to the topic of the following subsection: modal propositional logic.

3.1.3 Modal Propositional Logic

Although the notion of **possible world** dates back as early as to Leibniz[4], its modern development doesn't flourish until the 1960s with the works of Hintikka Hintikka (1957, 1961) and Kripke Kripke (1959, 1963), where **possible world semantics** was introduced. Basically, the term possible world semantics is used to designate semantic frameworks making use of the possible world model, in which the interaction, collaboration and transition among all the worlds are of vital importance. It is an extension of the standard model-theoretic semantics: there is a complete set of model configurations independently in each of the possible worlds. But what is a possible world anyway? Generally speaking, a possible world is a different yet complete way that the world might have been, simply a possibility, as in the literature:

> *(possible worlds are) simply alternative ways things might have been, with "things" construed very broadly as to include everything.* Abbott (2010)

In this thesis, we are not interested in the philosophical debates around possible worlds, such as whether they really exist or not[5]. Instead, we shall directly use this device as a vehicle to investigate modality, namely possibility, necessity, and other related notions.

Turning to linguistic examples, assume W is the set of all possible worlds, (61-a) is true iff there exists a possible world $w \in W$ such that Sandy is at home in it; (61-b) is true iff for any possible world $w \in W$, Sandy is at home in it. Hence the notion of meaning is now world-dependent: the truth value of a proposition is not absolute, it depends on the world where its truth is evaluated. In addition, modal expressions such as *might* and *must* can be considered as quantifiers which locate propositions (e.g., *Sandy is home*) in the space of the possible worlds set W: the former corresponds to an existential quantification, the latter to a universal quantification.

However, things are not quite so simple. If we understand the modal expressions in this way, we will run into trouble when dealing with modalized sentences, for instance:

(62) It is not possible for pigs to fly. Holton (2004)

[4]The term "possible world" is attributed to the phrase *the best of all possible worlds* (*le meilleur des mondes possibles* in French) in Leibniz's 1710 work on Theodicy. For the original French version, please refer to Leibniz (1840), for a translated version in English, please refer to Leibniz (2006).

[5]For a comprehensive discussion on this topic, please refer to Lewis (1986).

According to our above analysis, (62) is true iff there is no possible world $w \in W$ where pigs fly. However, this is not what we mean by uttering (62). In fact, one can always imagine a possible world where pigs do fly, e.g., in a fairy story, or in the outer space, but this does not prevent people from considering (62) to be true. The reason is that when uttering modalized sentences such as (61) and (62), some possible worlds in W are ignored. For instance, if (62) is to be interpreted true, then we confine ourselves to a subset of W, where the actual laws of physics hold.

Hence when talking about modality, there are some worlds which we can reach while others which we can not. This intuition is formalized as the **accessibility relation**, according to which the worlds under consideration are **accessible** from us, those out of the consideration are **inaccessible** from us. We've mentioned that modal operators can be seen as quantifiers over possible worlds, then accessibility relations function to restrict the set of possible worlds as the domain of quantification. As a result, on the one hand, the meaning of a modalized sentence depends on the modal expression, which provides the type of quantification; on the other hand, it depends on the interpretation of modally governed proposition in those possible worlds which are accessible from the **evaluation world**, namely the world where the sentence is uttered. We will come back to it in more detail in Chapter 7.

In summary, what one can deal with in possible world semantics, while not in the standard model-theoretic semantics such as PL and FOL, is the notion of modality (possibility and necessity). For the rest of this subsection, we present a logical system based on possible world: Modal Propositional Logic (MPL), which is an extension of the standard PL as introduced in section 3.1.1. As usual, we will start with the syntax and then proceed to its semantics.

The vocabulary of MPL enriches the one of PL with two additional symbols $\Diamond$, $\Box$: the former is called the **possibility modal operator**, the latter the **necessity modal operator**. Thus we append the following item to definition 3.1.1.

3. Modal operators: $\Diamond$, $\Box$.

Both of the two novel symbols: $\Diamond$ and $\Box$, are sentential/propositional operators. They take sentences to form new sentences. Hence, the syntax of PL is also a subset of the one of MPL, which is defined by appending the following item to definition 3.1.2

4. $(\Diamond\phi), (\Box\phi) \in \mathbb{F}$, whenever $\phi \in \mathbb{F}$.

The notations for alphabets and formulas are the same as before, see notation 3.1.1 and 3.1.2. The conventions to omit parentheses are the same as in PL, given that the modal operators have the same precedence as $\neg$.

Some other classical logical constants, such as $\rightarrow$ (implication) and $\vee$ (disjunction), are defined in exactly the same way as before, see formula 3.1 and 3.2. In addition, we can mutually define the two modal operators $\Diamond$ and $\Box$ as duals, with the primitive $\neg$:

$$\Diamond\phi \triangleq \neg(\Box(\neg\phi)) \tag{3.11}$$

$$\Box\phi \triangleq \neg(\Diamond(\neg\phi)) \tag{3.12}$$

The intuition behind formula 3.11 is that a proposition is possibly true iff it is not the case that it is necessarily false; similarly, the intuition behind formula 3.12 is that a proposition is necessarily true iff it is not the case that it is possibly false. We will verify the above definitions after presenting the semantics of MPL.

Since the notion of possible world is integrated, the semantics of MPL is quite different from that of PL. Before defining the model, which is also called a Kripke Model, we shall introduce the notion of frame.

Definition 3.1.20. A **frame** F of MPL is a pair $\langle W, \mathbf{R}\rangle$, where

1. W is a non-empty set of possible worlds;
2. $\mathbf{R} \subseteq W \times W$ is a binary relation on W, called the **accessibility relation**.

Notation 3.1.6. Lowercase letter w, together with some variants, such as w_1, w_2, ..., w', w''..., will denote possible worlds. We reserve the bold letter $\mathbf{R}$ for the accessibility relation, and $\mathbf{R}(w_1, w_2)$ denotes such a relation that w_2 is accessible from w_1, or equivalently, w_1 is accessible to w_2.

Now we present the notion of interpretation function in MPL.

Definition 3.1.21. An **interpretation function** I is a mapping such that it assigns every propositional variable a truth value at each possible world $w \in W$, namely at each $w \in W$, $I_w : \mathcal{A} \to \{0, 1\}$.

As we can see, different from the interpretation function in PL, see definition 3.1.3, which is a mapping from propositional variables to truth values, the one in MPL is a mapping from pairs of propositional variables and possible worlds to truth values.

Then we can define the model as follows:

Definition 3.1.22. A **(Kripke) model** M of MPL is a pair $\langle F, I\rangle$, where

1. $F = \langle W, \mathbf{R}\rangle$ is a frame;
2. I is an interpretation function.

Finally, the interpretation of a formula in MPL language is provided based upon the above knowledge, which can be seen as an extension of definition 3.1.4.

Definition 3.1.23. Let $M = \langle F, I\rangle$ be a Kripke model, where $F = \langle W, \mathbf{R}\rangle$ is a frame, I is an interpretation function, $w \in W$ a possible world, $\phi \in \mathbb{F}$ an MPL formula. The interpretation of ϕ at w under M, in notation $[\![\phi]\!]^{M,w}_{MPL}$, is defined inductively as follows:

1. $[\![p]\!]^{M,w}_{MPL} = I_w(p)$, if $p \in \mathcal{A}$;
2. $[\![\neg\phi]\!]^{M,w}_{MPL} = 1 - [\![\phi]\!]^{M,w}_{MPL}$;
3. $[\![\phi \wedge \psi]\!]^{M,w}_{MPL} = [\![\phi]\!]^{M,w}_{MPL} * [\![\psi]\!]^{M,w}_{MPL}$, where symbol "$*$" denotes the multiplication function;
4. $[\![\Diamond\phi]\!]^{M,w}_{MPL} = 1$ iff $\exists w' \in W : \mathbf{R}(w, w')$ and $[\![\phi]\!]^{M,w'}_{MPL} = 1$;
5. $[\![\Box\phi]\!]^{M,w}_{MPL} = 1$ iff $\forall w' \in W :$ if $\mathbf{R}(w, w')$ then $[\![\phi]\!]^{M,w'}_{MPL} = 1$.

The notion of truth, satisfiability and validity in MPL are similar as in PL, however, all corresponding notions in MPL are relativized to possible worlds. Compare the following definitions with definition 3.1.5 and 3.1.6.

Definition 3.1.24. Let $M = \langle F, I \rangle$ be a Kripke model, where $F = \langle W, \mathbf{R} \rangle$ is a frame, I is an interpretation function, $w \in W$ a possible world, $\phi \in \mathbb{F}$ an MPL formula. We say that ϕ is **true** at w under M, or equivalently, M **satisfies** ϕ at w, in notation $M, w \models_{MPL} \phi$, iff $[\![\phi]\!]^{M,w}_{MPL} = 1$.

Definition 3.1.25. Let $\phi \in \mathbb{F}$ be an MPL formula, $w \in W$ a possible world. ϕ is **satisfiable** at w iff there is some model M such that $M, w \models_{MPL} \phi$ (otherwise it is **unsatisfiable** at w); ϕ is **valid** at w if for any model M, $M, w \models_{MPL} \phi$ (otherwise it is **invalid** at w).

One remark on the accessibility relation $\mathbf{R}$: as explained above, $\mathbf{R}$ is a way to restrict the worlds over which the modal operators quantify, we can classify accessibility relations into various types, based on the impact that they have on possible worlds.

Definition 3.1.26. Let $F = \langle W, \mathbf{R} \rangle$ be a frame, where W is a set of possible worlds, $\mathbf{R}$ is the accessibility relation.

- $\mathbf{R}$ is **serial** iff $\forall w \in W$: there is a $w' \in W$ such that $\mathbf{R}(w, w')$;
- $\mathbf{R}$ is **reflexive** iff $\forall w \in W : \mathbf{R}(w, w)$;
- $\mathbf{R}$ is **transitive** iff $\forall w, w', w'' \in W$: if $\mathbf{R}(w, w')$ and $\mathbf{R}(w', w'')$ then $\mathbf{R}(w, w'')$;
- $\mathbf{R}$ is **symmetric** iff $\forall w, w' \in W$: if $\mathbf{R}(w, w')$ then $\mathbf{R}(w', w)$;
- $\mathbf{R}$ is **identical** iff $\forall w, w' \in W$: if $\mathbf{R}(w, w')$ then $w = w'$, or equivalently, $\mathbf{R}$ is reflexive, symmetric, and transitive.

Different combinations of the above properties will give rise to different logical systems, which contain their particular theorems. For instance, if $\mathbf{R}$ is reflexive, then $\Box\phi \rightarrow \phi$, $\phi \rightarrow \Diamond\phi$, and $\Box\phi \rightarrow \Diamond\phi$, are tautologies. We shall not go into detail here, see Forbes (1985) for more information.

In definition 3.1.23, we have directly given the semantic interpretations of the two modal operators. However, as we mentioned before, the two operators can be mutually defined, see formula 3.11 and 3.12. Now we would like to spell out the semantics of modal operators from their mutual definitions, and see whether it corresponds to the one we provided. Assume ϕ is a MPL formula, if we define $\Diamond\phi$ as $\neg(\Box(\neg\phi))$, then the interpretations of the two should coincide. Let M be a model, $w \in W$ a possible world, then:

$$[\![\Diamond\phi]\!]^{M,w}_{MPL} = [\![\neg(\Box(\neg\phi))]\!]^{M,w}_{MPL} = 1 - [\![\Box(\neg\phi)]\!]^{M,w}_{MPL}$$

Hence

$$\begin{aligned}
[\![\Diamond\phi]\!]^{M,w}_{MPL} = 1 \text{ iff } & 1 - [\![\Box(\neg\phi)]\!]^{M,w}_{MPL} = 1 \\
\text{iff } & [\![\Box(\neg\phi)]\!]^{M,w}_{MPL} = 0 \\
\text{iff } & \text{it is not the case that } \forall w' \in W: \text{ if } \mathbf{R}(w, w') \text{ then } [\![\neg\phi]\!]^{M,w'}_{MPL} = 1 \\
\text{iff } & \text{it is not the case that } \forall w' \in W: \text{ if } \mathbf{R}(w, w') \text{ then } [\![\phi]\!]^{M,w'}_{MPL} = 0 \\
\text{iff } & \exists w' \in W : \mathbf{R}(w, w') \text{ and it is not the case that } [\![\phi]\!]^{M,w'}_{MPL} = 0 \\
\text{iff } & \exists w' \in W : \mathbf{R}(w, w') \text{ and } [\![\phi]\!]^{M,w'}_{MPL} = 1
\end{aligned}$$

As a result, the definition 3.11 does correspond to the expected semantics of the possibility modal operator $\Diamond$, given that $\Box$ is provided. A similar test can be trivially done for 3.12, which we will carry out here.

As an illustration for MPL, we will use example (62). Assume the proposition expressed by *pigs fly* is ϕ, then the propositions expressed by (62) is $\neg(\Diamond\phi)$, we abbreviate it ψ. Let M be a model, $w \in W$ a possible world, its interpretation is as follows:

$$[\![\psi]\!]^{M,w}_{MPL} = [\![\neg(\Diamond\phi)]\!]^{M,w}_{MPL} = 1 - [\![\Diamond\phi]\!]^{M,w}_{MPL}$$

Hence

$$\begin{aligned}
[\![\psi]\!]^{M,w}_{MPL} = 1 \text{ iff } & 1 - [\![\Diamond\phi]\!]^{M,w}_{MPL} = 1 \\
\text{iff } & [\![\Diamond\phi]\!]^{M,w}_{MPL} = 0 \\
\text{iff } & \text{it is not the case that } \exists w' \in W : \mathbf{R}(w, w') \text{ and } [\![\phi]\!]^{M,w'}_{MPL} = 1 \\
\text{iff } & \nexists w' \in W : \mathbf{R}(w, w') \text{ and } [\![\phi]\!]^{M,w'}_{MPL} = 1
\end{aligned}$$

If we consider the accessible worlds are those where the actual laws of physics hold, then Sentence (62) is true at the current world w iff there is no possible worlds accessible from w, namely the ones where the actual laws of physics hold, and pigs fly at them.

A final remark on propositions: the propositions in standard model-theoretic frameworks, such as PL and FOL, and in MPL, are not the same semantic object. In PL and FOL, a proposition is interpreted as a truth value with respect to a model and an assignment function. However, the truth of a proposition in MPL is additionally relative to a possible world. Hence, the logical connectives are different in a corresponding way, for instance, the negation operator $\neg$ in PL and FOL takes a truth value and returns another truth value, while in MPL, it takes a truth value relative to a possible world, and returns another truth value relative to a possible world.

As we mentioned at the beginning, the above presented formal framework MPL is an extension of the standard PL. For a similar extension on predicate logic, please refer to Chapter 3 of Gamut (1991b), we will not go into its details. In the next section, we will present another formal system: simply typed λ-calculus, which has been extensively employed in natural language analysis, in particular in the field of semantics.

3.2 Simply Typed λ-Calculus

What is usually called λ-calculus is a formal system based on the notation introduced by Alonzo Church in the 1930s. Generally speaking, it is a system for manipulating functions as expressions. It is designed to describe the most basic ways that operators or functions can be combined to form other operators in a purely syntactic manner.

Although it was originally introduced to provide a foundation for mathematics, λ-calculus has a great impact in the development of computer science, in particular, the semantics of programming languages. Since the influential work of Montague, λ-calculus has been popularized as the major tool for analyzing natural language semantics. In this thesis, we shall use it in the same way as Montague, namely to construct semantic representations of natural language expressions. In the previous section, we provide the logical formulas to the corresponding natural language sentences simply based on our understanding of both language systems, there are no stepwise translation processes. However, with the help of λ-calculus, it is possible to establish formulas in logical languages in a compositional fashion, namely based on the syntactic information and the semantic representations of the components. This enables us to automate the process of associating semantic representations with expressions of natural language.

Various versions of λ-calculus mainly fall into two categories: **untyped** and **typed**. The latter restricts the former by imposing a type system, which associates types with certain λ-terms according to some typing rules. And it is the typed version that has been used by Montague in the analysis of natural language semantics. In this thesis, we will follow Montague, and focus on the typed one, in particular the simply typed λ-calculus, which, as its name implies, is one of the simplest among all typed λ-calculus.

For a better introduction on the simply typed λ-calculus, we break it down into two parts: the language of terms, and the language of types. Orderly, the two components will be discussed in the following two subsections. The presentation in this section mainly bases on Roger and Seldin (1986), some other comprehensive references on λ-calculus include Barendregt (1984); Barendregt et al. (2013); Girard et al. (1989).

3.2.1 Untyped λ-Calculus

In this subsection, we will focus on the untyped λ-calculus, this means, terms will be dealt with regardless of types. Same as before, we start introducing the syntax by giving the vocabulary.

Definition 3.2.1. The alphabet of simply typed λ-calculus consists of the following symbols:

1. Variables: x, y, z, ..., x_1, x_2, x_3, ...;
2. Constant symbols: $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, ..., $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, ...;
3. The abstraction symbol: λ;
4. The projection selectors: π_1, π_2.

Notation 3.2.1. We use $\mathcal{X}$ to denote the infinite set of variables; $\mathcal{C}$ to denote the countable set of constants symbols. Lowercase letters x, y, z will denote variables; lowercase bold letters $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$ will denote constant symbols.

The syntax of STLC, namely the way to form valid λ-terms, is provided below.

Definition 3.2.2. The set of λ**-terms**: Λ, is inductively defined as follows:

1. $x \in \Lambda$, whenever $x \in \mathcal{X}$;
2. $\mathbf{a} \in \Lambda$, whenever $\mathbf{a} \in \mathcal{C}$;
3. $(MN) \in \Lambda$, whenever $M, N \in \Lambda$;
4. $(\lambda x.M) \in \Lambda$, whenever $x \in \mathcal{X}, M \in \Lambda$;
5. $\langle M, N\rangle \in \Lambda$, whenever $M, N \in \Lambda$;
6. $(\pi_i M) \in \Lambda$, whenever $M \in \Lambda$, $i \in \{1, 2\}$.

Terms constructed from rule 1 and 2 are called **atoms**, terms constructed from rule 3 are called **(function) applications**, terms constructed from rule 4 are called **abstractions**, terms constructed from rule 5 are called **products**.

Notation 3.2.2. We use uppercase letters, such as M, N, P, Q to denote λ-terms.

In real practice, we omit brackets in λ-terms with respect to the following conventions.

Notation 3.2.3. First of all, we leave off the outermost parenthesis. Then, each sort of terms has its own rule.

For applications, parentheses will be omitted according to the "association to the left" convention, for instance:

- $MNPQ$ denotes $(((MN)P)Q)$.

For abstractions, parentheses, as well as λ-operators, will be omitted according to the "association to the right" convention, for instance:

- $\lambda xyz.M$ denotes $\lambda x.(\lambda y.(\lambda z.M))$.

For products, angle brackets will be omitted according to the "association to the right" convention, for instance:

- $\langle M, N, P, Q\rangle$ denotes $\langle M, \langle N, \langle P, Q\rangle\rangle\rangle$.

Just as in FOL, the notion of free variable can be employed for λ-terms as well. However, it is the λ-operator, rather than the quantifiers, which serves as binder.

Definition 3.2.3. The set of **free variables** of a λ-term $M \in \Lambda$, in notation $FV(M)$, is inductively defined as follows:

1. $FV(x) = \{x\}$;
2. $FV(\mathbf{a}) = \emptyset$;
3. $FV(MN) = FV(M) \cup FV(N)$;
4. $FV(\lambda x.M) = FV(M) - \{x\}$;
5. $FV(\langle M, N\rangle) = FV(M) \cup FV(N)$;
6. $FV(\pi_i M) = FV(M)$.

M is called a **closed λ-term** if $FV(M) = \emptyset$.

The λ-operator enables us to establish new terms from existing ones by abstracting over variables. And it allows us to hold out positions within a λ-term and to fill these positions later with some new terms. In other words, this means we can apply arguments to λ-abstractions, and occurrences of bound variables will get substituted correspondingly.

To illustrate that, we will present two fundamental operations: α-conversion and β-reduction, which are defined in λ-calculus for the purpose of automatic computation. Basically, the former provides a renaming rule, which changes the name of a bound variable in a λ-term, the latter provides a simplification rule, which reduces a λ-term. In what follows, we will first introduce the concept of substitution, then α-conversion and β-reduction.

Definition 3.2.4. Let M, N be λ-terms, x a variable. $[N/x]M$ is defined as the result of the operation that **substitutes** N for every free occurrence of x in M, and changes bound variables to avoid clashes. Then **substitution** is inductively defined on M as follows:

1. $[N/x]x = N$, for any $x \in \mathcal{X}$;
2. $[N/x]y = y$, for any $y \in \mathcal{X}$ such that $y \neq x$;
3. $[N/x]\mathbf{a} = \mathbf{a}$, for any $\mathbf{a} \in \mathcal{C}$;
4. $[N/x](PQ) = ([N/x]P)([N/x]Q)$;
5. $[N/x](\lambda x.P) = (\lambda x.P)$;
6. $[N/x](\lambda y.P) = \lambda y.[N/x]P$, if $y \neq x$, and $y \notin FV(N)$;
7. $[N/x](\lambda y.P) = \lambda z.[N/x][z/y]P$, if $y \neq x$, and $y \in FV(N)$, where z is chosen to be the first variable that is not in $FV(NP)$.

For instance, assume we have a λ-term $M = \lambda x.xy$, where x and y are variables. Then based on definition 3.2.4, the result of $[z/x]M$, $[z/y]M$ and $[x/y]M$ are computed as follows, respectively:

$$\begin{aligned}
[z/x]M &= [z/x]\lambda x.xy \\
&= \lambda x.xy && \text{by rule 5}
\end{aligned}$$

$$\begin{aligned}
[z/y]M &= [z/y]\lambda x.xy \\
&= \lambda x.[z/y](xy) && \text{by rule 6 since } x \notin FV(z) \\
&= \lambda x.([z/y]x)([z/y]y) && \text{by rule 4} \\
&= \lambda x.x([z/y]y) && \text{by rule 2 since } x \neq y \\
&= \lambda x.xz && \text{by rule 1}
\end{aligned}$$

$$\begin{aligned}
[x/y]M &= [x/y]\lambda x.xy \\
&= \lambda z.[x/y][z/x](xy) && \text{by rule 7 since } x \notin FV(x) \\
&= \lambda z.[x/y]([z/x]x)([z/x]y) && \text{by rule 4} \\
&= \lambda z.[x/y](zy) && \text{by rule 1 and 2} \\
&= \lambda z.([x/y]z)([x/y]y) && \text{by rule 4} \\
&= \lambda z.zx && \text{by rule 1 and 2}
\end{aligned}$$

Below, we will define the two operations which transform λ-terms.

Definition 3.2.5. Let P be a λ-term which contains an occurrence of $\lambda x.M$, and let $y \notin FV(M)$. An **α-conversion** in P is the act of replacing this $\lambda x.M$ by $\lambda y.[y/x]M$.

We say P **α-converts** to Q, in notation $P =_\alpha Q$, iff P can be changed to Q by a finite (perhaps empty) steps of α-conversion.

For instance, $\lambda x.xy =_\alpha \lambda z.zy$. The α-conversion admits the changing of bound variables as long as there is no capture of a free variable occurrence. Any λ-term and its α-converted counterpart can be considered as a pair of twins in λ-calculus, namely they are variant terms containing exactly the same information. The simplification operation, which is called β-contraction, is defined as follows:

Definition 3.2.6. Any term of form $(\lambda x.M)N$ is called a β**-redex**. The corresponding term $[N/x]M$ is called its **contractum**.

Iff a term P contains an occurrence of $(\lambda x.M)N$ and we replace that occurrence by $[N/x]M$, and the result is P', we say we have contracted the redex-occurrence in P, and P β**-contracts** to P', in notation $P \rightarrow_\beta P'$.

Iff P can be changed to a term Q by a finite (perhaps empty) steps of β-contractions and changes of bound variables, we say P β**-reduces** to Q, in notation $P \twoheadrightarrow_\beta Q$.

For instance, based on definition 3.2.6, the following λ-terms, where β-redexes are contained, can be simplified to the corresponding reduced forms:

- $(\lambda x.y)M \rightarrow_\beta y$
- $(\lambda x.xy)M \rightarrow_\beta My$
- $(\lambda x.(\lambda y.xy)x)z \rightarrow_\beta (\lambda y.zy)z \rightarrow_\beta zz$

The β-reduction terminates only when there are no redexes left in the term (called a β**-normal form**). It is the case for all the above examples, but sometimes, the reduction process will last forever without reaching a β-normal form. For instance, if we apply the term $\lambda x.xx$ to itself:

$$\begin{aligned}(\lambda x.xx)(\lambda x.xx) &\rightarrow_\beta [(\lambda x.xx)/x](xx) = (\lambda x.xx)(\lambda x.xx)\\ &\rightarrow_\beta [(\lambda x.xx)/x](xx) = (\lambda x.xx)(\lambda x.xx)\\ &\ldots\end{aligned}$$

As shown in the above definition, the rule of β-reduction essentially encompasses the process of function application, in other words, plugging arguments into functions. For instance, in the β-redex $(\lambda x.M)N$, $\lambda x.M$ is the function, N is the argument or parameter. By applying the former to the latter, we substitute all occurrences of variable x in M with N.

So far until now, we have generally introduced the language of terms in the simply typed λ-calculus, namely the untyped λ-calculus. There is no restriction on the usage of objects, e.g., any arbitrary term can serve as a function or an argument. However, this is undesirable in many fields of study. For instance, in mathematics, the trigonometric functions (e.g., sine, cosine, tangent) can only be applied to angles, it does not make any sense if we input a number; also, in programming languages, most functions are particular designed for some certain data type, such as the built-in function `reverse` in Python, which only works on lists, but not on other data structures like tuples or dictionaries. Hence, in the following context, we will present the second component of the simply typed λ-calculus: the type system, which ensures that operations are only applied to appropriate objects.

3.2.2 The Language of Types

As we mentioned, the simply typed λ-calculus restricts the untyped λ-calculus by incorporating a notion of type. This subsection is concerned with the language of types. We start the introduction by giving rules which determine the proper forms of types.

Definition 3.2.7. Assume we have a sequence of symbols called **atomic types**. The set of types: T, is inductively defined as follows:

1. every atomic type is a type;

2. $(\sigma \to \tau) \in T$, where $\tau, \sigma \in T$, called a **function type**;

3. $(\sigma \times \tau) \in T$, where $\tau, \sigma \in T$, called a **product type**.

Notation 3.2.4. As used in definition 3.2.7, lowercase greek letters, such as γ, τ, σ, ρ, and etc., will denote types.

The intuition behind a function type is that, when a term of type $(\sigma \to \tau)$ is applied to a term of σ, the result we obtain is another term of τ; the intuition behind a product type is relatively more straightforward, a term of type $(\sigma \times \tau)$ is an ordered pair $\langle M, N \rangle$, where M is of type σ and N is of type τ. Same as for λ-terms, we leave out unnecessary brackets when writing types.

Notation 3.2.5. Above all, outermost parentheses will be omitted. In addition, for both function types and product types, in case that parentheses are omitted, the constituent types are grouped from the right, for instance:

- $\sigma \to \tau$ denotes $(\sigma \to \tau)$;

- $\sigma_1 \to ... \to \sigma_n \to \tau$ denotes $(\sigma_1 \to (... \to (\sigma_n \to \tau)...))$;

- $\sigma \times \tau$ denotes $(\sigma \times \tau)$;

- $\sigma_1 \times ... \times \sigma_n \times \tau$ denotes $(\sigma_1 \times (... \times (\sigma_n \times \tau)...))$.

In what follows, we will define the notion of higher order signature De Groote (2001), where a finite set of constants are declared such that each of them is assigned a unique type. This makes the framework modular because we are able to reuse the technical core of the simply typed λ-calculus with different signatures. We will see some specific signature example at the end of this subsection.

Definition 3.2.8. A **Higher Order Signature** (**HOS**) Σ is a triple $\langle T_A, \mathcal{C}, t \rangle$, where:

- T_A is a finite set of atomic types, from which the set of types for Σ, in notation T_Σ, is built according to definition 3.2.7;

- $\mathcal{C}$ is a finite set of constant symbols;

- $t : \mathcal{C} \to T_\Sigma$ is a function that assigns to each constant in $\mathcal{C}$ a type in T_Σ.

As we mentioned, in the simply typed λ-calculus, each term is associated to a particular type. The type of a constant is provided in the signature, then how about the types of other terms? In fact, these types are properly assigned through a set of **typing rules**. We will first introduce some basic notion such as typing assumption, typing context, then present the typing rules. In the following context, the colon notation $M : \tau$ is used to mean that the λ-term M is of type τ.

Definition 3.2.9. Let $x \in \mathcal{X}$ be a variable, $\tau \in T$ a type. A **typing assumption**, in notation $x : \tau$, is a statement indicating that x is of type τ.

Definition 3.2.10. A **typing context** Γ is a set of typing assumptions such that for all variable $x \in \mathcal{X}$, if $x : \sigma \in \Gamma$ and $x : \tau \in \Gamma$ then $\sigma = \tau$.

Definition 3.2.11. Let Γ be a typing context, $M \in \Lambda$ be a λ-term, $\tau \in T$ a type. A **typing judgement**, in notation $\Gamma \vdash M : \tau$, is a statement indicating that term M is of type τ in context Γ.

Definition 3.2.12. Let Γ be a typing context. A typing judgement $\Gamma \vdash M : \tau$ is valid if it is derived through the following formal system, or equivalently, by obeying the set of **typing rules**:

1. $$\frac{x : \sigma \in \Gamma}{\Gamma \vdash x : \sigma}$$

2. $$\frac{\mathbf{a} \text{ is of type } \sigma}{\Gamma \vdash \mathbf{a} : \sigma}$$

3. $$\frac{\Gamma \vdash M : \sigma \to \tau \quad \Gamma \vdash N : \sigma}{\Gamma \vdash (MN) : \tau}$$

4. $$\frac{\Gamma, x : \sigma \vdash M : \tau}{\Gamma \vdash (\lambda x.M) : \sigma \to \tau}$$

5. $$\frac{\Gamma \vdash M : \sigma \quad \Gamma \vdash N : \tau}{\Gamma \vdash \langle M, N \rangle : (\sigma \times \tau)}$$

6. $$\frac{\Gamma \vdash M : (\sigma \times \tau)}{\Gamma \vdash \pi_1 M : \sigma} \qquad \frac{\Gamma \vdash M : (\sigma \times \tau)}{\Gamma \vdash \pi_2 M : \tau}$$

M is a **well-formed λ-term**, or equivalently, M is **typable**, if a typing judgement $\Gamma \vdash M : \tau$, indicating that the type of M is τ, can be derived using the above rules.

As one might notice, not all λ-terms are typable. For instance, xx and $\lambda x.xx$ are not well-formed since according to definition 3.2.10, a typing context will assign each variable a unique type. While when trying to type xx or $\lambda x.xx$, we will end up with different types for x. In addition, we are unable to assign types to terms such as $\pi_1(\lambda x.M)$ and $\langle M, N \rangle x$ as well. In STLC, every expression is always specified with a unique type, hence untypable expressions like the above are considered meaningless and should be avoided in the simply typed λ-calculus.

Up until now we have introduced the untyped λ-calculus and the language of types, whose incorporation will result in the simply typed λ-calculus. In order to examine most natural language examples presented so far, it is sufficient to do with the following signature Σ_0.

Definition 3.2.13. The signature Σ_0 is defined as follows:

$$\Sigma_0 = \langle \{\iota, o\}, \{\wedge, \neg, \exists\}, \{\wedge : o \to o \to o, \neg : o \to o, \exists : (\iota \to o) \to o\} \rangle$$

The two atomic types, ι and o, were first proposed in Church's simple theory of types Church (1940), they denote the type of individuals and the type of truth values (propositions), respectively. The set of all types in Σ_0, in notation T_{Σ_0}, can be constructed from the two primitive types based on definition 3.2.7. For instance, $\iota \to o$ is the type of sets of individuals (properties), when an expression of this type, such as **man**, **run**, is applied to an individual, a truth value will be returned (1 if the individual belongs to the set, 0 otherwise); $\iota \to \iota \to o$ is the type of 2-place predicates, when an expression of this type, such as **love**, **beat**, is applied to an individual, a property, as described above will be returned. Note that in the constant set of Σ_0, we only list three elements: $\neg$, $\wedge$ and $\exists$, they are called **logical constants**, in notation $\mathcal{C}_L$; other constants, such as **man**, **love**, etc., which are not explicitly specified in Σ_0, are called **non-logical constants**, in notation $\mathcal{C}_{NL}$. That is to say, $\mathcal{C} = \mathcal{C}_L \cup \mathcal{C}_{NL}$. Those non-logical constants will be declared on-site when they appear in the future illustrations.

In subsequent contexts, whenever we proceed to frameworks based on the simply typed λ-calculus, we only need to identify their particular signatures. Other notions such as λ-terms, free variables, closed λ-terms, substitution, α-conversion, and β-conversion will be taken for granted.

3.2.3 Semantics

Now let's have a look at the semantics of the simply typed λ-calculus, which appears to be a bit different from, but is essentially a more general form of the semantics of the logic systems in the previous section.

Definition 3.2.14. A **meaning function** is a mapping $\mathcal{M}$ assigning each atomic type τ a set $\mathcal{M}(\tau)$, which is called the **domain** of τ.

Take Σ_0 for instance, $\mathcal{M}(\iota) = D_\iota$, which is a set of individuals; $\mathcal{M}(o) = D_o = \{1, 0\}$, which is the set of truth values.

Definition 3.2.15. Let $\tau, \sigma \in T$ be types. The interpretation of τ, in notation $[\![\tau]\!]_\lambda$ is defined as follows:

- $[\![\tau]\!]_\lambda = \mathcal{M}(\tau)$, if τ is an atomic type;
- $[\![\sigma \to \tau]\!]_\lambda = [\![\tau]\!]_\lambda^{[\![\sigma]\!]_\lambda}$;
- $[\![\sigma \times \tau]\!]_\lambda = [\![\sigma]\!]_\lambda \times [\![\tau]\!]_\lambda$.

On the right hand side of the above formulas, notation B^A denotes the set of all functions from A to B, notation $A \times B$ denotes the set of all ordered pairs whose elements are from A and B respectively.

Definition 3.2.16. Let Γ be a typing context. A **model** on Γ is a pair $M = \langle D, I\rangle$ such that:

1. D is a family of D_τ, for every atomic type τ, called the **domain**;

2. I is an **interpretation function** such that $I(\mathbf{a}) \in [\![\tau]\!]_\lambda$, for every $\mathbf{a} \in \mathcal{C}$ and $\Gamma \vdash \mathbf{a} : \tau$.

Definition 3.2.17. Let Γ be a typing context. An **assignment function** on Γ is a mapping g such that $g(x) \in [\![\tau]\!]_\lambda$, for every $x \in \mathcal{X}$ and $\Gamma \vdash x : \tau$.

For the conventional notation on assignment functions, see notation 3.1.5. In addition, assume $h, g \in \mathcal{G}$ are assignment functions, the notation $h[X]g$ is used in the same way as in FOL, for its meaning, see definition 3.1.15. Finally, we can define the semantics of the simply typed λ-calculus, namely the interpretation of λ-terms.

Definition 3.2.18. Let Γ be a typing context, $M = \langle D, I\rangle$ a model, g an assignment function. The interpretation of a λ-term N in M with respect to g, such that $\Gamma \vdash N : \tau$, in notation $[\![N]\!]_\lambda^{M,g}$, is defined inductively as follows:

1. $[\![x]\!]_\lambda^{M,g} = g(x)$;

2. $[\![\mathbf{a}]\!]_\lambda^{M,g} = I(\mathbf{a})$;

3. $[\![N_1 N_2]\!]_\lambda^{M,g} = [\![N_1]\!]_\lambda^{M,g}([\![N_2]\!]_\lambda^{M,g})$, where $\Gamma \vdash N_1 : \sigma \to \tau$ and $\Gamma \vdash N_2 : \sigma$, the notation $A(B)$ denotes the result of passing B to A, or applying A to B;

4. $[\![\lambda x.N]\!]_\lambda^{M,g} \in [\![\tau]\!]_\lambda^{[\![\sigma]\!]_\lambda}$ is the function f such that for all $d \in D_\sigma$: $f(d) = [\![N]\!]_\lambda^{M,h}$, where h is an assignment function such that $h(x) = d$ and $h[\{x\}]g$;

5. $[\![\langle N_1, N_2\rangle]\!]_\lambda^{M,g} \in [\![\sigma]\!]_\lambda \times [\![\tau]\!]_\lambda$ such that $[\![\langle N_1, N_2\rangle]\!]_\lambda^{M,g} = \langle [\![N_1]\!]_\lambda^{M,g}, [\![N_2]\!]_\lambda^{M,g}\rangle$, where $\Gamma \vdash N_1 : \sigma$ and $\Gamma \vdash N_2 : \tau$, the notation $\langle a, b\rangle$ denotes the ordered pair whose elements are a and b respectively;

6. $[\![\pi_i N]\!]_\lambda^{M,g} = [\![N_i]\!]_\lambda^{M,g}$, where $i \in \{1, 2\}$, $\Gamma \vdash N : \sigma \times \tau$, N_i denotes the i-th element in N.

Focusing on the semantics of specific constants, such as $\wedge$, $\neg$ and $\exists$ in Σ_0, we can assign them the following particular interpretations:

Definition 3.2.19. The logical constants in Σ_0 (i.e., $\wedge$, $\neg$ and $\exists$) are interpreted as follows:

1. $I(\wedge) \in ([\![o]\!]_\lambda^{[\![o]\!]_\lambda})^{[\![o]\!]_\lambda}$ is the function that maps $\langle 1, 1\rangle$ to 1, and other pairs of truth values to 0;

2. $I(\neg) \in [\![o]\!]_\lambda^{[\![o]\!]_\lambda}$ is the function that maps 0 to 1, and 1 to 0;

3. $I(\exists) \in [\![o]\!]_\lambda^{([\![o]\!]_\lambda^{[\![\iota]\!]_\lambda})}$ is the function that maps a function $f \in [\![o]\!]_\lambda^{[\![\iota]\!]_\lambda}$ to 1 iff there is a $d \in D_\iota$ such that $f(d) = 1$.

As we can see, the above definition ensures that the logical constants are interpreted exactly the same as in FOL. As we said at the beginning of this section, λ-calculus enables us to construct semantic representations of complex expressions in a compositional way. To illustrate that, we will again take Sentence (59) as an example.

For the syntax of natural language, we will employ a simpler version of the Categorial Grammar developed in Lambek (1958)[6]. Basically, each linguistic item is associated with a syntactic category. The set of categories are defined in a similar way as types in section 3.2.2, see definition 3.2.7: there is a finite set of primitive syntactic categories, for instance, n (noun), np (noun phrase) and s (sentence), other categories are established from them through the only constructor $\rightarrow$, they are thus called derived categories, for instance, $n \rightarrow np$ (determiner), $np \rightarrow s$ (intransitive verb), $np \rightarrow np \rightarrow s$ (transitive verb), etc. With the above categories, we can construct grammatical expressions based on the following syntactic rule/type inference rule, which is similar to Rule 3 in definition 3.2.12: if A is an expression of category α, B is an expression of category $\alpha \rightarrow \beta$, then the function application BA is an expression of category β. If the categories of A and B do not conform to the above rule, the application BA will otherwise be an ungrammatical expression. In this way, logic indeed serves as a grammatical formalism.

Turn to a particular example, the parse tree of (59), with the categorial information marked at each node, is shown as in figure 3.2. With the specific type indicated for each elementary lexical item, we can see that *every man* is a grammatical expression of category np, similarly, *loves Mary* is a grammatical expression of category $np \rightarrow s$, finally, the whole sentence is a grammatical expression of category s.

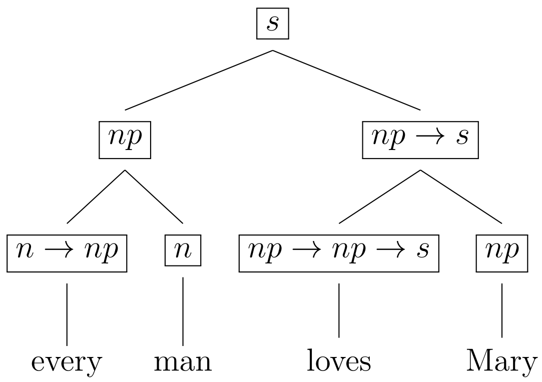


Fig. 3.2 Syntactic Tree of Example (59)

One important theoretical advantage of employing the type-theoretic grammar as our syntactic formalism is that the Curry-Howard correspondence, which holds between syntactic categories and semantic types, or equivalently, between proofs and λ-terms, can be adapted to describe the strict correspondence between syntax and semantics van Benthem (1986, 1988), because the syntactic derivation is simply a logical proof. Thus, by parsing a sentence, we automatically restrict how the semantic recipes, expressed as

[6] The Categorial Grammar developed by Lambek is order-sensitive. It contains two function types, namely $\alpha \backslash \beta$ and β / α, where α and β are syntactic categories. The former is applied to argument on the left, the latter is applied to argument on the right. In this thesis, we shall use the order-insensitive version by confining ourselves only with standard function type (i.e., $\alpha \rightarrow \beta$). The advantage is that we can directly use λ-terms at the syntactic level.

typed λ-terms, are combined, and this results the semantic representation for the whole sentence.

In what follows, we give the semantic lexical entry (a typed λ-term) for each expression in example (59). Notation $[\![\ ]\!]$ is used to indicate the logical representation corresponding to the expression within the bracket:

$$\begin{aligned}
[\![\mathit{every}]\!] &= \lambda PQ.\forall x.(P(x) \to Q(x)) \\
[\![\mathit{man}]\!] &= \lambda x.\mathbf{man}\ x \\
[\![\mathit{love}]\!] &= \lambda OS.S(\lambda x.O(\lambda y.\mathbf{love}\ x\ y)) \\
[\![\mathit{Mary}]\!] &= \lambda P.P(\mathbf{mary})
\end{aligned}$$

Note that all the above λ-terms are well-typed. The three non-logical constants, which are not specified in definition 3.2.13, contain the following typing information: $\mathbf{mary} : \iota$, $\mathbf{man} : \iota \to o$, and $\mathbf{love} : \iota \to \iota \to o$. As a result, the semantic type of each lexical entry is as follows:

$$\begin{aligned}
[\![\mathit{every}]\!] &: (\iota \to o) \to (\iota \to o) \to o \\
[\![\mathit{man}]\!] &: (\iota \to o) \to o \\
[\![\mathit{love}]\!] &: ((\iota \to o) \to o) \to ((\iota \to o) \to o) \to o \\
[\![\mathit{Mary}]\!] &: (\iota \to o) \to o
\end{aligned}$$

Then by referring back to figure 3.2, we know that the logical representation of the NP *every man* can be obtained by applying $[\![\mathit{every}]\!]$ to $[\![\mathit{man}]\!]$, the only operations involved are β-reductions (possibly α-conversions as well):

$$\begin{aligned}
[\![\mathit{every_man}]\!] &= [\![\mathit{every}]\!][\![\mathit{man}]\!] \\
&= \lambda PQ.\forall x.(P(x) \to Q(x))(\lambda x.\mathbf{man}\ x) \\
&\twoheadrightarrow_\beta \lambda Q.\forall x.(\mathbf{man}\ x \to Q(x))
\end{aligned}$$

The representation of the VP *loves Mary* can be obtained in a similar manner:

$$\begin{aligned}
[\![\mathit{loves_Mary}]\!] &= [\![\mathit{love}]\!][\![\mathit{Mary}]\!] \\
&= \lambda OS.S(\lambda x.O(\lambda y.\mathbf{love}\ x\ y))(\lambda P.P(\mathbf{mary})) \\
&\twoheadrightarrow_\beta \lambda S.S(\lambda x.\mathbf{love}\ x\ \mathbf{mary})
\end{aligned}$$

Finally we apply the representation of the VP to that of the subject NP:

$$\begin{aligned}
[\![(59)]\!] &= [\![\mathit{loves_Mary}]\!][\![\mathit{every_man}]\!] \\
&= \lambda Q.\forall x.(\mathbf{man}\ x \to Q(x))(\lambda S.S(\lambda x.\mathbf{love}\ x\ \mathbf{mary})) \\
&\twoheadrightarrow_\beta \forall x.(\mathbf{man}\ x \to \mathbf{love}\ x\ \mathbf{mary})
\end{aligned}$$

As one might see, the last λ-term: $[\![(59)]\!]$, is identical to the FOL translation of example (59) as we gave out of the blue in section 3.1.2. Further more, $[\![(59)]\!]$ is of type o, it will be interpreted as a proposition. Since logical constants in the simply typed λ-calculus (e.g., $\wedge$, $\neg$, $\exists$) are assigned an identical semantics as in FOL (compare definition 3.1.17 and 3.2.19), the interpretation of $[\![(59)]\!]$ is also same as before. For more information, please refer to section 3.1.2.

As a summary, with the simply typed λ-calculus, constituents of a sentence are represented by λ-terms, and function application combines these terms into FOL expressions through β-reduction. In this way, the link between syntax and semantics is captured compositionally: the semantics of each linguistic item is specified by its corresponding λ-term, and the grammatical structure instructs in what way each lexical entry contributes to the final semantics.

Chapter 4

Dynamic Frameworks

Contents

> *Discourse meanings are more than plain conjunctions of sentence meanings. And this 'more' is often the effect of interpretation principles that are an integral part of linguistic knowledge, and thus legitimate objects of linguistic study.* Kamp (2000)

In this chapter, we will mainly focus on **dynamic semantics**, which interprets a sentence in terms of contribution it makes to an existing discourse. We shall start by giving motivations for dynamic semantics, then present two representative frameworks in the field, namely Discourse Representation Theory (DRT) Kamp (1981) and Dynamic Predicate Logic (DPL) Groenendijk and Stokhof (1991). Finally, we will introduce a more recently proposed dynamic framework de Groote (2006), which is based on continuation Strachey and Wadsworth (1974). This framework follows the tradition of MG technically and is completely compositional. It will serve as the technical background of this thesis.

4.1 Why Dynamics?

As we've shown in the previous chapter, classical logical semantics such as MG interprets sentences in terms of their truth conditions. However, despite being a revolutionary work in the field of formal semantics, MG was designed to treat natural language utterances in isolation. Thus linguistic phenomena which cross sentence boundaries, such as inter-sentential anaphora, donkey anaphora, presupposition, etc., are obviously beyond the scope of MG. Let's have a look at the following discourses, which are originally from Barbara Partee:

(63) a. I dropped ten marbles and found all of them, except for one. It is probably under the sofa.
b. I dropped ten marbles and found only nine of them. ?It is probably under the sofa. Heim (1982)

From a truth conditional point of view, the first parts of (63-a) and (63-b) are semantically equivalent, namely the state of affairs described by them are exactly the same: ten marbles were dropped, and nine of them were found, one was missing. However, the acceptability of an identical continuation *it is probably under the sofa* is admitted in (63-a) while disputed in (63-b)[1]. Namely the anaphoric relation is successfully constructed between *it* and the missing marble in the former sentence, while it is not the case in the latter. There is also another pair of similar examples:

(64) a. A delegate arrived. She registered.
b. It is not the case that every delegate failed to arrive. *She registered. Kamp et al. (2011)

As presented in section 3.1.2, in FOL, universal quantifier is defined in terms of existential quantifier and negation (formula 3.5). Since a double negation can be eliminated in standard predicate logic, the following logical equivalence always holds:

$$\exists x.\phi = \neg\forall x.\neg\phi \tag{4.1}$$

From formula 4.1, we can infer that the first sentence of (64-a) and (64-b) are logically equivalent: they have the same truth conditions. However, same as for example (63), the utterance *she registered*, which involves a pronominal anaphor, is a felicitous continuation in (64-a) but is ruled out in (64-b). So the two first sentences ought not to be regarded as identical, at least on the aspect that they have different potentials to license subsequent anaphors. As a result, the truth conditional approach is not rich enough to capture the semantics of sentences such as (63) and (64). This gives rise to a more dynamic notion of meaning relative to context.

In the late 1970s, a new generation of semantic theories, known as **dynamic semantics**[2], emerged for the purpose of investigating discourses rather than isolated sentences. Works in this camp have their roots in various research disciplines, such as formal se-

[1] Geurts (2011) argues that (63-b) is actually not completely unacceptable, the author proposes such a situation where the marbles are of great values, and the topic of the discourse is on the missing marble, hence it is salient in the discourse and a discourse anaphoric relation does not sound that bad. But as far as we are concerned, this hypothesis is too pragmatically biased. In our thesis, we consider (63-b) as infelicitous.

[2] The term is used in contrast to the classical logical semantics such as MG, which is considered to be "static".

mantics, philosophy and logic, pragmatics, computational linguistics, etc. In classical semantic theories, such as MG, the meaning of an expression is its truth conditions. However in dynamic semantics, it is assumed that the interpretation of an expression brings about a change to an existing context. Hence the meaning of the expression is identified with its contribution to the change. In a slogan, meaning is the "context change potential" Heim (1983). It is this new notion of interpretation, which is concerned with some kind of change, that gives dynamic semantics its name.

The idea was inspired from two perspectives. On the one hand, the meaning of a sentence depends on the context where it occurs, this can be exemplified with a wide range of context-dependent phenomena Halliday and Hasan (1976). On the other hand, the sentence in turn enriches or updates the semantics of the context. Thus the resulting context, which incorporates the contribution of the processed sentence, may affect the interpretation of sentences which come after. Consequently, the interaction between sentence and context is reciprocal. In such way, the semantics of a sentence is not what it describes **statically** about the world, rather it is what changes it brings about **dynamically** to the whole context.

One of the most salient context dependent phenomena, anaphora, in particular pronominal anaphora, has been extensively investigated in dynamic semantics. In the rest of this subsection, we will concentrate on two anaphoric phenomena, which illustrate the motivations for the development of dynamic semantics.

4.1.1 Inter-Sentential Anaphora

The first instance we examine is the inter-sentential anaphora, where a pronoun is anaphorically related to an indefinite NP in a preceding sentence. As we discussed in section 2.3.1, from the semantic point of view, a pronoun either co-refers with its antecedent (e.g., example (14), (15), etc.), or it functions as a variable bound by its antecedent (e.g., example (19), (20), etc.). But it seems that neither possibility can account for the anaphoric relation in the following classical example:

(6) $\underline{\text{A man}}_i$ walks in the park. $\underline{\text{He}}_i$ whistles.

Ever since Bertrand Russell, it has been widely acknowledged that indefinite NPs are quantifier phrases (existential) rather than referring expressions[3]. Following Russellian logicians, the anaphora in (6) can not be co-referential. Then the only option left to account for the anaphoric relation in (6) is to translate the pronoun *he* as a bound variable. However, this does not appear to work, either. One has to ensure that the antecedent quantifier phrase precedes and c-commands the anaphor (see section 2.3.2) in order to adopt the bound variable solution. While in the case of (6), the two NPs, *he* and *a man*, are distributed in different sentences. Hence they do not bear the c-commanding relation, which implies that the pronoun can by no means be bound. We will illustrate this with specific representations.

Looking at the two sentences in (6) independently, they can be mapped into the following FOL formulas, respectively (the internal structure of the VP *walk in the park* is abbreviated as a single predicate):

$$\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x) \tag{4.2}$$

[3] Although Russell's theory has been advocated for nearly half a century Heim (1982); Russell (1905, 2008), there are various challenges since the 1950s, in particular Donnellan (1966); Strawson (1950).

$$\textbf{whistle}\ x \tag{4.3}$$

By representing sentence sequencing as conjunction, we can build up the semantic representation of discourse (6) straightforwardly as follows (based on the above two formulas 4.2 and 4.3):

$$(\exists x.(\textbf{man}\ x \land \textbf{walk\_in\_the\_park}\ x)) \land \textbf{whistle}\ x \tag{4.4}$$

In accordance to what we analyzed, the last occurrence of variable x in 4.4 is free. Consequently, the representation will not receive the desired interpretation for (6) under the standard FOL semantics: the anaphoric link between the pronoun and the indefinite is not captured.

Intuitively, the discourse (6) can be paraphrased as either of the following single sentences without changing its meaning:

(65) a. A man who walks in the park whistles
b. A man$_i$ walks in the park and he$_i$ whistles.

So Geach (1962) proposes to assign it the following closed formula as semantic representation:

$$\exists x.(\textbf{man}\ x \land \textbf{walk\_in\_the\_park}\ x \land \textbf{whistle}\ x) \tag{4.5}$$

Contrasting formula 4.4 to 4.5, the scope of the existential quantifier in the latter is extended, the pronoun *he* can thus be treated as a variable bound cross-sententially. This analysis does correspond to what the original discourse expresses. However, the ad-hoc approach, which extends the scope of operators, suffers from some serious problems. Firstly, this approach is based on the assumption that indefinite NPs are quantificational expressions, inter-sentential pronouns are variable-like elements. Then one would expect example (66-a) and (66-b) to be treated in a similar way.

(66) a. * Every dog$_i$ came in. It$_i$ lay down under the table.
b. * No dog$_i$ came in. It$_i$ lay down under the table. Heim (1982)

Namely, according to the scope-extending approach, the semantic representations of the two discourses are respectively:

$$\forall x.((\textbf{dog}\ x) \rightarrow (\textbf{come\_in}\ x \land \textbf{lie\_under\_the\_table}\ x))$$

$$\neg\exists x.(\textbf{dog}\ x \land \textbf{come\_in}\ x \land \textbf{lie\_under\_the\_table}\ x)$$

However, as we can see, neither are the anaphoric links in (66) felicitous, nor are the logical representations proper for the semantics of the corresponding discourses. Hence the scope-extending approach is not general enough. Moreover, we will run into trouble as soon as the following example is considered:

(67) A man$_i$ walks in the park. He$_i$ whistles. He$_i$ smokes.

Discourse (67) is the result of continuing (6) with an utterance, which contains an anaphoric expression referring back to the indefinite *a man*. Based on the representation in 4.5, a step by step discourse processing will yield the following logical translation for example (67):

$$(\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x \wedge \mathbf{whistle}\ x)) \wedge \mathbf{smoke}\ x \quad (4.6)$$

Same as 4.4, formula 4.6 contains a free occurrence of variable, thus it fails to reflect the expected semantics of the extended discourse (67). As a result, regardless of the problem of lacking generality, the scope-extending approach is only feasible when truth conditions are assigned all at once to the entire discourse, rather than little by little to single sentences. This causes a side-effect: compositionality (meaning of a discourse is composed from meanings of its parts) is lost. Following the scope-extending mechanism, we have to ensure that a discourse being terminated before interpreting it. That is to say, sentences can not be interpreted as soon as they are uttered. This does not correspond to the intuitive way of understanding discourses. When a discourse unfolds, its interpretation should be constructed incrementally: every new sentence is updated to an existing piece of interpreted discourse.

As a summary, inter-sentential anaphora with an indefinite as the antecedent, such as (6), can be treated neither as a co-reference nor as a bound variable. This does pose problems to standard logical semantics.

4.1.2 Donkey Sentence

The second phenomenon we will discuss is the so-called **donkey sentence**. Donkey sentences, which are notorious examples in theories of anaphora, date back to a medieval English philosopher Walter Burleigh. As recorded in one of his works on reference theory[4] around 1328:

(68) Omnis homo habens asinum videt illum.
(Every man owning a donkey sees it.)

Example (68) was originally introduced for a problem of another nature: the relative position between an anaphor and its antecedent. From that time on, nobody has paid special attention to this sentence any more. In the middle of the 20th century, another british philosopher, Peter Geach, originated some interesting discussions in linguistics and logics by making use of a sequence of natural language examples with donkeys Geach (1962). Since then, donkey sentences have become famous in the literature of modern semantics attracted extensive attentions from linguists and philosophers.

However, the debate on it has never ended since the first day it was introduced. In modern semantic literatures, donkey sentences are defined as:

> *(Donkey sentences are) sentences that contain an indefinite NP which is inside an if-clause or relative clause, and a pronoun which is outside that if-clause or relative clause, but is related anaphorically to the indefinite NP.* Heim (1982)

The above definition reveals the two canonical forms of donkey sentences that people are familiar with nowadays:

(69) Every farmer who owns $\underline{\text{a donkey}_i}$ beats $\underline{\text{it}_i}$.

[4] The original work is "*De puritate artis logicae (On the Purity of the Art of Logic)*", for more comments, see Seuren (2009).

(7) If a $\underline{\text{farmer}}_i$ owns $\underline{\text{a donkey}}_j$, $\underline{\text{he}}_i$ beats $\underline{\text{it}}_j$.

The two above sentences are known as **the quantified version** (69) and **the conditional version** (7), respectively. They are generally considered the paraphrase of one other, and the linguistic questions which are concerned with them (interaction between anaphora and quantification) are almost overlapping, we will mainly use (7) as an illustration for the rest of this thesis.

At first sight, although donkey anaphora is confined within sentence boundaries, it seems to be problematic in the similar way as the inter-sentential anaphora (6): neither of the two semantic options can account for it. On the one hand, the anaphoric relation can not be co-referential, because the pronoun *it* and the indefinite *a donkey* do not refer to any specific donkey. On the other hand, the pronoun *it* can not be treated a bound variable, because it is not in the scope of the existential quantifier.

Now let's take a closer look at the semantics of example (7), which is the heart of the problem. A naive attempt to translate it in FOL would yield the following representation, where indefinite NPs are unitarily treated as existential quantifiers:

$$(\exists x \exists y.(\mathbf{farmer}\ x \wedge \mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y)) \to (\mathbf{beat}\ x\ y) \tag{4.7}$$

As we can see, same as 4.4 and 4.6, formula 4.7 also contains free occurrences of variable, i.e., x and y in the sub-formula ($\mathbf{beat}\ x\ y$). Hence 4.7 will not reflect the correct semantics of example (7) under the standard FOL interpretation. A potential remedy is to adopt the ad-hoc scope-extending approach, which Geach proposes to handle the discourse anaphora. Thus the following formula is achieved:

$$\exists x \exists y.((\mathbf{farmer}\ x \wedge \mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y) \to \mathbf{beat}\ x\ y) \tag{4.8}$$

However, the above representation 4.8 still fails to render the correct meaning of (69). For instance, imagine a model such that there is a farmer, a donkey and a pig, the farmer owns both the donkey and the pig, and he beats the pig but not the donkey. This model satisfies 4.8, however, obviously example (69) will be false in it. The appropriate FOL translation of (69) ought to be as follows[5]:

$$\forall x \forall y.((\mathbf{farmer}\ x \wedge \mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y) \to \mathbf{beat}\ x\ y) \tag{4.9}$$

As a result, in order to obtain the desired reading of donkey sentences, the indefinite antecedents, *a farmer* and *a donkey*, should be represented as universal quantifiers, rather than as existential ones. This, from the semantic point of view, is rather counterintuitive.

As a summary, the problem on donkey sentences is more complicated than the one on discourse anaphora. On the one hand, neither is the anaphor in donkey sentence an individual constant co-referential with the antecedent, nor is it a variable bound by the antecedent. On the other hand, the type of quantifier introduced by the indefinite NP is of a universal type. This deviates from the standard treatment, where indefinite NPs are uniformly treated as existentially quantified expressions. Hence like inter-sentential anaphora, donkey sentences also pose challenge to standard logical semantics.

[5]The precise interpretation of donkey sentence is a complicated question still full of debates. We adopt the most widely recognized one, namely the universal reading, or the strong reading in this thesis. Interested readers can refer to Chierchia (1995); Elworthy (1992); Geurts (2002); Kanazawa (1994) for more discussions.

In the subsequent sections, we will formally yet briefly present two well-known dynamic frameworks: discourse representation theory Kamp (1981); Kamp and Reyle (1993) and dynamic predicate logic Groenendijk and Stokhof (1991), serving as an illustration for dynamic semantics. After that we will present a recent framework de Groote (2006), which is based on the notion of continuation and successfully integrates the dynamic concept of "context" into the standard MG.

4.2 Discourse Representation Theory (DRT)

Discourse Representation Theory (**DRT**) refers to the semantic theory originally proposed by Hans Kamp. The theory was first introduced in Kamp (1981), a maturer version was given in Kamp and Reyle (1993), which the following presentation will mainly base on. Note that a very similar framework, File Change Semantics (FCS) Heim (1982), was developed by Irene Heim independently at the same time of DRT. Basically, FCS aims to address the same sort of problems as DRT, and the empirical predictions obtained by the two systems are also similar. Since DRT leads to a wider range of subsequent works, such as the extension for presupposition Geurts (1999); Van der Sandt (1992), and it is more familiar to semanticists and logicians nowadays, we will focus on it here in this thesis.

Below, we first give a brief introduction to DRT, then introduce its formal details. This will be followed by some linguistic illustrations.

4.2.1 Introduction

As its name implies, DRT is a framework proposed to deal with the semantics of discourses, not of single sentences as MG does. It is essentially motivated to solve the problems presented in section 4.1.1 and 4.1.2.

In DRT, every discourse, namely a sequence of sentences, is translated into a system, which is called the **discourse representational structure** (**DRS**). According to Kamp, a DRS is regarded as:

> *The mental representations which speakers form in response to the verbal inputs they receive.* Kamp (1981)

Basically, a DRS consists of two parts: a universe, which contains a set of discourse referents representing the individuals or entities under discussion; and a set of DRS-conditions, or simply conditions, which encode the logical properties or relations on discourse referents. Following the Geachean tradition, DRT uniformly treats all sorts of anaphors (co-referential and bound, see section 2.3.1), as well as indefinite NPs, by translating them into discourse referents. Different kinds of NPs, e.g., indefinite, pronominal, and definite, are distinguished in such a way that the discourse referent from an indefinite NP is fresh in the context, while the one from a pronoun or a definite NP[6] ought to be linked to some existing referent in the context. Thus, indefinite NPs have the potential to change the dynamic meaning of a discourse, while pronouns (including definite NPs) do not.

[6]Here we only consider the anaphoric usage of definite NPs, although this usage is not emphasized in classical theories of NP Russell (1905); Strawson (1950).

Conforming to their dynamic nature as described in section 4.1, DRSs are established in an incremental way. A sentence is interpreted with respect to the DRS of a prior discourse. Particularly, the anaphoric expressions in the sentence are associated with antecedents which are already present in the context. Moreover, in turn, the semantics of the sentence will contribute in resulting a new DRS, which is an updated version of the previous one and will determine the interpretation of subsequent sentences.

Although DRT was originally designed to deal with the problem of anaphora, it is not restricted on that. Subsequent developments have extended DRT to cover a wide range of linguistic phenomena, such as tense, plurality, generalized quantifiers, rhetorical structure, presupposition, modal subordination, etc. However, they are out of the horizon of this thesis, interested readers can refer to Asher (1993); Geurts (1999); Roberts (1989); Van der Sandt (1992).

In the next subsection, we will formally investigate DRT from both the syntactic and semantic perspectives.

4.2.2 Formal Framework

In this subsection, the formal details of DRT will be presented. We start off with its syntax, then proceed to its semantic, namely the way to interpret DRS and conditions. Finally we will discuss the notion of accessibility, which plays an important role in DRT's account on anaphora.

Definition 4.2.1. The alphabet for Discourse Representation Theory (DRT) consists of the following symbols:

1. Constant symbols: $\mathbf{a}$, $\mathbf{b}$, $\mathbf{c}$, ..., $\mathbf{a}_1$, $\mathbf{a}_2$, $\mathbf{a}_3$, ...;
2. Variables: x, y, z, ..., x_1, x_2, x_3, ...;
3. Predicate symbols: $\mathbf{P}$, $\mathbf{Q}$, $\mathbf{R}$...;
4. Logical connectives: $\neg$ (negation);
5. Identity symbol: $\doteq$.

The notation on vocabulary is the same as in FOL, see notation 3.1.3. The notion of terms (variables or constants) in DRT is the same as in FOL as well, see definition 3.1.8. As we mentioned in section 4.2.1, DRSs are pairs of sets of discourse referents and sets of DRS-conditions. In what follows we provide the syntax of DRT, where DRSs and DRS-conditions are simultaneously defined on each other by recursion.

Definition 4.2.2. A Discourse Representation Structure (DRS) $K = \langle R_K, Con_K \rangle$ is a pair such that:

1. R_K is a finite set of discourse referents, called the **universe** of K, $R_K \subseteq \mathcal{X}$;
2. Con_K is a finite set of DRS-conditions, called the **condition set** of K.

The set of **DRS-conditions**, in notation Con, is defined as follows:

1. $\mathbf{P}t_1, ..., t_n \in Con$, whenever $\mathbf{P} \in \mathcal{P}$, $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
2. $\neg K \in Con$, whenever K is a DRS;

3. $x \doteq t \in Con$, whenever $x \in \mathcal{X}$, $t \in \mathcal{T}$, $x \neq t$.

Conditions that are constructed from Rule 1 are called **atomic conditions**, those from Rule 2 are called **complex conditions**, those from Rule 3 are called **links**. DRSs which are included in complex conditions are called **sub-DRSs**.

Notation 4.2.1. As used in definition 4.2.2, uppercase characters (with an optional subscript) K, K_1, K_2, K_3, etc., will denote DRSs. Lowercase greek letters ϕ, ψ, ρ will denote DRS-conditions. Given a DRS K, we use R_K and Con_K to denote its universe and conditions, respectively.

Other conventional logical connectives, such as $\vee$ (disjunction) and $\rightarrow$ (implication), are defined in terms of negation as follows. Let $K_1 = \langle R_{K_1}, Con_{K_1} \rangle$, K_2 be two DRSs, then:

$$K_1 \vee K_2 \triangleq \neg\langle \emptyset, \neg K_1 \cup \neg K_2 \rangle \tag{4.10}$$

$$K_1 \rightarrow K_2 \triangleq \neg\langle R_{K_1}, Con_{K_1} \cup \neg K_2 \rangle \tag{4.11}$$

According to the above formulas, both $K_1 \rightarrow K_2$ and $K_1 \vee K_2$ are DRS-conditions. Compare the syntax of DRT (definition 4.2.2) with that of FOL (definition 3.1.9), one may find that conjunction and existential quantifier is not defined in DRT, that is because they are both included in the standard setups. We will look at them one by one.

Firstly, conjunction is the default logical connective in DRT. That is to say, all conditions in a DRS are assumed to be connected through conjunction. Hence, in order to conjoin two DRSs, we simply put their universes and their conditions together respectively: the merge of two DRSs is their pointwise union. This prompts the following definition:

Definition 4.2.3. Let $K_1 = \langle R_{K_1}, Con_{K_1} \rangle$, $K_2 = \langle R_{K_2}, Con_{K_2} \rangle$ be DRSs. The **merge operation** of K_1 and K_2, in notation $K_1 \oplus K_2$, is defined as follows:

$$K_1 \oplus K_2 = \langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2} \rangle$$

According to definition 4.2.3, the merge operation $\oplus$ is both commutative and associative. Let K_1, K_2, and K_3 be DRSs, then we have the following relations:

- $K_1 \oplus K_2 = K_2 \oplus K_1$
- $K_1 \oplus (K_2 \oplus K_3) = (K_1 \oplus K_2) \oplus K_3$

Since both discourses and sentences are represented by DRSs, the merge operation can directly be applied to achieve discourse incrementation: the DRS of a sentence is updated to the one of a prior discourse with the operator $\oplus$. Besides definition 4.2.3, there are also various other versions of merge operation (symmetric or non-symmetric). For an elaborated study, please refer to Fernando (1994); Van Eijck and Kamp (1997); Vermeulen (1995).

In addition, quantifier is not specified in definition 4.2.2 because discourse referents receive an existential interpretation by default. By way of example, let's have a look at the first sentence in (6), namely *a man walks in the park*. It can be mapped into the following DRS structure[7]:

$K_{(6)\text{-}1}$:

x
man x **walk_in_the_park** x

There are explicit construction algorithms (either top-down or bottom-up) which show how a DRS can be established step by step from a syntactic parse tree, we shall not go into detail here. For more discussion, see Asher (1993); Kamp (1981); Kamp and Reyle (1993); Muskens (1996); Van Eijck and Kamp (1997).

Now let's take a closer look at $K_{(6)\text{-}1}$. The discourse referent x in the universe is introduced by the indefinite NP *a man*, which also brings about the atomic condition **man** x. The other condition, **walk_in_the_park** x, is contributed by the VP of the sentence. An intuitive interpretation of $K_{(6)\text{-}1}$ would be as follows. $K_{(6)\text{-}1}$ is a representation which models the situation that (6)-1 describes. In this situation, only one individual is involved, represented by the discourse referent x. In addition, the two atomic conditions, one indicating that he is a man, the other indicating that he walks in the park, are the logic properties that the individual bears.

Technically, a DRS can be viewed as a list of conditions together with (some of) the variables that occur in them. From the standard predicate logic point of view, these conditions are open formulas connected by conjunction. Hence $K_{(6)\text{-}1}$ is no different from $(\mathbf{man}\ x) \wedge (\mathbf{walk\_in\_the\_park}\ x)$. The open formula will be true as long as there exists an individual for x in the domain, with respect to which the formula is satisfied. It is in this way that discourse referents are existentially quantified. Moreover, complex conditions may result in other sorts of interpretation, we will see this in detail shortly afterwards.

Up until now, we've finished introducing the syntax of DRT. In the following, we will present its semantics, namely the way to interpret DRSs and conditions. Similar as before, the semantics involves the concept of model and assignment function. To interpret DRSs, we adopt the usual first-order model $M = \langle D, I\rangle$, as in definition 3.1.13. The assignment function and relevant notations are also the same as in FOL, see definition 3.1.14 and notation 3.1.5. In addition, the notation $h[X]g$ is used in the same way as in FOL, see definition 3.1.15.

With the above knowledge, we will then discuss the semantic notions for DRT, such as **interpretation** (for both terms and DRSs) and **satisfiability**. Firstly, the interpretation of terms in DRT is exactly the same as the one in FOL, see definition 3.1.16. Then, as to the semantics of DRT (interpretation of DRSs and conditions), it is provided as follows.

Definition 4.2.4. Let $K = \langle R_K, Con_K\rangle$ be a DRS, g and h assignment functions, M a model. The **interpretation** of K in M, in notation $[\![K]\!]^M_{DRT}$, is defined as follows:

$$[\![K]\!]^M_{DRT} = \{\langle g, h\rangle \mid h[R_K]g \text{ and } \forall \phi \in Con_K : h \in [\![\phi]\!]^M_{DRT}\}.$$

[7]In this thesis, we will use both the linear/set-based notation and the pictorial box notation for DRSs. The former, as we have seen in definition 4.2.2 and 4.2.3, saves space and will be concise when presenting the formal details of DRT. The latter, where the referents and conditions are respectively listed on the top and lower part of a box, is visually appealing. It provides a better readability when presenting DRSs for specific linguistic examples, particularly, the anaphoric possibility can be observed at a glance.

The **interpretation** of a DRS-condition $\phi \in Con$ in M, in notation $[\![\phi]\!]^M_{DRT}$, is defined inductively on $[\![K]\!]^M_{DRT}$ as follows:

1. $[\![\mathbf{P}t_1, ..., t_n]\!]^M_{DRT} = \{g \mid \langle [\![t_1]\!]^{M,g}, ..., [\![t_n]\!]^{M,g}\rangle \in I(\mathbf{P})\}$;
2. $[\![\neg K]\!]^M_{DRT} = \{g \mid \neg\exists h : \langle g, h\rangle \in [\![K]\!]^M_{DRT}\}$;
3. $[\![x \doteq t]\!]^M_{DRT} = \{g \mid g(x) = [\![t]\!]^{M,g}\}$.

As we can see, for a DRS K, all its DRS-conditions in Con_K are indeed interpreted in parallel to conjunction in standard predicate logic. Additionally, the notion of satisfiability is defined in a similar way as in FOL:

Definition 4.2.5. Let $K = \langle R_K, Con_K\rangle$ be a DRS, M a model, $g \in \mathcal{G}$ an assignment function, $C \in Con_K$ a DRS-condition.

- We say that M **satisfies** C with respect to g, or equivalently, g **verifies** C in M, in notation $M, g \models_{DRT} C$, iff $g \in [\![C]\!]^M_{DRT}$;
- We say that M **satisfies** K with respect to g, or equivalently, g **verifies** K in M, in notation $M, g \models_{DRT} K$, iff $\exists h \in \mathcal{G} : \langle g, h\rangle \in [\![K]\!]^M_{DRT}$, namely there is an assignment function h such that $h[R_K]g$, and h verifies every DRS-condition $C \in Con_K$.

As stated in the previous section, the difference between an indefinite NP and an anaphoric pronoun in DRT is that the former introduces a discourse-new referent, while the latter a discourse-old referent. Further more, the referent from a pronoun needs to be identified with an existing one in order to resolve the anaphora. However, it is not the case that all established referents are potential antecedent, take discourse (64-b) for instance. Hence, a discourse referent has its lifespan: only an "alive" or accessible referent are available for resolving anaphoras. This gives rise to another crucial ingredient of DRT: **accessibility**. Before presenting it, we first introduce the notion of **subordination**, which is a fundamental structural relation between DRSs.

Definition 4.2.6. Let $K_1 = \langle R_{K_1}, Con_{K_1}\rangle$ and K_2 be DRSs, K_1 **weakly subordinates** K_2, in notation $K_1 \geq K_2$, iff either:

1. $K_1 = K_2$;
2. $\neg K_2 \in Con_{K_1}$;
3. there is a DRS K_3 such that $K_1 \geq K_3$ and $K_3 \geq K_2$.

K_1 **strongly subordinates** K_2, in notation $K_1 > K_2$, iff either:

1. $\neg K_2 \in Con_{K_1}$;
2. there is a DRS K_3 such that $K_1 > K_3$ and $K_3 > K_2$.

The accessibility of a discourse referent is subject to the way in which it is introduced. Hence the definition of accessibility depends on the subordination relation, which determines where the referent is situated in the DRS.

Definition 4.2.7. Let K, K_1 be DRSs such that $K \geq K_1$, $x \in \mathcal{X}$ a discourse referent. We say that x is **accessible from** K_1 in K iff there is a DRS $K_2 = \langle R_{K_2}, Con_{K_2}\rangle$ such that $x \in R_{K_2}$ and $K \geq K_2$ and $K_2 \geq K_1$.

Because only accessible referents can be anaphorically linked to an anaphor, the notion of accessibility can be applied to explain the awkwardness of the anaphora in example (64-b), as well as in the following ones:

(70) *If $\underline{\text{every man}}_i$ meets $\underline{\text{a nice woman}}_j$, $\underline{\text{he}}_i$ smiles at $\underline{\text{her}}_j$. Van Eijck and Kamp (1997)

(71) *If John owns $\underline{\text{no donkey}}_i$, he wants $\underline{\text{it}}_i$. Chierchia (1995)

By translating the above discourses into the DRT language (the implications in the following DRSs are kept for a more straightforward illustration, they can be transformed based on formula 4.11), we might see that in example (70), the discourse referents introduced by *every man* and *a nice woman* are not accessible from the referents of *he* and *she*. So it is the case for example (71), where the referent from *no donkey* is inaccessible from the anaphoric pronoun *it*.

$$K_{(70)}: \quad \begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline x \\ \hline \textbf{man } x \\ \hline \end{array} \rightarrow \begin{array}{|c|}\hline y \\ \hline \textbf{nice\_woman } y \\ \textbf{meet } x\ y \\ \hline \end{array} \\ \hline \end{array} \rightarrow \begin{array}{|c|}\hline \\ \hline \textbf{smile\_at } x\ y \\ \hline \end{array} \\ \hline \end{array}$$

$$K_{(71)}: \quad \begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline \\ \hline \neg \begin{array}{|c|}\hline x \\ \hline \textbf{donkey } x \\ \textbf{own john } x \\ \hline \end{array} \\ \hline \end{array} \rightarrow \begin{array}{|c|}\hline \\ \hline \textbf{want john } x \\ \hline \end{array} \\ \hline \end{array}$$

We will see more examples which involve infelicitous anaphoric links in Chapter 5. In the following subsection, the detailed treatments for inter-sentential anaphora and donkey anaphora in DRT will be presented.

4.2.3 Illustration

In this subsection, we will illustrate with a couple of examples, showing how DRT works in discourse processing. The examples that we shall use are the two typical ones involving puzzling anaphora, namely the inter-sentential anaphora in section 4.1.1 and the donkey anaphora in section 4.1.2. We will look at them one by one.

Inter-Sentential Anaphora

Firstly, we examine the example of discourse anaphora (6), repeated as follows:

(6) $\underline{\text{A man}}_i$ walks in the park. $\underline{\text{He}}_i$ whistles.

In section 4.2.2, we have already presented the DRS for (6)-1, namely $K_{(6)\text{-}1}$, which we shall not restate. For the second sentence in (6), its DRS is also straightforward:

$$K_{(6)\text{-}2}: \quad \begin{array}{|c|}\hline y \\ \hline \textbf{whistle } y \\ \hline \end{array}$$

Then we can obtain the DRS of the whole discourse by merging the DRSs of its two component sentences. According to definition 4.2.3, we have:

$$K_{(6)\text{-}1}: \begin{array}{|c|}\hline x \\ \hline \textbf{man } x \\ \textbf{walk\_in\_the\_park } x \\ \hline \end{array} \quad \oplus \quad K_{(6)\text{-}2}: \begin{array}{|c|}\hline y \\ \hline \textbf{whistle } y \\ \hline \end{array} \quad =$$

$$K_{(6)}: \quad \begin{array}{|c|}\hline x,\ y \\ \hline \textbf{man } x \\ \textbf{walk\_in\_the\_park } x \\ \textbf{whistle } y \\ \hline \end{array}$$

Since the discourse referent y is introduced by an anaphoric pronoun, we may link it to an appropriate existing referent, which is x in the above case. Then by inserting the condition $y \doteq x$ into the above DRS, we end up with the final representation, where anaphora is properly resolved:

$$K_{(6)}: \quad \begin{array}{|c|}\hline x,\ y \\ \hline \textbf{man } x \\ \textbf{walk\_in\_the\_park } x \\ \textbf{whistle } y \\ y \doteq x \\ \hline \end{array}$$

Now let's try to interpret the DRS $K_{(6)}$ with respect to definition 4.2.4. Let $M = \langle D, I \rangle$ be a model. The interpretation of $K_{(6)}$ in M is as follows:

$$\begin{aligned}
[\![K_{(6)}]\!]^M_{DRT} &= \{\langle g, h\rangle \mid h[R_{K_{(6)}}]g \text{ and } \forall C \in Con_{K_{(6)}} : h \in [\![C]\!]^M_{DRT}\} \\
&= \{\langle g, h\rangle \mid h[\{x, y\}]g \text{ and } h \in [\![\textbf{man } x]\!]^M_{DRT} \text{ and } h \in [\![\textbf{walk_in_the_park } x]\!]^M_{DRT} \\
&\qquad \text{and } h \in [\![\textbf{whistle } y]\!]^M_{DRT} \text{ and } h \in [\![y \doteq x]\!]^M_{DRT}\} \\
&= \{\langle g, h\rangle \mid h[\{x, y\}]g \text{ and } h(x) \in I(\textbf{man}) \text{ and } h(x) \in I(\textbf{walk_in_the_park}) \\
&\qquad \text{and } h(y) \in I(\textbf{whistle}) \text{ and } h(y) = h(x)\}
\end{aligned}$$

As a result, M satisfies $K_{(6)}$ with respect to g iff there is some individual in the domain such that he is a man, he walks in the park, and he whistles, this correctly corresponds to what example (6) expresses. Further more, let's look back formula 4.5, which is the expected FOL translation of (6), we call it $Rep_{FOL}(6)$:

$$Rep_{FOL}(6) = \exists x.(\textbf{man } x \wedge \textbf{walk\_in\_the\_park } x \wedge \textbf{whistle } x) \tag{4.12}$$

If we interpret Rep_{FOL}(6) under the semantics of FOL (definition 3.1.17), we will obtain exactly the same truth conditions as what $K_{(6)}$ achieves in DRT. Hence:

$$M, g \models_{DRT} K_{(6)} \text{ iff } M, g \models_{FOL} Rep_{FOL}(6) \tag{4.13}$$

Donkey Sentence

Now let's conduct the same process for the conditional donkey sentence (7), repeated as follows:

(7) If <u>a farmer</u>$_i$ owns <u>a donkey</u>$_j$, <u>he</u>$_i$ beats <u>it</u>$_j$.

Similarly, the DRS of the whole sentence is obtained by combining the DRSs of the two components with implication, which has already been defined in the syntax of DRT, see formula 4.11. Thus:

$$\begin{array}{|c|}\hline \\ \hline K_{(7)\text{-}1}: \begin{array}{|c|}\hline x,\ y \\ \hline \textbf{farmer}\ x \\ \textbf{donkey}\ y \\ \textbf{own}\ x\ y \\ \hline \end{array} \rightarrow K_{(7)\text{-}2}: \begin{array}{|c|}\hline z,\ u \\ \hline \textbf{beat}\ z\ u \\ \hline \end{array} \\ \hline \end{array} = K_{(7)}: \begin{array}{|c|}\hline \\ \hline \neg \begin{array}{|c|}\hline x,\ y \\ \hline \textbf{farmer}\ x \\ \textbf{donkey}\ y \\ \textbf{own}\ x\ y \\ \neg \begin{array}{|c|}\hline z,\ u \\ \hline \textbf{beat}\ z\ u \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array}$$

Again, both discourse referents z and u in the deeply embedded sub-DRS are established by anaphoric pronouns (i.e., *he* and *it* respectively). In order to resolve the anaphora, we have to associate the corresponding referent with an appropriate antecedent. Hence by inserting the following links in the sub-DRS: $z \doteq x$, $u \doteq y$, we obtain the final representation, where both anaphoras are resolved:

$$K_{(7)}: \quad \begin{array}{|c|}\hline \\ \hline \neg \begin{array}{|c|}\hline x,\ y \\ \hline \textbf{farmer}\ x \\ \textbf{donkey}\ y \\ \textbf{own}\ x\ y \\ \neg \begin{array}{|c|}\hline z,\ u \\ \hline \textbf{beat}\ z\ u \\ z \doteq x \\ u \doteq y \\ \hline \end{array} \\ \hline \end{array} \\ \hline \end{array}$$

Now we can examine how $K_{(7)}$ is interpreted semantically. Let $M = \langle D, I \rangle$ be a model. The interpretation of $K_{(7)}$ in M is as follows:

$$
\begin{aligned}
[\![K_{(7)}]\!]^M_{DRT} &= \{\langle g,h\rangle \mid h[R_{K_{(7)}}]g \text{ and } \forall C \in Con_{K_{(7)}} : h \in [\![C]\!]^M_{DRT}\} \\
&= \{\langle g,h\rangle \mid h[\{\}]g \text{ and } \neg\exists f : f[\{x,y\}]h \text{ and } f(x) \in I(\mathbf{farmer}) \text{ and } f(y) \in I(\mathbf{donkey}) \\
&\qquad \text{and } \langle f(x), f(y)\rangle \in I(\mathbf{own}) \text{ and } \neg\exists k : k[\{z,u\}]f \\
&\qquad \text{and } \langle k(z), k(u)\rangle \in I(\mathbf{beat}) \text{ and } k(z) = k(x) \text{ and } k(u) = k(y)\} \\
&= \{\langle g,g\rangle \mid \neg\exists f : f[\{x,y\}]g \text{ and } f(x) \in I(\mathbf{farmer}) \text{ and } f(y) \in I(\mathbf{donkey}) \\
&\qquad \text{and } \langle f(x), f(y)\rangle \in I(\mathbf{own}) \text{ and } \langle f(x), f(y)\rangle \notin I(\mathbf{beat})\} \\
&= \{\langle g,g\rangle \mid \forall f : \text{ if } (f[\{x,y\}]g \text{ and } f(x) \in I(\mathbf{farmer}) \text{ and } f(y) \in I(\mathbf{donkey}) \\
&\qquad \text{and } \langle f(x), f(y)\rangle \in I(\mathbf{own})) \text{ then } \langle f(x), f(y)\rangle \in I(\mathbf{beat})\}
\end{aligned}
$$

As we can see, as predicted by DRT, example (7) is satisfied in M with respect to g iff there is no such farmer-donkey pair which bears the owning relation, and the farmer does not beat the donkey. In other words, for any farmer-donkey pair that bears the owning relation, the farmer beats the donkey, this correctly reflects the meaning of the donkey sentence. Further more, let's look back at formula 4.9, which is the expected FOL translation of (7), we call it $Rep_{FOL}(7)$:

$$Rep_{FOL}(7) = \forall x \forall y.((\mathbf{farmer}\ x \wedge \mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y) \rightarrow \mathbf{beat}\ x\ y) \tag{4.14}$$

If we interpret $Rep_{FOL}(7)$ in the semantics of FOL, we will obtain the same truth conditions as $K_{(7)}$ in DRT. Hence:

$$M, g \models_{DRT} K_{(7)} \text{ iff } M, g \models_{FOL} Rep_{FOL}(7) \tag{4.15}$$

4.3 Dynamic Predicate Logic (DPL)

In this section, we are going to present another classical dynamic framework: Dynamic Predicate Logic (DPL). Likewise, we shall start with a brief introduction, then dive into the formal details of the theory, including its syntax and semantics. Finally, its applications on the two problematic anaphoric phenomena will be presented.

4.3.1 Introduction

DRT is one of the first systems which adopt the dynamic point of view towards meaning. Although there exists a translation between DRSs and the traditional FOL formulas Kamp and Reyle (1993); Van Eijck and Kamp (1997), researchers are trying to find solutions which express the dynamic meaning in a more canonical syntax. In addition, the defect of lacking compositionality in early versions of DRT[8] has been widely criticized in the literature Geurts (1999); Geurts and Beaver (2011); Groenendijk and Stokhof (1991); Kracht (2007).

Dynamic Predicate Logic (DPL) Groenendijk and Stokhof (1991), as another representative work in the family of dynamic semantics, was introduced as a compositional alternative to DRT. Hence the typical phenomena which DPL works on are also the same

[8]There are a range of subsequent works which attempt to make DRT compositional, for instance Muskens (1996); Zeevat (1989).

as in DRT, namely the inter-sentential anaphora in section 4.1.1 and donkey anaphora in section 4.1.2. However, different from DRT, DPL aims to express the dynamics within classical logical systems. As purported by the authors, DPL achieves the following improvement over DRT:

> *It (DPL) gives a compositional semantic treatment of the relevant phenomena, while the syntax of the language used, being that of standard predicate logic, is an orthodox one.* Groenendijk and Stokhof (1991)

The philosophy of DPL lies in dynamic logic Pratt (1976), which was developed to account for the semantics of imperative programming languages. Generally speaking, the meaning of a computer program can be associated to two machine states, namely the state before the program is executed, and the one after the execution Harel (1984). Similarly, by paralleling a sentence to a computer program, the meaning of a sentence can be identified as a pair of (input/output) contexts, namely the one before the sentence is processed, and the updated one after discourse incrementation. Because of that, sentences are also seen as "context change devices" Geurts (1999).

Briefly speaking, the main idea of DPL, on the one hand, is to stick to the principle of compositionality during the discourse incrementation; on the other hand, is to preserve as much as possible the shape of logical representations in the standard logical semantics style. To achieve this, DPL employs an identical syntax as FOL, while a new set of dynamic interpretations are assigned to standard logical constants, i.e., connectives and quantifiers. A predicate logical formula is thus interpreted in DPL as a set of pairs of input and output states (assignment functions), which respectively represent the appropriate input and output contexts where the corresponding utterance occurs.

This change of semantics gives rises to a number of consequences. For instance, an existential quantifier in DPL has the potential to bind variables outside its normal scope, namely the free variables in FOL as defined in 3.1.12, will be bound in DPL. As a result, formula 4.4 and 4.5 will receive the same interpretation under DPL. Namely they have the same potential to change the context. This characterization properly reflects the semantics of example (6). We shall see this in more detail in section 4.3.3.

In a nutshell, it is the novel interpretations of logical constants in DPL that plays the essential role in achieving the dynamics. In the following subsections, we will briefly present the technical details of DPL, then provide some examples.

4.3.2 Formal Framework

In this subsection, we will look into the technical details of FOL. Same as before, we will investigate the framework from both syntactic and semantic perspectives.

As we mentioned, DPL is claimed to be "orthodox" in the sense that it inherits the syntax of the standard FOL. Hence for the formal definitions of vocabulary, term, and formula in DPL, we refer back to definitions 3.1.7, 3.1.8 and 3.1.9. Relevant notations are also the same, see notation 3.1.3 and 3.1.4. Some other conventional logical connectives, such as $\rightarrow$ (implication), $\vee$ (disjunction), and $\forall$ (universal quantifier), can be defined through De Morgan's laws with the primitive connectives ($\wedge$, $\neg$ and $\exists$), exactly the same as in FOL, see formula 3.1, 3.2 and 3.5.

Now let's focus on the semantics of DPL, which fundamentally sets DPL apart from FOL. We adopt the usual first-order model $M = \langle D, I \rangle$, as in definition 3.1.13, where D is called the domain of M, and the elements of D are called individuals as well. Other preliminaries, such as the notion of assignment function, its notation, are also as usual, see definition 3.1.14 and notation 3.1.5 for more details. In addition, notation $h[X]g$ in DPL preserves a same meaning as in definition 3.1.15, which says the assignment function h agrees with g except possibly with respect to the value they assign to elements of X.

The semantics of terms in DPL is identical to that in FOL, as in definition 3.1.16. In what follows, we present the semantic interpretation of DPL formulas.

Definition 4.3.1. Let $M = \langle D, I\rangle$ be a model, $\phi \in \mathbb{F}$ a formula. The **interpretation** of ϕ in M, in notation $[\![\phi]\!]^M_{DPL}$, is defined inductively as follows:

1. $[\![\mathbf{P}t_1, ..., t_n]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \langle [\![t_1]\!]^{M,h}, ..., [\![t_n]\!]^{M,h}\rangle \in I(\mathbf{P})\}$, where $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;

2. $[\![(\neg\phi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \langle h, k\rangle \in [\![\phi]\!]^M_{DPL}\}$;

3. $[\![(\phi \wedge \psi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid \exists k : \langle g, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ and } \langle k, h\rangle \in [\![\psi]\!]^M_{DPL}\}$;

4. $[\![(\exists x.\phi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid \exists k : k[\{x\}]g \text{ and } \langle k, h\rangle \in [\![\phi]\!]^M_{DPL}\}$.

As shown above, a DPL formula ϕ is indeed interpreted as a set of ordered pairs of assignment functions. The member of each pair $\langle g, h\rangle$, as we explained, can be contrasted against the input and output machine state of a computer program, respectively. That is to say, when ϕ is interpreted with respect to the input assignment g, h is the output assignment after its interpretation. Following are some futher elaborations on the above semantics.

- An atomic formula ϕ does not have a dynamic effect. As a result, the interpretation of ϕ merely checks whether the input assignment function g verifies the formula in the static sense, and g still serves as the output;

- A negation $\neg\phi$ does not have a dynamic nature either. It returns an input assignment g as output iff ϕ can by no means be verified with respect to g;

- The interpretation of a conjunction $\phi \wedge \psi$ is carried in a sequential way: we first interpret the left hand conjunct ϕ with respect to g, then identify the output assignment function k with the input assignment of the right hand conjunct ψ. In other words, a conjunction is satisfied if there is an assignment function which is successfully resulted from verifying ϕ and also initiating the verification of ψ;

- The interpretation of an existential formula $\exists x.\phi$ finds an assignment function k which possibly differs from the input assignment g at most on the value of the bound variable x. Then k is set as the input assignment and ϕ is interpreted with respect to k. Then the output assignment function of the overall interpretation, namely h, will at least possibly differ from g on the value of x.

As for the semantics of other conventional logical connectives, such as $\rightarrow$ (implication), $\vee$ (disjunction), and $\forall$ (universal quantifier), we can first transform them based on formula 3.1, 3.2 and 3.5; then compute their interpretations by applying the corresponding rules in definition 4.3.1, like what we did in formula 3.3, 3.4, and 3.8. In the following, let's take implication as an example, and deduce its semantics step by step.

$$
\begin{aligned}
&[\![(\phi \to \psi)]\!]^M_{DPL} \\
=&[\![\neg(\phi \wedge \neg\psi)]\!]^M_{DPL} \\
=&\{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \langle h, k\rangle \in [\![\phi \wedge \neg\psi]\!]^M_{DPL}\} \\
=&\{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \exists j : \langle h, j\rangle \in [\![\phi]\!]^M_{DPL} \text{ and } \langle j, k\rangle \in [\![\neg\psi]\!]^M_{DPL}\} \\
=&\{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \exists j : \langle h, j\rangle \in [\![\phi]\!]^M_{DPL} \text{ and } j = k \text{ and} \\
&\qquad\qquad \neg\exists i : \langle k, i\rangle \in [\![\psi]\!]^M_{DPL}\} \\
=&\{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \langle h, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ and } \neg\exists j : \langle k, j\rangle \in [\![\psi]\!]^M_{DPL}\} \\
=&\{\langle g, h\rangle \mid h = g \text{ and } \forall k : \text{ if } \langle h, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ then } \exists j : \langle k, j\rangle \in [\![\psi]\!]^M_{DPL}\}
\end{aligned}
\tag{4.16}
$$

The detailed semantics for $\vee$ and $\forall$ can be obtained in exactly the same way, they will not be spelled out here. In the next chapter, we will come back to this topic in more detail. Based on the semantics of DPL formulas, we define the notion of satisfiability, which is similar to the one of DRT as in definition 4.2.5:

Definition 4.3.2. Let M be a model, $g \in \mathcal{G}$ an assignment function, and $\phi \in \mathbb{F}$ a formula. We say that M **satisfies** ϕ with respect to g, or equivalently, g **verifies** ϕ in M, in notation $M, g \models_{DPL} \phi$, iff $\exists h \in \mathcal{G} : \langle g, h\rangle \in [\![\phi]\!]^M_{DPL}$.

The notion of validity in DPL is defined in a completely similar way as in FOL, we will not repeat it here, please refer back to definition 3.1.19. In DPL, conjunction $\wedge$ is used to represent sentence sequencing/discourse incrementation. In contrast to the merge operation $\oplus$ in DRT (definition 4.2.3), $\wedge$ is non-commutative. This property of DPL can apply to account for the unacceptable anaphoric link in the following discourse, where the antecedent comes after the anaphor:

(72) *He$_i$ whistles. A man$_i$ walks in the park. Geurts (1999)

Finally, we would like to define the semantic notion of **test**, which will be used in the next chapter when we are going to discuss accessibility in DPL.

Definition 4.3.3. Let M be a model, $g, h \in \mathcal{G}$ assignment functions, $\phi \in \mathbb{F}$ a formula. ϕ is a **test** iff $\forall M \forall g \forall h : \langle g, h\rangle \in [\![\phi]\!]^M_{DPL} \to g = h$.

A test either returns the input assignment or fails. It is dynamically meaningless because it does not pass any context information to future sentences. According to the above definition, a test could be an atomic formula, a negation, a disjunction, an implication, or a universally quantified formula[9]. In addition, the conjunction of two tests is also a test. Among the logical constants, only the existential quantifier $\exists$ and the conjunction $\wedge$, have the potential to update the context. In particular, the former assigns a new value to the corresponding variable, the latter passes the information from

[9]It is obvious that disjunction, implication and universally quantified formula are tests from their definitions: all of them are transformed into a negation, see formula 3.1 and 3.2, and 3.5.

the first conjunct to the second. Remark that in standard predicate logic such as FOL, we have the following logical equivalences:

$$\phi \wedge \psi = \neg(\phi \to \neg\psi) \tag{4.17}$$

$$\phi \wedge \psi = \neg(\neg\phi \vee \neg\psi) \tag{4.18}$$

$$\exists x.\phi = \neg\forall x.\neg\phi \tag{4.19}$$

However, it is not the case in DPL. That is because logical constants bear a new set of interpretations in DPL. The failure of the above relations can be easily shown with the semantics of DPL. Take formula 4.17 for instance. Its left hand side is a conjunction, which passes context information (assignment functions) from the first conjunct to the second, then to subsequent utterances. Its right hand side is a negation, which is a test and does not pass any context information for future utterances. As a result, unless both ϕ and ψ are dynamically meaningless, the equivalence 4.17 shall not hold in DPL. The rest two formulas are ruled out in an analogous way. For more peculiar logical facts in DPL, please refer to the original reference Groenendijk and Stokhof (1991).

In DPL, there are also some constraints that an anaphoric expression should follow when selecting its antecedent. Hence, a notion similar to the accessibility in DRT has been proposed: active quantifier occurrences.

Definition 4.3.4. Let $\phi \in \mathbb{F}$ be a formula. The set of **active quantifier occurrences** in ϕ, in notation $\mathbf{aq}(\phi)$ is defined as follows:

1. $\mathbf{aq}(\mathbf{P}t_1, ..., t_n) = \emptyset$, where $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
2. $\mathbf{aq}(\neg\phi) = \emptyset$;
3. $\mathbf{aq}(\phi \wedge \psi) = \mathbf{aq}(\psi) \cup \{\exists x \in \mathbf{aq}(\phi) \mid \exists x \notin \mathbf{aq}(\psi)\}$;
4. $\mathbf{aq}(\exists x.\phi) = \begin{cases} \mathbf{aq}(\phi) \cup \{\exists x\} & \text{if } \exists x \notin \mathbf{aq}(\phi), \\ \mathbf{aq}(\phi) & \text{otherwise.} \end{cases}$.

Basically, an active quantifier occurrence means that the corresponding variable is able to be accessed by subsequent sentences. As we can see, those formulas, which are tests, such as atomic formulas, negations (including disjunctions and implications), do not imply any active quantifier occurrence. Hence same as DRT, DPL also predicts the awkwardness of anaphora in examples such as (64-b), (70), and (71). We will see this in more detail in the next chapter.

Note that the accessibility in DRT is stipulated, however, the notion of active quantifier occurrences is semantically based. That is to say, definition 4.3.4 can be induced directly from the semantics of DPL (definition 4.3.1).

4.3.3 Illustration

DPL is designed as a compositional alternative of DRT, the two frameworks also make similar empirical predictions. In this subsection, we will show how DPL works with the two puzzling anaphoras, namely the inter-sentential anaphora in section 4.1.1 and the donkey anaphora in section 4.1.2.

Inter-Sentential Anaphora

We start with the former, again, example (6) is repeated as follows:

(6) A man_i walks in the park. He_i whistles.

Based on the syntax of DPL, or more precisely, the syntax of FOL, the logical representation of the first sentence in (6) is:

$$Rep_{DPL}(6)\text{-}1 = \exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x) \tag{4.20}$$

The representation of the second sentence is also intuitively straightforward:

$$Rep_{DPL}(6)\text{-}2 = \mathbf{whistle}\ x \tag{4.21}$$

To obtain the logical form of the whole discourse in a compositional way, we simply combine the above two formula with logical conjunction:

$$Rep_{DPL}(6) = (\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x)) \wedge \mathbf{whistle}\ x \tag{4.22}$$

Note that $Rep_{DPL}(6)$ is exactly the same as the previous formula 4.4, where the last occurrence of x is free with respect to standard predicate logic. We have already shown that under FOL, it won't yield a correct semantics for the original discourse (6). Then what is its interpretation under DPL? This is what we shall investigate below. Let $M = \langle D, I\rangle$ be a model. The Interpretation of $Rep_{DPL}(6)$ in M is computed as follows based on definition 4.3.1:

$$
\begin{aligned}
&[\![Rep_{DPL}(6)]\!]^M_{DPL}\\
=&\{\langle g,h\rangle \mid \exists k : \langle g,k\rangle \in [\![Rep_{DPL}(6)\text{-}1]\!]^M_{DPL} \text{ and } \langle k,h\rangle \in [\![Rep_{DPL}(6)\text{-}2]\!]^M_{DPL}\}\\
=&\{\langle g,h\rangle \mid \exists k : \langle g,k\rangle \in [\![Rep_{DPL}(6)\text{-}1]\!]^M_{DPL} \text{ and } k = h \text{ and } h(x) \in I(\mathbf{whistle})\}\\
=&\{\langle g,h\rangle \mid \langle g,h\rangle \in [\![Rep_{DPL}(6)\text{-}1]\!]^M_{DPL} \text{ and } h(x) \in I(\mathbf{whistle})\}\\
=&\{\langle g,h\rangle \mid \exists k : k[\{x\}]g \text{ and } \langle k,h\rangle \in [\![\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x]\!]^M_{DPL}\\
&\qquad\qquad \text{and } h(x) \in I(\mathbf{whistle})\}\\
=&\{\langle g,h\rangle \mid \exists k : k[\{x\}]g \text{ and } \exists k' : \langle k,k'\rangle \in [\![\mathbf{man}\ x]\!]^M_{DPL} \text{ and}\\
&\qquad\qquad \langle k',h\rangle \in [\![\mathbf{walk\_in\_the\_park}\ x]\!]^M_{DPL} \text{ and } h(x) \in I(\mathbf{whistle})\}\\
=&\{\langle g,h\rangle \mid \exists k : k[\{x\}]g \text{ and } \exists k' : k = k' \text{ and } k'(x) \in I(\mathbf{man}) \text{ and } k = h\\
&\qquad\qquad \text{and } h(x) \in I(\mathbf{walk\_in\_the\_park}) \text{ and } h(x) \in I(\mathbf{whistle})\}\\
=&\{\langle g,h\rangle \mid h[\{x\}]g \text{ and } h(x) \in I(\mathbf{man}) \text{ and } h(x) \in I(\mathbf{walk\_in\_the\_park})\\
&\qquad\qquad \text{and } h(x) \in I(\mathbf{whistle})\}
\end{aligned}
$$

As we can see, the scope of the existential quantifier is extended to cover the subformula **whistle** x, due to the dynamic interpretation of $\wedge$ and $\exists$. Hence, although $Rep_{DPL}(6)$ is not a closed formula in view of FOL, it perfectly reflects to the semantics of discourse (6) under DPL. Further more, if we interpret $Rep_{FOL}(6)$ (formula 4.12) under the semantics of DPL, we will achieve the same result as above, namely:

$$[\![Rep_{DPL}(6)]\!]^M_{DPL} = [\![Rep_{FOL}(6)]\!]^M_{DPL}$$

As a result, the whole discourse (6) is satisfied in M with respect to g iff there is some individual in the domain such that he is a man, he walks in the park, and he whistles, this is exactly what example (6) means.

As we can see, the conditions obtained from the interpretation of (6) in DPL are the same as the one from DRT and FOL. Hence we can further extend the relation 4.13 as follows:

$$M, g \models_{DPL} Rep_{DPL}(6) \text{ iff } M, g \models_{DRT} K_{(6)} \text{ iff } M, g \models_{FOL} Rep_{FOL}(6) \tag{4.23}$$

Donkey Sentence

Now let's turn to the semantics of the conditional donkey sentence (7), repeated as follows:

(7) If $\underline{\text{a farmer}}_i$ owns $\underline{\text{a donkey}}_j$, $\underline{\text{he}}_i$ beats $\underline{\text{it}}_j$.

First of all, the DPL logical representation for the two component sentences, which we call $Rep_{DPL}(7)$-1 and $Rep_{DPL}(7)$-2 are respectively as follows:

$$Rep_{DPL}(7)\text{-}1 = \exists x \exists y.(\textbf{farmer } x \wedge \textbf{donkey } y \wedge \textbf{own } x\ y) \tag{4.24}$$

$$Rep_{DPL}(7)\text{-}2 = \textbf{beat } x\ y \tag{4.25}$$

Thus by combining the above two formulas with logical implication $\rightarrow$, we compositionally obtained the semantic representation for the whole donkey sentence (7), namely:

$$Rep_{DPL}(7) = (\exists x \exists y.(\textbf{farmer } x \wedge \textbf{donkey } y \wedge \textbf{own } x\ y)) \rightarrow \textbf{beat } x\ y \tag{4.26}$$

As we might see, $Rep_{DPL}(7)$ is exactly the same as the previous formula 4.7. Again, from standard predicate logic point of view, the last two variable occurrences are both free. Hence under the interpretation of FOL (definition 3.1.17), $Rep_{DPL}(7)$ will not yield the expected semantics of the original sentence (7). In the following, we will show how $Rep_{DPL}(7)$ is interpreted under the semantics of DPL step by step. The current logical representation $Rep_{DPL}(7)$ contains a derived connective $\rightarrow$ as in formula 4.26, we transform $Rep_{DPL}(7)$ as follows according to formula 3.1 before the interpretation. Hence it is updated as follows:

$$Rep_{DPL}(7) = \neg((\exists x \exists y.(\textbf{farmer } x \wedge \textbf{donkey } y \wedge \textbf{own } x\ y)) \wedge \neg(\textbf{beat } x\ y)) \tag{4.27}$$

Namely, $Rep_{DPL}(7) = \neg(Rep_{DPL}(7)\text{-}1 \wedge \neg Rep_{DPL}(7)\text{-}2)$. In the following, we will first compute the interpretations of $Rep_{DPL}(7)$-1 and $Rep_{DPL}(7)$-2, then incorporate them compositionally. Let $M = \langle D, I \rangle$ be a model, the interpretation of $Rep_{DPL}(7)$-1 in M is as follows:

$$
\begin{aligned}
&[\![Rep_{DPL}(7)\text{-}1]\!]^M_{DPL}\\
=&\{\langle g,h\rangle \mid \exists k : k[\{x,y\}]g \text{ and } \langle k,h\rangle \in [\![\mathbf{farmer}\ x \wedge \mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y]\!]^M_{DPL}\}\\
=&\{\langle g,h\rangle \mid \exists k : k[\{x,y\}]g \text{ and } k = h \text{ and } h(x) \in I(\mathbf{farmer}) \text{ and}\\
&\qquad h(y) \in I(\mathbf{donkey}) \text{ and } \langle h(x),h(y)\rangle \in I(\mathbf{own})\}\\
=&\{\langle g,h\rangle \mid h[\{x,y\}]g \text{ and } h(x) \in I(\mathbf{farmer}) \text{ and } h(y) \in I(\mathbf{donkey}) \text{ and}\\
&\qquad \langle h(x),h(y)\rangle \in I(\mathbf{own})\}
\end{aligned}
$$

Since Rep_{DPL}(7)-2 is an atomic formula, its interpretation in M is rather straightforward:

$$[\![Rep_{DPL}(7)\text{-}2]\!]^M_{DPL} = \{\langle g,h\rangle \mid h = g \text{ and } \langle h(x),h(y)\rangle \in I(\mathbf{beat})\}$$

Finally, the interpretation of the overall logical formula Rep_{DPL}(7) can be achieved compositionally as follows:

$$
\begin{aligned}
&[\![Rep_{DPL}(7)]\!]^M_{DPL}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : \langle h,k\rangle \in [\![Rep_{DPL}(7)\text{-}1 \wedge \neg Rep_{DPL}(7)\text{-}2]\!]^M_{DPL}\}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : \exists f : \langle h,f\rangle \in [\![Rep_{DPL}(7)\text{-}1]\!]^M_{DPL} \text{ and}\\
&\qquad \langle f,k\rangle \in [\![\neg Rep_{DPL}(7)\text{-}2]\!]^M_{DPL}\}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : (\exists f : f[x,y]h \text{ and } f(x) \in I(\mathbf{farmer}) \text{ and}\\
&\qquad f(y) \in I(\mathbf{donkey}) \text{ and } \langle f(x),f(y)\rangle \in I(\mathbf{own}) \text{ and}\\
&\qquad f = k \text{ and } (\neg\exists j : \langle k,j\rangle \in [\![Rep_{DPL}(7)\text{-}2]\!]^M_{DPL}))\}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : k[x,y]h \text{ and } k(x) \in I(\mathbf{farmer}) \text{ and } k(y) \in I(\mathbf{donkey})\\
&\qquad \text{and } \langle k(x),k(y)\rangle \in I(\mathbf{own}) \text{ and } (\neg\exists j : \langle k,j\rangle \in [\![Rep_{DPL}(7)\text{-}2]\!]^M_{DPL})\}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : k[x,y]h \text{ and } k(x) \in I(\mathbf{farmer}) \text{ and } k(y) \in I(\mathbf{donkey})\\
&\qquad \text{and } \langle k(x),k(y)\rangle \in I(\mathbf{own}) \text{ and } (\neg\exists j : k = j \text{ and } \langle k(x),k(y)\rangle \in I(\mathbf{beat}))\}\\
=&\{\langle g,h\rangle \mid h = g \text{ and } \neg\exists k : k[x,y]h \text{ and } k(x) \in I(\mathbf{farmer}) \text{ and } k(y) \in I(\mathbf{donkey})\\
&\qquad \text{and } \langle k(x),k(y)\rangle \in I(\mathbf{own}) \text{ and } \langle k(x),k(y)\rangle \notin I(\mathbf{beat})\}
\end{aligned}
$$

As a result, example (7) is satisfied in a DPL model M with respect to g iff there is no such a farmer-donkey individual pair that if the farmer owns the donkey, he does not beat it. In other words, every farmer beats every donkey he owns. This correctly reflects the meaning of the donkey sentence.

Analogously, by comparing the treatment in FOL, DRT, and DPL, we can further extend the relation 4.15 as follows:

$$M,g \models_{DPL} Rep_{DPL}(7) \text{ iff } M,g \models_{DRT} K_{(7)} \text{ iff } M,g \models_{FOL} Rep_{FOL}(7) \tag{4.28}$$

4.4 Type Theoretic Dynamic Logic (TTDL)

In the last section of this chapter, we will present a more recently proposed dynamic semantic framework de Groote (2006). This framework, which is based on the notion of **continuation**, serves as the technical foundation of this thesis. Same as in previous sections, we first provide a brief introduction to this framework, including the background and some preliminary notions. Then we will introduce the framework in a formal way. Finally we end up with some illustrations.

4.4.1 Introduction

So far we have already presented two dynamic frameworks. DRT provides a novel representational structure (DRS) to express the dynamics, but it is criticized for lacking compositionality. DPL sticks to the canonical syntax of FOL, and interpret predicate logical formulas in terms of how they change the contexts, rather than their truth conditions. However, both systems suffer from the so-called destructive assignment problem. Hence variable naming should be conducted with a full load of carefulness in order to avoid crash. Because of that, it is natural to come up with the question: whether it is possible to encode the dynamics, namely the potential to change the context, and preserve the spirit of MG at the same time? The answer is yes, and it can be technically achieved through continuation.

The notion of **continuation** was proposed as a device for formalizing control flows in programming languages Strachey and Wadsworth (1974). Within this method, a term is evaluated in a context which represents the rest of the computation. Hence, functions written in continuation-passing style (CPS) are given an extra argument (the continuation) representing what rests to be done. This extra argument is itself a function, it takes the would-be-return value of the original function. The technique of continuation has been incorporated into natural language semantics for various linguistic phenomena Barker (2002, 2004); De Groote (2001); Shan (2004). In what follows, we will present the framework proposed by de Groote (2006, 2007), which we call **Type-Theoretical Dynamic Logic** (**TTDL**).

By providing a notion of **context** to the traditional MG, TTDL successfully handles dynamic phenomena in an (both syntactically and semantically) orthodox way. Basically, given a sentence, its **left context** denotes the discourse that precedes it, namely what has already been processed; its **right context** denotes the discourse that follows it, namely what is to be processed in future. A sentence is interpreted with respect to both its left and right contexts, and its semantics is abstracted over the two contexts. This is different from DRT and DPL, where only the preceding discourse (the left context) is taken into consideration when interpreting a sentence.

Like in other dynamic frameworks, discourses in TTDL are also processed in an incremental way: when a sentence is composed with a preceding discourse, namely its left context, information in the context can help to interpret the sentence, in particular when it is concerned with anaphoras; after the interpretation, the right context of the original discourse will be updated such that the sentence is interpolated. This dual-procedure is exactly the standpoint of dynamic semantics, which has been described in section 4.1.

Technically, TTDL sticks to the tradition of MG. It only makes use of standard mathematical and logical tools, such as λ-calculus and theory of types. Logical notions such as free and bound variables, quantifier scopes, are as usual, and the only operations involved are standard α-conversions and β-reductions. This property enables it to directly inherit all the nice properties in mathematics and logics, which have been thoroughly studied in the last century.

TTDL differentiates itself from both dynamic frameworks we have already seen. Compared to DRT, TTDL only clings to classical mathematical and logical techniques, in particular the simply typed λ-calculus as presented in section 3.2. A favorable consequence is that the framework is totally compositional and intuitive. Compared to DPL, TTDL not only inherits the syntax from the standard FOL, but persists with original semantic interpretations as well. In this sense, TTDL is more elegant because it explains the same problem in terms of an existing theory. In addition, variable naming is a crucial issue for both DRT and DPL: in DRT, discourse referents introduced by various NPs are stored as free variables lacking explicit quantification force; in DPL, although the quantifier scope can be extended subject to the particular interpretation of some logical constants, the binder only grabs those variables having the same name, hence one should be really careful in choosing the name for a variable. However, as we mentioned above, because TTDL uses the same concepts of variable (free or bound) and quantifier scope as in standard mathematical and logical systems, variable naming (during the process of semantic derivation) is merely a trivial task for it.

In the next subsection, we will turn to the formal details of TTDL.

4.4.2 Formal Framework

As we explained, TTDL is a framework based on standard mathematical and logical tools such as λ-calculus and theory of types. Hence TTDL is a parameterized version of the simply typed λ-calculus as presented in the previous chapter. For the formal details, please refer back to section 3.2. Note that we will not use product types (i.e., $\tau \times \sigma$) in TTDL, hence all the rules that are concerned with product will be temporarily ignored.

In what follows, we will specify the signature of TTDL, which sets it apart from other simply typed λ-calculus based frameworks.

Definition 4.4.1. The signature Σ_{TTDL} is defined as follows:

$$\begin{aligned}\Sigma_{TTDL} = \langle \{\iota, o, \gamma\}, &\{\top, \wedge, \neg, \exists, \_::\_, \mathsf{sel}, \mathsf{nil}\},\\ &\{\top : o, \wedge : o \to o \to o, \neg : o \to o, \exists : (\iota \to o) \to o,\\ &\ \_::\_ : \iota \to \gamma \to \gamma, \mathsf{sel} : \gamma \to \iota, \mathsf{nil} : \gamma\}\rangle\end{aligned}$$

Same as in definition 3.2.13, only logical constants $\mathcal{C}_L$ are specified in the Σ_{TTDL}. The non-logical constants, such as **farmer**, **beat**, etc., will be declared on-site in the applications. Some other common logical connectives, such as $\to$ (implication), $\vee$ (disjunction), and $\forall$ (universal quantifier), can be defined in terms of the primitive constants in Σ_{TTDL}: $\wedge$, $\neg$ and $\exists$. Their definitions are exactly the same way as in FOL, see formula 3.1, 3.2 and 3.5 for more information.

Now let's take a closer look at the typing information in TTDL. Among the set of atomic types as provided by Σ_{TTDL}, namely $T_A = \{\iota, o, \gamma\}$, ι and o should be rather familiar. They are exactly the same as in Church's simple type theory Church (1940): ι is the type of individuals, o is the type of propositions. As to the third atomic type γ, it is added to denote the type of the left context. Then the right context, which is interpreted as a continuation of the sentence, is a function from left contexts to truth values. So, its type ought to be $\gamma \to o$.

The left context as introduced in de Groote (2006) is a list of individuals. This proposition explains how the types are assigned to various logical constants in the signature Σ_{TTDL}. For instance, the type of the constructor :: is $\iota \to \gamma \to \gamma$, it takes an individual and a left context, and returns an updated left context; the type of the choice operator sel is $\gamma \to \iota$, its function is to pick out an appropriate individual (of type ι) from a left context (of type γ). Remark that sel can be implemented with various resolution algorithms. But since real anaphoric resolution procedure is not a focus of the current work, we shall not go into detail on how sel makes the choice. Every time when sel is present, we assume that it picks up the desired referent.

Then how can sentences be interpreted with the above setups? In standard truth-conditional semantics, a sentence expresses a proposition, which is of type o. While as explained in section 4.4.1, under the framework of TTDL, a sentence will be interpreted with respect to both its left and right contexts, which are of type γ and $\gamma \to o$, respectively. Further more, its meaning is abstracted over the two contexts. Then if we use s to denote the syntactic category of sentences, then the semantic representation of s is as follows:

$$[\![s]\!]_{TTDL} = \gamma \to (\gamma \to o) \to o \tag{4.29}$$

Discourses, which also express propositions, are interpreted in the same way as single sentences. So, let d be the syntactic category of discourses, we have:

$$[\![d]\!]_{TTDL} = \gamma \to (\gamma \to o) \to o \tag{4.30}$$

In order to contrast with o, which is the type of (standard/static) propositions, we call $\gamma \to (\gamma \to o) \to o$ the type of **dynamic propositions**. Hereinafter, we will use Ω as an abbreviation for $\gamma \to (\gamma \to o) \to o$, namely:

$$\Omega \triangleq \gamma \to (\gamma \to o) \to o \tag{4.31}$$

After presenting the typing information in TTDL, we will proceed to the logics of the framework. Same as in other dynamic systems, sentences in TTDL are not studied in isolation, they are incrementally updated into a prior discourse. Assume there is a discourse D and a sentence S, whose logical representations are $[\![D]\!]$ and $[\![S]\!]$, respectively. In order to obtain the semantics of $D.S$, which is the new discourse with S appended to D, we can employ the following rule of composition:

$$[\![D.S]\!] = \lambda e\phi.[\![D]\!]e(\lambda e'.[\![S]\!]e'\phi) \tag{4.32}$$

Same as S and D, the composed discourse $D.S$ is also interpreted as a dynamic proposition, hence its semantic type is Ω. Accordingly in formula 4.32, variable e and e' are of type γ, variable ϕ is of type $\gamma \to o$. But how can we understand the right hand side of the above composition rule? Let's have a look at the figure 4.1.

First of all, since the semantics of $D.S$ is contributed by D and S, this is why $[\![D]\!]$ and $[\![S]\!]$ are both involved in the composition. In addition, from figure 4.1, it is clear that e and ϕ are the left and right contexts of $D.S$, respectively. They are also called the current left and right contexts. Further more, the left context of D is the current left context, this is why e is passed to $[\![D]\!]$; the right context of S is the current right context, this is why ϕ is passed to $[\![S]\!]$. Finally, the right context of D is made up of S and the current right context, this explains why $\lambda e'.[\![S]\!]e'\phi$ is passed to $[\![D]\!]$; the left context of S is made

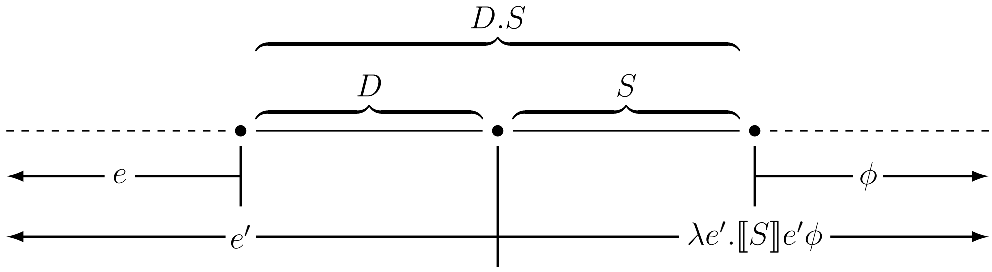


Fig. 4.1 Rule of Composition in TTDL

up of D and the current left context, this explains why e', which forms a λ-abstraction and will be substituted by a complex structure of type γ (consisting of e and information in D), is passed to $[\![S]\!]$.

In fact, we can rephrase the rule of composition 4.32 in terms of a λ-abstraction, this gives rise to the function $\mathbf{update}_{TTDL}$. It takes the representations of a discourse and a sentence as argument, and returns the representation of the compound discourse:

$$\mathbf{update}_{TTDL} \triangleq \lambda DSe\phi.De(\lambda e'.Se'\phi) \tag{4.33}$$

Same as in DPL, sentence sequencing is by default represented as conjunction. So the function $\mathbf{update}_{TTDL}$ can directly be used as the dynamic conjunction $\wedge^d_{TTDL}$ in TTDL, which conjoins two dynamic propositions, namely:

$$\wedge^d_{TTDL} \triangleq \lambda ABe\phi.Ae(\lambda e'.Be'\phi) \tag{4.34}$$

In order to negate a dynamic proposition in TTDL, we define the dynamic negation operator $\neg^d_{TTDL}$ as follows:

$$\neg^d_{TTDL} \triangleq \lambda Ae\phi.\neg(Ae(\lambda e'.\top)) \wedge \phi e \tag{4.35}$$

The operator $\neg^d_{TTDL}$ takes a dynamic proposition A and returns its dynamically negated counterpart, hence it is of type $\Omega \to \Omega$. The right hand side of formula 4.35 can be understood as follows. Firstly, the left context of the to-be-negated proposition A is the current left context, this is why e is passed to A. Further more, we do not want negation to take scope over any future part of the discourse, so the empty right context $\lambda e'.\top$, rather than the current right context ϕ, is passed to A. Finally, a dynamic negation does not have the potential to update the left context, this is why ϕe, the function-application of the original left and right contexts, appears as a conjunct at the end of the formula. In order to provide a better readability, we define the empty continuation as a compact term **stop**:

$$\mathbf{stop} \triangleq \lambda e.\top \tag{4.36}$$

The term **stop** is used to cease the availability of all variables in a left context. Accordingly, the dynamic negation $\neg^d_{TTDL}$ in formula 4.35 can be equivalently rewritten as follows:

$$\neg^d_{TTDL} \triangleq \lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e \tag{4.37}$$

As to the dynamic existential quantifier in TTDL $\exists^d_{TTDL}$, it is defined as:

$$\exists^d_{TTDL} \triangleq \lambda Pe\phi.\exists(\lambda x.Px(x :: e)\phi) \tag{4.38}$$

The dynamic quantifier $\exists^d_{TTDL}$ takes a dynamic property P of type $\iota \to \Omega$, and returns a existentially quantified dynamic proposition. Hence the semantic type of the operator $\exists^d_{TTDL}$ is $(\iota \to \Omega) \to \Omega$. The right hand side of formula 4.38 can be understood as follows. In an existentially quantified dynamic proposition, variables which are bound by the existential quantifier shall update the current left context, this is why the updated context $(x :: e)$ is passed to the proposition within the scope of $\exists$.

Above we have generally presented the typing information in TTDL, in particular, the dynamic way to interpret sentences and discourses. In fact, there exists a **dynamic translation**, which systematically associates (standard/static) logical expressions to their dynamic counterparts. The translation process is concerned with both types and λ-terms. We will examine them one by one below.

Notation 4.4.1. We use the bar notation, for instance, $\overline{\tau}$ or $\overline{M}$, to denote the **dynamic translation** of a type τ or a λ-term M in TTDL.

Definition 4.4.2. The **dynamic translation of a type** $\tau \in T$: $\overline{\tau}$, is defined inductively as follows:

1. $\overline{\iota} = \iota$;

2. $\overline{o} = \Omega$;

3. $\overline{\sigma \to \tau} = \overline{\sigma} \to \overline{\tau}$, where $\tau, \sigma \in T$.

As we can see, the static and dynamic types of individuals are both ι. While the static and dynamic type of propositions, as we explained above, are respectively o and Ω. The dynamic translation of a function type is still a function type, with the argument type and the result type being translated independently.

In order to present the dynamic translation of λ-terms, it is useful to first introduce the following two functions: the dynamization function $\mathbb{D}$ and the staticization function $\mathbb{S}$, whose definitions are mutually dependent. They will be used to translate non-logical constants.

Definition 4.4.3. The **dynamization function** $\mathbb{D}_\tau$, which takes an input λ-term A of type $(\gamma \to \tau)$, returns an output λ-term A' of type $\overline{\tau}$; the **staticization function** $\mathbb{S}_\tau$, which takes an input λ-term A' of type $\overline{\tau}$, returns an output λ-term A of type $(\gamma \to \tau)$.

- $\mathbb{D}_\tau$ is defined inductively on type τ as follows:
  1. $\mathbb{D}_\iota A = A\ \mathsf{nil}$;
  2. $\mathbb{D}_o A = \lambda e\phi.(Ae \wedge \phi e)$;
  3. $\mathbb{D}_{\alpha\to\beta} A = \lambda x.\mathbb{D}_\beta(\lambda e.Ae(\mathbb{S}_\alpha xe))$.
- $\mathbb{S}_\tau$ is defined inductively on type τ as follows:
  1. $\mathbb{S}_\iota A' = \lambda e.A'$;
  2. $\mathbb{S}_o A' = \lambda e.A'\ e\ \mathbf{stop}$;

3. $\mathbb{S}_{\alpha\to\beta}A' = \lambda e.(\lambda x.\mathbb{S}_\beta(A'(\mathbb{D}_\alpha(\lambda e'.x)))e)$.

It is relatively straightforward when only individuals (of type ι) or truth values (of type o) are concerned with. In what follows, some explanations on the general form of the two functions, namely $\mathbb{D}_{\alpha\to\beta}$ and $\mathbb{S}_{\alpha\to\beta}$, will be provided.

- The general dynamization function $\mathbb{D}_{\alpha\to\beta}$

  As we know, function $\mathbb{D}_{\alpha\to\beta}$ transforms a static expression A (of type $\gamma \to (\alpha \to \beta)$) into a dynamic one A' (of type $\overline{\alpha} \to \overline{\beta}$). From the above typing information, we can infer that the output, A' or $\mathbb{D}_{\alpha\to\beta}A$, must be a λ-abstraction $\lambda x.\Psi$, such that x is a variable of type $\overline{\alpha}$, Ψ is a dynamic term of type $\overline{\beta}$, namely:

$$\underbrace{A'}_{\overline{\alpha}\to\overline{\beta}} = \underbrace{\mathbb{D}_{\alpha\to\beta}\underbrace{A}_{\gamma\to(\alpha\to\beta)}}_{\overline{\alpha}\to\overline{\beta}} = \underbrace{\lambda\underbrace{x}_{\overline{\alpha}}.\underbrace{\Psi}_{\overline{\beta}}}_{\overline{\alpha}\to\overline{\beta}}$$

  In order to represent Ψ, we can re-employ the $\mathbb{D}$ function, by passing an argument of type $\gamma \to \beta$ to $\mathbb{D}_\beta$. Now let's examine whether the complex sub-formula $\lambda e.Ae(\mathbb{S}_\alpha xe)$ has the expected type or not. Since x is of type $\overline{\alpha}$, e is of type γ, according to definition 4.4.3, $\mathbb{S}_\alpha xe$ is a term of type α. As a result, $Ae(\mathbb{S}_\alpha xe)$ is of type β. Hence $\lambda e.Ae(\mathbb{S}_\alpha xe)$ is of type $\gamma \to \beta$, and it forms a valid input for $\mathbb{D}_\beta$.

  We repeat the complete definition below, with the explicit typing information for each sub-part of the formula. This should yield a better illustration:

$$\underbrace{A'}_{\overline{\alpha}\to\overline{\beta}} = \underbrace{\mathbb{D}_{\alpha\to\beta}\underbrace{A}_{\gamma\to(\alpha\to\beta)}}_{\overline{\alpha}\to\overline{\beta}} = \underbrace{\lambda\underbrace{x}_{\overline{\alpha}}.\underbrace{\mathbb{D}_\beta(\underbrace{\lambda e.\underbrace{\underbrace{\underbrace{A}_{\gamma\to\alpha\to\beta}e}_{\alpha\to\beta}\underbrace{(\mathbb{S}_\alpha\underbrace{x}_{\overline{\alpha}}e)}_{\alpha}}_{\beta}}_{\gamma\to\beta})}_{\overline{\beta}}}_{\overline{\alpha}\to\overline{\beta}}$$

- The general staticization $\mathbb{S}_{\alpha\to\beta}$

  Opposite to $\mathbb{D}_{\alpha\to\beta}$, the function $\mathbb{S}_{\alpha\to\beta}$ transforms a dynamic expression A' (of type $\overline{\alpha} \to \overline{\beta}$) into a static one A (of type $\gamma \to (\overline{\alpha} \to \overline{\beta})$). From the above typing information, we can infer that the output, A or $\mathbb{S}_{\alpha\to\beta}A'$, must be a λ-abstraction $\lambda e.(\lambda x.\psi)$, such that e is a variable of type γ, x is a variable of type α, ψ is a static term of type β, namely:

$$\underbrace{A}_{\gamma\to(\alpha\to\beta)} = \underbrace{\mathbb{S}_{\alpha\to\beta}\underbrace{A'}_{\overline{\alpha}\to\overline{\beta}}}_{\gamma\to(\alpha\to\beta)} = \underbrace{\lambda\underbrace{e}_{\gamma}.\underbrace{(\lambda\underbrace{x}_{\alpha}.\underbrace{\psi}_{\beta})}_{\alpha\to\beta}}_{\gamma\to(\alpha\to\beta)}$$

  In order to represent ψ, we can re-employ the $\mathbb{S}$ function, by sequentially passing an argument of type $\overline{\beta}$ and an argument of type γ to $\mathbb{S}_\beta$. Now let's examine whether the complex sub-formula $\mathbb{S}_\beta(A'(\mathbb{D}_\alpha(\lambda e'.x)))e$ has the expected type or not. Since x is of type $\overline{\alpha}$, e' is of type γ, according to definition 4.4.3, $\mathbb{D}_\alpha(\lambda e'.x)$ is a term of type $\overline{\alpha}$. As a result, $A'(\mathbb{D}_\alpha(\lambda e'.x))$ is of type $\overline{\beta}$. In addition, because e is of type γ, the sub-formula $\mathbb{S}_\beta(A'(\mathbb{D}_\alpha(\lambda e'.x)))e$, which is represented by ψ, is of type β.

Same as above, we repeat the corresponding complete definition of $\mathbb{S}_{\alpha\to\beta}$ below, with the explicit typing information attached:

$$\underbrace{A}_{\gamma\to(\alpha\to\beta)} = \underbrace{\mathbb{S}_{\alpha\to\beta}\underbrace{A'}_{\overline{\alpha}\to\overline{\beta}}}_{\alpha\to\beta} = \underbrace{\lambda \underbrace{e}_{\gamma}.\underbrace{(\lambda \underbrace{x}_{\alpha}.\underbrace{\underbrace{\mathbb{S}_\beta(\underbrace{A'(\underbrace{\mathbb{D}_\alpha(\underbrace{\lambda e.\underbrace{x}_{\alpha}}_{\gamma\to\alpha})}_{\overline{\alpha}})}_{\overline{\beta}})}_{\gamma\to\beta}\underbrace{e}_{\gamma}}_{\beta})}_{\alpha\to\beta}}_{\gamma\to(\alpha\to\beta)}$$

Now we can define the dynamic translation of λ-terms.

Definition 4.4.4. The **dynamic translation of a λ-term** M (of type τ): $\overline{M}$, which is another λ-term of type $\overline{\tau}$, is defined as follows:

1. $\overline{x} = x$, if $x \in \mathcal{X}$;
2. $\overline{\mathbf{a}} = \mathbb{D}_\tau(\lambda e.\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}_{NL}$ and $\mathbf{a} : \tau$;
3. $\overline{\wedge} = \wedge^d_{TTDL}$, see formula 4.34;
4. $\overline{\neg} = \neg^d_{TTDL}$, see formula 4.37;
5. $\overline{\exists} = \exists^d_{TTDL}$, see formula 4.38;
6. $\overline{(MN)} = (\overline{M}\,\overline{N})$;
7. $\overline{(\lambda x.M)} = (\lambda x.\overline{M})$.

Since the derived operators $\to$ (implication), $\vee$ (disjunction), and $\forall$ (universal quantifier) are defined in primitive logical constants (see formula 3.1, 3.2, and 3.5), their dynamic translations can be deduced by applying the corresponding rules in definition 4.4.4. Take implication for instance, assume A and B are FOL formulas:

$$\begin{aligned}
\overline{A \to B} &= \overline{\neg(A \wedge \neg B)} \\
&= \overline{\neg}(\overline{A}\overline{\wedge}(\overline{\neg}\overline{B})) \\
&= \neg^d_{TTDL}(\overline{A} \wedge^d_{TTDL} (\neg^d_{TTDL}\overline{B})) \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\overline{A}e(\lambda e'.\neg(\overline{B}\ e'\ \mathbf{stop}))) \wedge \phi e
\end{aligned} \tag{4.39}$$

As to the other two operators (disjunction and universal quantifier), their dynamic translations will be provided in the next chapter. The semantics of TTDL is basically the same as in FOL. Logical constants are interpreted in the standard way, see definition 3.2.19. In the next subsection, we will present the applications of TTDL on linguistic examples.

4.4.3 Illustration

As presented in the previous subsection, the translation process in definition 4.4.4 systematically converts static λ-terms into their dynamic counterparts. In this subsection, before going to the illustrations of sentences and discourses, we will first show how to obtain the dynamic lexical entries.

Lexical Entries

We will take the transitive verb (e.g., *beat*) as an example and conduct its translation step by step.

1. The standard entry for *beat*:

$$[\![beat]\!] = \lambda OS.S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))$$

   It takes two NPs and yields a proposition, its type is $((\iota \to o) \to o) \to ((\iota \to o) \to o) \to o$.

2. According to definition 4.4.4:

$$\begin{aligned} \overline{[\![beat]\!]} &= \overline{\lambda OS.S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))} \\ &= \lambda OS.\overline{S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))} \\ &= \lambda OS.S(\lambda x.O(\lambda y.\overline{\mathbf{beat}}\ x\ y)) \end{aligned} \tag{4.40}$$

3. The predicate constant **beat** is of type $\iota \to \iota \to o$, hence according to definition 4.4.3:

$$\begin{aligned} \overline{\mathbf{beat}} &= \mathbb{D}_{\iota\to\iota\to o}(\lambda e.\mathbf{beat}) \\ &= \lambda x.\mathbb{D}_{\iota\to o}(\lambda e.(\lambda e'.\mathbf{beat})e(\mathbb{S}_\iota xe)) \\ &\to_\beta \lambda x.\mathbb{D}_{\iota\to o}(\lambda e.\mathbf{beat}\ x) \\ &= \lambda xy.\mathbb{D}_o(\lambda e.(\lambda e'.\mathbf{beat}\ x)e(\mathbb{S}_\iota ye)) \\ &\to_\beta \lambda xy.\mathbb{D}_o(\lambda e.\mathbf{beat}\ x\ y) \\ &= \lambda xy.(\lambda e\phi.((\lambda e'.\mathbf{beat}\ x\ y)e \wedge \phi e)) \\ &\to_\beta \lambda xye\phi.(\mathbf{beat}\ x\ y \wedge \phi e) \end{aligned} \tag{4.41}$$

4. As a result, by substituting the $\overline{\mathbf{beat}}$ in formula 4.40 with the result of formula 4.41, we can obtain:

$$\begin{aligned} \overline{[\![beat]\!]} &= \lambda OS.S(\lambda x.O(\lambda y.\overline{\mathbf{beat}}\ x\ y)) \\ &= \lambda OS.S(\lambda x.O(\lambda y.(\lambda x'y'e\phi.(\mathbf{beat}\ x'\ y' \wedge \phi e))(x, y))) \\ &\twoheadrightarrow_\beta \lambda OS.S(\lambda x.O(\lambda ye\phi.(\mathbf{beat}\ x\ y \wedge \phi e))) \end{aligned}$$

   O and S are both of the dynamized NP type, namely $(\iota \to \Omega) \to \Omega$, x and y are both of type ι, hence $\overline{[\![beat]\!]}$ is of type $((\iota \to \Omega) \to \Omega) \to ((\iota \to \Omega) \to \Omega) \to \Omega$.

The same procedure can be carried out for any other syntactic category. In Appendix A.1, we provide the systematic dynamizations of more lexical entries in TTDL.

Sentences & Discourses

Now that we know how to obtain the dynamic lexical entries, we will use them to compute the representations of sentences and discourses. Same as for DRT and DPL, we will illustrate TTDL with the two classical examples, namely the inter-sentential anaphora (6) and donkey anaphora (7). We shall look at them one by one.

(6) A man$_i$ walks in the park. He$_i$ whistles.

Same as what we have done in section 3.2, the logical representations of (6)-1 and (6)-2 can be obtained with the function-application information from the syntactic structures. In the following context, we use the notation $[\![\ ]\!]_{TTDL}$ to indicate the logical representation of an expression under TTDL.

$$\begin{aligned}[\![(6)\text{-}1]\!]_{TTDL} &= \overline{[\![walk]\!]([\![a]\!][\![man]\!])} \\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x \wedge \phi(x :: e))\end{aligned}$$

$$\begin{aligned}[\![(6)\text{-}2]\!]_{TTDL} &= \overline{[\![whistle]\!][\![he]\!]} \\ &\twoheadrightarrow_\beta \lambda e\phi.(\mathbf{whistle}(\mathsf{sel}\ e) \wedge \phi e)\end{aligned}$$

To obtain the semantic representation for the overall discourse (6), we simply use the update function (formula 4.33), or equivalently, the dynamic conjunction (formula 4.34), to combine the two component sentences.

$$\begin{aligned}[\![(6)]\!]_{TTDL} &= [\![(6)\text{-}1]\!]_{TTDL} \wedge^d_{TTDL} [\![(6)\text{-}2]\!]_{TTDL} \\ &= \lambda ABe\phi.Ae(\lambda e'.Be'\phi) \\ &\qquad (\lambda e\phi.\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x \wedge \phi(x :: e))) \\ &\qquad (\lambda e\phi.(\mathbf{whistle}(\mathsf{sel}\ e) \wedge \phi e)) \\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x \wedge \mathbf{whistle}(\mathsf{sel}(x :: e)) \wedge \phi(x :: e))\end{aligned}$$

Assume the choice operator sel selects variable x in the current left context $x :: e$), the anaphoric link between the pronoun *he* and the indefinite *a man* is successfully established. By passing the empty left context nil together with the empty right context **stop** to the above formula, we can end up the following representation:

$$[\![(6)]\!]_{TTDL}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_\beta \exists x.(\mathbf{man}\ x \wedge \mathbf{walk\_in\_the\_park}\ x \wedge \mathbf{whistle}\ x)$$

As we can see, the finally obtained result, which is achieved in a purely compositional way, is exactly the same as the expected FOL translation proposed by Geach (1962). Compare the above formula with formula 4.5.

Now let's turn to the conditional donkey sentence (7), repeated as follows:

(7) If a farmer$_i$ owns a donkey$_j$, he$_i$ beats it$_j$.

With the corresponding lexical entries and the syntactic information, we can obtain the semantic representations of the antecedent and the consequent of the conditional in a straightforward way:

$$\begin{aligned}[\![(7)\text{-}1]\!]_{TTDL} &= \overline{[\![own]\!]([\![a]\!][\![donkey]\!])([\![a]\!][\![farmer]\!])} \\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y \wedge \phi(y :: (x :: e))))\end{aligned}$$

$$\begin{aligned}[\![(7)\text{-}2]\!]_{TTDL} &= \overline{([\![beat]\!][\![it]\!])[\![he]\!]} \\ &\twoheadrightarrow_\beta \lambda e\phi.(\mathbf{beat}\ (\mathsf{sel_{he}}\ e)\ (\mathsf{sel_{it}}\ e) \wedge \phi e)\end{aligned}$$

In $[\![(7)\text{-}2]\!]_{TTDL}$, the choice operators introduced by the two pronouns *he* and *it* are distinguished with corresponding subscripts. Different from the previous example, (7) is a complex single sentence (conditional). So we need to use implication, instead of conjunction, to sequence the two component sentences. Based on formula 4.39, we carry out the following computation:

$$\begin{aligned}[\![(7)]\!]_{TTDL} &= \overline{[\![(7)\text{-}1]\!] \rightarrow [\![(7)\text{-}2]\!]} \\ &= \bar{\neg}([\![(7)\text{-}1]\!]_{TTDL} \bar{\wedge} (\bar{\neg}[\![(7)\text{-}1]\!]_{TTDL})) \\ &\twoheadrightarrow_\beta (\lambda ABe\phi.\neg(Ae(\lambda e'.\neg(B\ e'\ \mathbf{stop}))) \wedge \phi e) \\ &\qquad (\lambda e\phi.\exists x.(\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y \wedge \phi(y :: (x :: e))))) \\ &\qquad (\lambda e\phi.(\mathbf{beat}\ (\mathsf{sel_{he}}\ e)\ (\mathsf{sel_{it}}\ e) \wedge \phi e)) \\ &\twoheadrightarrow_\beta \lambda e\phi.\neg\exists x.(\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge (\mathbf{own}\ x\ y \wedge \\ &\qquad\qquad \neg\mathbf{beat}\ (\mathsf{sel_{he}}(y :: (x :: e)))\ (\mathsf{sel_{it}}(y :: (x :: e)))))) \wedge \phi e \\ &= \lambda e\phi.\forall x.(\mathbf{farmer}\ x \rightarrow \forall y.(\mathbf{donkey}\ y \rightarrow (\mathbf{own}\ x\ y \rightarrow \\ &\qquad\qquad \mathbf{beat}\ (\mathsf{sel_{he}}(y :: (x :: e)))\ (\mathsf{sel_{it}}(y :: (x :: e)))))) \wedge \phi e\end{aligned}$$

Let ϕ and ψ be predicate logical formulas. Because of the De Morgan's laws, we have the following logical equivalence:

$$\neg\exists x.(\phi \wedge \psi) = \forall x.(\phi \rightarrow \neg\psi)$$

This explains why at the last step of the above computation, both existential quantifiers are transformed into universal ones. Assume each of the choice operators $\mathsf{sel_{he}}$ and $\mathsf{sel_{it}}$ selects the appropriate variable, namely x and y, respectively. Again, by passing the empty left context nil and the empty right context **stop** to $[\![(7)]\!]_{TTDL}$, we can obtain the following representation:

$$[\![(7)]\!]_{TTDL}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_\beta \forall x.(\mathbf{farmer}\ x \rightarrow \forall y.(\mathbf{donkey}\ y \rightarrow (\mathbf{own}\ x\ y \rightarrow \mathbf{beat}\ x\ y)))$$

This is the exactly the desired reading for donkey sentences. Compare the above formula with formula 4.9, they are logically equivalent and each can be transformed into the other through the De Morgan's laws. Because TTDL follows the simply typed λ-calculus in a way that the logical constants receive their conventional semantics, so the interpretations of the above formulas are exactly the same as in FOL, which we will not

not proceed.

One final remark on the left context. In order to resolve the problematic anaphoras for classical semantic theories (i.e., the inter-sentential anaphora and the donkey sentence), de Groote (2006) designs the left context as a list of individuals. That is to say, a left context contains potential antecedents for an upcoming anaphoric expression. It is thus analogous to the set of accessible discourse referents in DRT, and the active quantifier occurrences in DPL. We will see this in detail in the next chapter. However, the left context in TTDL is in fact a parameter which is flexible enough to express various sorts of discourse information. With the manipulations on the types of the left context, this same framework has been proved to be fairly adaptive in many other linguistic phenomena, such as modal subordination Asher and Pogodalla (2011a), rhetorical structure Asher and Pogodalla (2011b), presupposition and implicature De Groote and Lebedeva (2010); Lebedeva (2012).

So far as we've shown, TTDL has proved to be completely feasible in dealing with the set of problems that other dynamic frameworks such as DRT and DPL are aiming for, but in a more classical way.

Chapter 5

Discourse Referent and Exceptions

Contents

> *In general, an indefinite NP establishes a permanent discourse referent just in case the quantifier associated with it is attached to a sentence that is asserted, implied, or presupposed to be true and there are no higher quantifiers involved.* Karttunen (1969)

In the previous chapter, we have introduced three dynamic frameworks, and illustrated their potential in handling inter-sentential anaphora, as in example (6), and donkey anaphora, as in example (7). However, a discourse referent/variable is not always accessible for upcoming anaphoric expressions, see example (70), and (71).

The perspective of this chapter is to review a number of exceptions which do not conform to the prediction of standard dynamic frameworks. This will form the motivations for the next two chapters. We will start with comparing the accessibility constraints of the three dynamic frameworks, which have been presented in chapter 4. Then we shall elaborate the notion of discourse referent. At the same time, some other accessibility constraints, which can be treated in a similar way as negation in dynamic semantics, will be illustrated. Finally, we focus on the exceptional cases, and explain in detail why they fail to receive an appropriate account with the current setup of dynamic frameworks.

5.1 Accessibility Constraints

As shown in the previous chapter, various dynamic semantic frameworks have their own constraints on anaphoric relations, for example, accessibility in DRT (definition 4.2.7), active quantifier in DPL (definition 4.3.4), left context in TTDL. In this subsection, we will concentrate on the notion of accessibility, and make a general comparison between the three frameworks: DRT, DPL and TTDL.

5.1.1 Accessibility in DRT

As shown in Section 4.2.3, DRT can successfully account for inter-sentential anaphora (6) and donkey anaphora (7). As indicated in definition 4.2.7, if a DRS subordinates another, then the discourse referents of the former are accessible from the latter. That is to say, an anaphoric expression can only be linked with potential antecedents at the same level or above. For instance, the discourse referent introduced by an indefinite NP in the scope of a negation operator is not available for subsequent anaphors that are outside the negation. Take the following examples:

(73) Bill doesn't have a car$_i$.
  a. *It$_i$ is black.
  b. *The car$_i$ is black.
  c. *Bill's car$_i$ is black. Karttunen (1969)

(74) a. It is not the case that a man$_i$ walks in the park. *He$_i$ whistles.
  b. No man$_i$ walks in the park. *He$_i$ whistles. Groenendijk and Stokhof (1991)

All anaphoras in example (73) and (74) appear awkward because in each discourse, the indefinite NP in the first sentence occurs within the scope of a negation. This apparent inaccessibility falls under DRT's prediction because negation gives rise to a subordinating relation (see definition 4.2.6).

As to disjunction, it is defined in terms of negation (formula 4.10). We repeat the original formula as follows:

$$K_1 \vee K_2 \triangleq \neg\langle \emptyset, \neg K_1 \cup \neg K_2 \rangle \tag{4.10}$$

where K_1 and K_2 are DRSs. In DRT, disjunction is treated as a DRS-condition. Assume $K_1 \vee K_2$ occurs in a DRS $K = \langle R_K, Con_K \rangle$, namely $K_1 \vee K_2 \in Con_K$. Then according to definition 4.2.6, neither K_1 subordinates K_2, nor the other way around; further more, both K_1 and K_2 do not subordinate any DRS which is at the same level or a superior level of K. This implies that, on the one hand, discourse referents of neither disjunct are accessible from the other; on the other hand, none the referents introduced in the disjunction is accessible from external context. These two features can be used to account for infelicitous anaphras in the following examples:

(75) a. ?Jones owns a car$_i$ or he hides it$_i$. Kamp and Reyle (1993)
  b. *John has a new car$_i$ or Mary has it$_i$. Chierchia (1995)
  c. *Either Jones owns a bicycle$_i$, or it$_i$'s broken. Simons (1996)

(76) a. Either John is home or he went out to get a coke$_i$. *It$_i$ is sugar free. Chierchia (1995)

b. Jane either borrowed a car$_i$ or rented a truck$_j$ to get to Boston. *It$_{i/j}$ broke down on the way. Simons (1996)

Example (75) shows that no anaphoras are possible between the two parts of a disjunction. And example (76) shows that when antecedents are in the disjunction while anaphors are outside, the anaphoric links are unacceptable, as well.

Now let's turn to implication. In DRT, it has also been defined based upon the negation operator as well, see formula 4.11, which is repeated as follows:

$$K_1 \to K_2 \triangleq \neg\langle R_{K_1}, Con_{K_1} \cup \neg K_2\rangle \tag{4.11}$$

where K_1 and K_2 are DRSs. Let's use $K'_{1,2}$ as an abbreviation for the DRS $\langle R_{K_1}, Con_{K_1} \cup \neg K_2\rangle$, namely:

$$K_1 \to K_2 \triangleq \neg\langle R_{K_1}, Con_{K_1} \cup \neg K_2\rangle = \neg K'_{1,2}$$

Then according to definition 4.2.6, $K'_{1,2}$ subordinates K_2; in addition, $K'_{1,2}$ and K_2 are not able to subordinate subsequent DRSs because both of them are in the scope of a negation. Hence, on the one hand, discourse referents in the antecedent of an implication, namely R_{K_1} are accessible from the consequent of the implication, namely K_2, on the other hand, referents introduced from either the antecedent or the consequent, namely $R_{K_1} \cup R_{K_2}$, are not accessible from subsequent structures. The first property can well predict donkey sentences, either the quantified version (69) or the conditional version (7); the second property is used to explain the following sets of examples:

(77) a. If a farmer$_i$ owns a pedigree donkey$_j$, he$_i$ is rich. *It$_j$ lives on caviar.
b. Every farmer owns a donkey$_i$. *It$_i$ brays in distress. Elworthy (1992)

(78) a. Every man$_i$ walks in the park. *He$_i$ whistles.
b. Every farmer$_i$ who owns a donkey$_j$ beats it$_j$. *He$_i$ hates it$_j$. Groenendijk and Stokhof (1991)

As shown in examples (77) and (78), anaphoric relations where the anaphors lay outside the scope of an implication are infelicitous.

In summary, we recap the accessibility constraints in DRT as follows:

1. Negation blocks the accessibility of discourse referents within its scope;

2. Disjunction blocks the accessibility of discourse referents from either disjunct, as well as from outside its scope;

3. Implication admits the accessibility of discourse referents in the antecedent from the consequent, but not from outside its scope.

5.1.2 Internal and External Dynamicity in DPL

As introduced in section 4.3, DPL is a dynamic system which uses the standard syntax of FOL, but formulas are interpreted in a completely different way. In FOL, the semantics of a formula is a set of assignment functions, which makes the formula true; while in DPL, it is identified with a set of transitions between assignment functions, which represents the formula's potential to change the context. With the new semantics, one may obtain a couple of interesting logical properties, for instance:

$$(\exists x.\phi) \wedge \psi = \exists x.(\phi \wedge \psi) \tag{5.1}$$

$$(\exists x.\phi) \rightarrow \psi = \forall x.(\phi \rightarrow \psi) \tag{5.2}$$

Assume ϕ and ψ are FOL/DPL formulas, equivalence 5.1 and 5.2 hold in FOL only when $x \notin FV(\psi)$. However in DPL, even if x occurs free in ψ, 5.1 and 5.2 are still satisfied. As one may see, the above two logical equivalences can respectively be used to explain why the inter-sentential anaphora, as in example (6), and donkey anaphora, as in example (7), are correctly justified. Unlike in FOL, the scope of the (existential) quantifier in DPL has the potential to extend beyond sentence boundary in order to bind subsequent occurrences of variable. This is exactly due to its non-classical semantics. In what follows, we will induce the notion of accessibility in DPL based on its semantics.

Looking into the details of the semantic interpretation of DPL (definition 4.3.1), we can classify the logical connectives into different groups, with respect to the feature of **dynamicity**. Assume M is a model, ϕ and ψ are formulas. Then the conjunction $\phi \wedge \psi$ is interpreted in the following way (rule 3 in definition 4.3.1):

$$[\![(\phi \wedge \psi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid \exists k : \langle g, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ and } \langle k, h\rangle \in [\![\psi]\!]^M_{DPL}\}$$

Firstly, the input assignment g is changed into the output assignment k for interpreting the left conjunct ϕ. Then k is used as the input for the interpretation of the right conjunct ψ, this returns the output assignment h, which is also the output assignment of the whole conjunction. On the one hand, information from ϕ is passed to ψ (through the "intermediate" assignment k), hence we call $\wedge$ **internally dynamic**; on the other hand, information from the whole conjunction, which is contained in the overall output h, will continue to be passed to subsequent sentences (assume that they are composed by conjunction), hence we call $\wedge$ **externally dynamic**.

Further more, let's look at the negation. Again, assume ϕ is a DPL formula, the interpretation of $\neg\phi$ has been defined in rule 2 in definition 4.3.1:

$$[\![(\neg\phi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \neg\exists k : \langle h, k\rangle \in [\![\phi]\!]^M_{DPL}\}$$

According to the above definition, if a pair of assignments $\langle g, h\rangle$ is among the interpretation of $\neg\phi$, then the input assignment g is required to be identical to the output assignment h. This means, negation does not pass any new (variable assignment) information to subsequent sentences, variables within its scope are "invisible/inaccessible" from external sub-formulas. Hence we call the negation operator $\neg$ **externally static**. In fact, all tests, as defined in definition 4.3.3, are externally static.

As to existential quantified propositions such as $\exists x.\phi$, where x is a variable, ϕ is a DPL formula, its interpretation is as follows (rule 4 in definition 4.3.1):

$$[\![(\exists x.\phi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid \exists k : k[\{x\}]g \text{ and } \langle k, h\rangle \in [\![\phi]\!]^M_{DPL}\}$$

One might see that it has a similar pattern as conjunction: within the scope of $\exists$, all occurrences of x are bound; in addition, assignment information on x will be passed to sentences yet to come, namely $\exists$ can bind variables outside its scope. See the following interpretation. So like $\wedge$, the existential quantifier $\exists$ is both internally and externally dynamic, as well.

As to the dynamicity of other connectives, such as $\vee$, $\rightarrow$, and $\forall$, we will first have to transform them according to formula 3.1, 3.2 and 3.5, then check the corresponding semantics exactly as what we did above for $\neg$, $\wedge$ and $\exists$. For instance, we can obtain the following interpretations[1]:

$$[\![(\phi \vee \psi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \exists k : \langle h, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ or } \langle h, k\rangle \in [\![\psi]\!]^M_{DPL}\}$$

$$[\![(\phi \rightarrow \psi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \forall k : \text{ if } \langle h, k\rangle \in [\![\phi]\!]^M_{DPL} \text{ then } \exists j : \langle k, j\rangle \in [\![\psi]\!]^M_{DPL}\}$$

$$[\![(\forall x.\phi)]\!]^M_{DPL} = \{\langle g, h\rangle \mid h = g \text{ and } \forall k : \text{ if } k[\{x\}]h \text{ then } \exists j : \langle k, j\rangle \in [\![\phi]\!]^M_{DPL}\}$$

As a result, based on our previous analysis on $\wedge$, $\neg$ and $\exists$, we can infer that $\vee$ is both externally and internally static, $\rightarrow$ and $\forall$ are internally dynamic while externally static. We sum up the above presented dynamicity of all logical connectives in Table 5.1. In fact, dynamicity reflects the potential of a variable being bound in subsequent formulas, it is equivalent to the notion of active quantifier occurrence as defined in definition 4.3.4.

Connectives	**Internally**	**Externally**
$\neg$	—	static
$\wedge$	dynamic	dynamic
$\vee$	static	static
$\rightarrow$	dynamic	static
$\exists$	dynamic	dynamic
$\forall$	dynamic	static

Table 5.1 Dynamicity of Connectives in DPL

As a compositional alternative of DRT, DPL makes a similar set of empirical predications on referent accessibility. While in DPL, it is achieved via the dynamicity of connectives, rather than the structural configurations as in DRT. For instance, anaphoras in example (73) and (74) are anomalous because negation is externally static: the variable assignment information within the scope of negation is not passed on to subsequent sentences, namely these "internal" variables are inaccessible from outside; in addition, anaphoric links in examples such as (75) and (76), are infelicitous as well, because disjunction is both internally and externally static, which means anaphoras are impossible between the disjuncts, also they are not allowed from subsequent sentences; finally, intra-sentential anaphoras are admitted, while inter-sentential ones are denied, when implication is involved, as in example (77) and (78), which can be accounted for by the fact that universal quantifier shares with implication the characteristic of being internally dynamic but externally static.

In summary, dynamicity of connectives in DPL denotes the same notion of accessibility as in DRT. We recap the accessibility constraints of variables in DPL as follows:

1. Negation disables assignments of existentially bound variables within its scope to be passed to anaphoric expressions outside it;

[1]The stepwise computations are omitted, interested readers may refer to the illustrations in section 3.1.1, 3.1.2 and 3.1.3, where we have achieved the semantics of various logical constants, e.g., $\vee$, $\rightarrow$, $\forall$, $\diamond$, through the corresponding primitives.

2. Conjunction passes assignments of existentially bound variables from the left conjunct to the right one; in addition, assignments of existentially bound variables of both conjuncts are passed to upcoming clauses;

3. Disjunction disables assignments of existentially bound variables of either disjunct to be passed to the other; in addition, assignments of existentially bound variables of neither conjunct can be passed in upcoming clauses;

4. Implication passes the assignments of existentially bound variables of the antecedent to the consequent, and modifies the quantificational force to universal; in addition, assignments of existentially bound variables of neither component can be passed to upcoming clauses;

5. Existential quantifier can extend its scope to subsequent clauses, which is subject to the constraints from the logical connectives that lay between sub-formulas;

6. Universal quantifier does not extend its scope.

5.1.3 Left Context in TTDL

As we mentioned in Section 4.4, by introducing a third atomic type γ besides the two conventional ones: ι and o, TTDL successfully integrates the notion of context in MG. A sentence is now interpreted with respect to both its left and right context.

The left context, which is of type γ, consists of the context information of the already-processed discourse. Technically, the left context in TTDL is a list of variables that subsequent anaphors can refer to, which is similar to the notion of accessible discourse referents in DRT, and the dynamicity or active quantifier occurrence in DPL. As a result, in order to investigate the accessibility constraints in TTDL, we ought to have a look at the impacts that different logical constructs have on the left context.

Let's start with the three primitive dynamic logical connectives: $\wedge^d_{TTDL}$, $\neg^d_{TTDL}$, and $\exists^d_{TTDL}$. As for the dynamic conjunction $\wedge^d_{TTDL}$, its definition has been provided by formula 4.34, repeated as follows:

$$\wedge^d_{TTDL} \triangleq \lambda ABe\phi.Ae(\lambda e'.Be'\phi) \tag{4.34}$$

The dynamic connective $\wedge^d_{TTDL}$ takes two dynamic propositions A and B, and returns another dynamic proposition. During the composition, the second conjunct B is interpolated into the continuation of the first conjunct A. This gives rise to such an effect that the context information of A is updated into the left context of B. Hence anaphoric expressions in B can select variables introduced in A. Further more, after the whole conjunction is interpreted, the variables in the left contexts of A and B will remain accessible. They will thus contribute to the left context of subsequent sentences. In terms of DPL, $\wedge^d_{TTDL}$ is both internally and externally dynamic.

Concerning the dynamic negation $\neg^d_{TTDL}$, its definition has been given in formula 4.37, repeated as follows:

$$\neg^d_{TTDL} \triangleq \lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e \tag{4.37}$$

The dynamic negation $\neg^d_{TTDL}$ is a unary connective, it takes a dynamic proposition A, and returns its negated form, which is also a dynamic proposition. More specifically, the logical content of A is negated by operator $\neg$, besides, the empty continuation $\lambda e'.\top$ is passed to A. Hence, dynamic negation terminates the discourse updating based upon its argument. Namely it prohibits any variable within its scope to be accessed from outside, and subsequent anaphoric elements can only refer to variables introduced prior to the negation, if there is any. Hence, same as the negation in DPL, $\neg^d_{TTDL}$ is externally static. Because of that, TTDL provides the correctly interpretations for examples involving anaphora under negation, such as (73) and (74).

As to the dynamic existential quantifier $\exists^d_{TTDL}$, it has been defined in formula 4.38, repeated as follows:

$$\exists^d_{TTDL} \triangleq \lambda P e \phi . \exists (\lambda x . P x (x :: e) \phi) \tag{4.38}$$

It takes a dynamic property P, which is of type $\iota \rightarrow \Omega$, and returns an existentially quantified dynamic proposition. As we mentioned before in section 4.4.2, $(x :: e)$ in formula 4.38 indicates that an existentially quantified proposition has the potential to update the context by making variable x accessible for subsequent anaphoric expressions, which are outside its scope. Accordingly, same as $\wedge^d_{TTDL}$, $\exists^d_{TTDL}$ is both internally and externally dynamic. That is why discourses involving inter-sentential anaphoras, such as example (6), can be successfully accounted for in TTDL.

Now let's turn to the derived connectives. The dynamic implication is not defined directly. Rather, as shown in section 4.4.2, it can be obtained through a stepwise dynamic translation of formula 3.1. The detailed computations have been provided by formula 4.39, repeated as follows, where A and B are FOL formulas:

$$\begin{aligned}
\overline{A \rightarrow B} &= \overline{\neg(A \wedge \neg B)} \\
&= \overline{\neg}(\overline{A} \overline{\wedge} (\overline{\neg} \overline{B})) \\
&= \neg^d_{TTDL}(\overline{A} \wedge^d_{TTDL} (\neg^d_{TTDL} \overline{B})) \\
&\twoheadrightarrow_\beta \lambda e \phi . \neg(\overline{A} e (\lambda e' . \neg(\overline{B}\ e'\ \mathbf{stop}))) \wedge \phi e
\end{aligned} \tag{4.39}$$

The dynamic implication passes the dynamic negation of the consequent proposition $\overline{B}$ into the continuation of the antecedent proposition $\overline{A}$, then the empty right context **stop** is passed to the whole conditional proposition. Hence, variables introduced in the antecedent can be accessed by the consequent, but no variables in the implication are accessible from outside. As a result, the dynamic implication is internally dynamic while externally static.

To obtain the representations for the dynamic disjunction and dynamic universal quantification, an analogous analysis can be carried out. In what follows, we dynamically translate formulas 3.2 and 3.5. De Morgan's laws are used in the last step of the computation:

$$\begin{aligned}
\overline{A \vee B} &= \overline{\neg(\neg A \wedge \neg B)} \\
&= \overline{\neg}(\overline{\neg} \overline{A} \overline{\wedge} \overline{\neg} \overline{B}) \\
&\twoheadrightarrow_\beta \lambda e \phi . \neg(\neg(\overline{A}\ e\ \mathbf{stop}) \wedge \neg(\overline{B}\ e\ \mathbf{stop})) \wedge \phi e \\
&= \lambda e \phi . ((\overline{A}\ e\ \mathbf{stop}) \vee (\overline{B}\ e\ \mathbf{stop})) \wedge \phi e
\end{aligned} \tag{5.3}$$

$$\begin{aligned}
\overline{\forall(\lambda x.A)} &= \overline{\neg\exists(\lambda x.\neg A)} \\
&= \overline{\neg}\overline{\exists}(\lambda x.\overline{\neg}\overline{A}) \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg\exists x.(\neg(\overline{A}(x :: e)\mathbf{stop})) \wedge \phi e \\
&= \lambda e\phi.\forall x.(\overline{A}(x :: e)\mathbf{stop}) \wedge \phi e
\end{aligned} \tag{5.4}$$

We shall examine the two logical connectives one by one. Firstly, the dynamic disjunction takes two dynamic propositions $\overline{A}$ and $\overline{B}$, and returns another dynamic proposition. The empty right context **stop** is passed to each disjunct. That is to say, variables introduced in both disjuncts are not allowed to be accessed from outside. Hence neither are variables of the two disjuncts passed to each other (internally static), nor are they updated to the current left context ϕ (externally static). Namely, the dynamic disjunction is both internally and externally static. This explains the problematic anaphoric examples in (75) and (76).

Turn to the dynamic universal quantification. Due to the sub-part $(x :: e)$ in formula 5.4, bound variables, such as x, are accessible within the scope of the quantification. However, the empty right context **stop** is passed to the dynamic proposition $\overline{A}$. Hence no subsequent anaphoric expression can refer to variables introduced within the scope of the quantification. In terms of DPL, the dynamic universal quantification is internally dynamic but externally static. This offers a correct account for examples such as (77) and (78).

In summary, the logical constants in TTDL share the same dynamic features as their counterparts in DPL. We recap the accessibility constraints in TTDL as follows:

1. Dynamic conjunction passes the right conjunct to the continuation of the left conjunct; further more, upcoming clauses can access an updated left context, which contains variables from both conjuncts;

2. Dynamic negation passes the empty right context to the (dynamic) proposition within its scope, and it does not modify the current left context, which upcoming clauses will access;

3. Dynamic existential quantifier appends the variables it binds to the current left context, and passes the updated left context to upcoming clauses;

4. Dynamic disjunction passes the empty right context to both disjuncts, it does not modify the current left context, which upcoming clauses will access;

5. Dynamic implication passes the consequent proposition to the continuation of the antecedent proposition; then it passes the empty right context to the whole conditional, and the current left context is not modified;

6. Dynamic universal quantifier passes an updated left context, which contains variables it binds, to the proposition within its scope; then it passes the empty right context to the proposition. Hence the current left context, which upcoming clauses will access, is not modified.

Above, we have shown that all the three dynamic frameworks, namely DRT, DPL and TTPL, have the same predictions on referent/variable accessibility, though they are expressed in different forms: the structural configuration (i.e., subordination) in DRT, the dynamicity in DPL, the left context in TTDL.

5.2 Discourse Referent

In the late 1960s, Karttunen has researched on indefinites and their potential to serve as antecedent in discourse. In Karttunen (1969), the author introduced the notion of **discourse referent**, which was later incorporated in dynamic semantics, in particular, DRT. Moreover, this notion offers a reconciliation of the referential and bound analysis of anaphora (see section 2.3.1): anaphoric expressions are uniformly treated as variable-like entities.

Karttunen suggested that novel entities (e.g., individuals, events, objects, etc.) would be introduced during the discourse processing. These entities are the prototypes of discourse referent. It is the function of indefinite NPs to establish them. And anaphoric expressions will be identified with existing entities in the discourse. This is the way such that antecedents and anaphors are related. But how can discourse referent be formally defined? According to Karttunen, the closest description is as follows:

> *Let us say that the appearance of an indefinite noun phrase establishes a discourse referent just in case it justifies the occurrence of a coreferential pronoun or a definite noun phrase later in the text.* Karttunen (1969)

Thus discourse referent is not exactly the referent of a referring expression, and it does not even necessarily correlate with reference (see definition 2.1.1). As remarked by Heim:

> *I even think that "discourse referent" is a misleading term, aside from being superfluous, because reference has nothing to do with it.* Heim (1982)

In fact, Karttunen himself was also not much concerned with the metaphysics of discourse referent, which is a philosophical issue. Now let's follow Karttunen and leave the formal definition of discourse referent aside. To put it simply, we assume that if an indefinite can serve as antecedent in an anaphoric relation, it introduces a discourse referent; otherwise, it does not. In this thesis, we will concentrate on the conditions under which indefinites introduce discourse referents, more importantly, the environments where the antecedent-potential of indefinite may cease. In section 2.3.2, we have briefly discussed Chomsky's GB Theory, which gives an account on various possibility of anaphora in the syntactic tradition. However, the GB Theory only works for intra-sentential anaphora. This thesis will extend to discourse anaphora, and it is the semantic constraints that are to be focused on. For the rest of this section, the concept of specificity will be discussed first. Then we will provide a range of Karttunen's observations on anaphora, which can be properly handled by the dynamic frameworks presented so far.

5.2.1 Specificity

The goal of this subsection is to present the notion of **specificity**. After that, a range of interpretations which this thesis is not interested in will be excluded.

In the previous section, we have seen various semantic constraints that have an impact on the occurrence of anaphoric expressions. For instance, negation, disjunction, and implication will not allow anaphors to be resolved within their scopes, see examples (73), (74), (75), (76), (77), and (78) for demonstrations. While there are cases where the above rules might run into trouble:

(79) Bill didn't find a misprint$_i$. Can you find it$_i$? Karttunen (1969)

The first part of (73) and (79) are in a parallel syntactic structure. However, it is permitted to continue the latter with an anaphoric pronoun, but not the former. To explicate that, we have to look into the notion of specificity. Specificity is a property used to determine different interpretations of a NP Farkas (1994, 2002); Quine (1956); Von Heusinger (2002). It is essentially concerned only with indefinite NPs, although on rare occasions people also talk about specificity for definite expressions. Other NPs, such as quantified NPs, are basically out of the scope of specificity. Semantically, specificity is often related to the theories of reference. In this thesis, we simply consider it as the interplay between existential quantification (from indefinite NP) and other scope-bearing operators, such as universal quantification, negation, modals, etc. Let's illustrate the concept with the first sentence of (79):

(80) Bill didn't see a misprint.
  a. "There is a misprint which Bill didn't see."
  b. "Bill saw no misprints." Karttunen (1969)

sentence (80) is ambiguous in two ways, with respect to which operator (existential quantifier or negation) takes scope over the other. The two readings are spelled out in (80-a) and (80-b), respectively. For a more explicit illustration on the difference between the two readings, we provide their semantic representations (in standard predicate logic) as follows:

- $[\![(80\text{-a})]\!] = \exists x.(\textbf{misprint}\ x \wedge \neg\textbf{see bill}\ x)$
- $[\![(80\text{-b})]\!] = \neg(\exists x.(\textbf{misprint}(x) \wedge \textbf{see bill}\ x))$

In the former case, the existential quantifier is given wider scope than the negation. We say that the indefinite *a misprint* is interpreted **specifically** such that there exists a particular misprint which Bill didn't see. While in the latter case, the negation has scope over the existential quantifier[2]. Then the indefinite is interpreted **non-specifically** such that Bill did not see any misprint at all, in fact, it does not even imply the existence of any misprint.

Similar observations can be made with respect to intensional predicates, e.g., propositional attitude verbs, modals. Let's look at the following example:

(81) John wants to catch a fish.
  a. "There is a particular fish which John wants to catch."
  b. "What John wants to catch is a fish." Karttunen (1969)

Same as (80), sentence (81) is also ambiguous. The analysis is not difficult: we only need to take the relative position of various scopes from existing operators in consideration. The two readings of sentence (81), namely (81-a) and (81-b) are logically represented

[2]Many languages, including English, have a particular marker for indefinite NP if it is under the immediate scope of negation (i.e., non-specific), *any* for instance. This particular marker will not give rise to the specific/non-specific ambiguity. But it is not the case for (80).

as follows. We treat *want* in a similar way as the "higher-order" verb *try* in Montague (1973). Thus, **want** is a predicate relation between individuals and properties:

- $[\![(81\text{-a})]\!] = \exists x.(\mathbf{fish}\ x \wedge \mathbf{want}\ \mathbf{john}\ (\lambda y.\mathbf{catch}\ y\ x))$
- $[\![(81\text{-b})]\!] = \mathbf{want}\ \mathbf{john}\ \lambda y.(\exists x.(\mathbf{fish}\ x \wedge \mathbf{catch}\ y\ x))$

In the former reading, the existential quantifier is read with wider scope than the propositional attitude verb *want*. The indefinite *a fish* is hence in its specific interpretation. In the latter reading, the existential quantifier is under the scope of *want*. So the indefinite is interpreted non-specifically. In the semantic literature, cases like (81), where intensional predicates (or opaque predicates in terms of Quine (1956)) are involved, are termed the *de re*/*de dicto* ambiguity as well. In particular, *de re* corresponds to the specific interpretation, *de dicto* corresponds to the non-specific one.

Both examples that we have presented so far on specificity: (80) and (81), are two-way ambiguous, namely the indefinite is either specific or non-specific. However, specificity is a relative concept in nature, rather than an absolute one. That is to say, when more than two scopes are involved in the context, an indefinite NP can be either specific or non-specific, with respect to different operators. Take the following example:

(82) Mary may want to marry a Swede.
  a. "There is some Swede whom Mary may want to marry."
  b. "It may be the case that there is some Swede whom Mary wants to marry."
  c. "It may be the case that Mary wants her future husband to be a Swede." Karttunen (1969)

In sentence (82), there are three semantic scopes: one introduced by the modal *may*, one introduced by the propositional attitude verb *want*, and one introduced by the indefinite *a Swede*. By various permutations of the scopes, we can interpret (82) at least in three ways, listed as (82-a), (82-b), and (82-c), respectively. Their corresponding semantic representations are provided as follows, where the modal *may* is treated as a sentential operator such that it takes a proposition and returns another proposition:

- $[\![(82\text{-a})]\!] = \exists x.(\mathbf{Swede}\ x \wedge \mathbf{may}(\mathbf{want}\ \mathbf{mary}\ (\mathbf{marry}\ \mathbf{mary}\ x)))$
- $[\![(82\text{-b})]\!] = \mathbf{may}(\exists x.(\mathbf{Swede}\ x \wedge \mathbf{want}\ \mathbf{mary}\ (\mathbf{marry}\ \mathbf{mary}\ x)))$
- $[\![(82\text{-c})]\!] = \mathbf{may}(\mathbf{want}\ \mathbf{mary}\ \exists x.(\mathbf{Swede}\ x \wedge \mathbf{marry}\ \mathbf{mary}\ x))$

As one might see, with the same characterization for previous examples (permutation of scopes), we can account for the various readings of (82) without much difficulty. Firstly, in (82-a), the indefinite NP *a Swede* takes scope over the other two operators. Hence this reading is about some particular Swede, and the indefinite is specific (with respect to all other elements of the sentence). Then in (82-b), *a Swede* is under the scope of *may*, while it is given wider scope than *want*. This time, the indefinite is specific with respect to *want* while non-specific with respect to *may*. Finally in (82-c), the indefinite takes the narrowest scope, so it is non-specific (with respect to all other elements of the sentence). A completely similar analysis can be carried out for the following example, where there are three scopes involved as well. We shall not elaborate on it any more.

(83) Bill intends to visit a museum every day.

a. "There is a certain museum that Bill intends to visit every day."
b. "Bill intends that there be some museum that he visits every day."
c. "Bill intends to do a museum visit every day." Karttunen (1969)

As a summary, specificity is concerned with whether an indefinite NP is within or out of the scopes of some other scope-bearing expressions, e.g., quantifications, negation, intensional predicates, etc. Thus we treat it as a pure scope ambiguity[3]: if the indefinite takes scope over another expression, it is specific relative to that expression and its referent will be a constant from the point of view of that expression; otherwise, if it is under the scope of another expression, then the indefinite is non-specific relative to that expression, and its referent will be dependent on the corresponding expression.

The reason for presenting the notion of specificity is that a specific indefinite will not be affected by the accessibility constraints developed in dynamic frameworks. If an indefinite is interpreted specifically, it can always serve as antecedent for anaphors in subsequent contexts. This explains the acceptability of discourse (79). Also, we may perfectly continue (81) and (82) with anaphoric expressions:

(84) a. John wants to catch <u>a fish</u>$_i$. You can see <u>the fish</u>$_i$ from here.
b. Mary may want to marry <u>a Swede</u>$_i$. She introduced <u>him</u>$_i$ to her mother yesterday. Karttunen (1969)

Discourses in (84) will only be felicitous if the indefinites (i.e., *a fish* and *a Swede*) are specific, namely the two first sentences are read as (81-a), and (82-a). Otherwise, if the indefinites have narrower scope than some other scopes, the anaphoras will fail to have a proper resolution.

The dynamic frameworks presented so far may well account for the accessibility of specific indefinite without difficulty. Take TTDL for instance, one possible technique is to integrate it with the quantifying-in rule devised by Montague. In Montague (1973), the author assigned different derivational structures to sentences, where interaction of scope-bearing elements may take place. However, this solution is not intuitively well-motivated because sentences such as (80), (81) and (82) are generally considered as syntactically (structurally) unambiguous. Another strategy which could be incorporated in TTDL is the Cooper storage Cooper (1975); Keller (1988). It describes the scope issue without introducing the syntactic ambiguity. Under this strategy, the interpretations of NPs are kept in a store. These interpretations could later be retrieved from the store in different orders. Then the wide scope interpretation of the indefinite can be achieved.

Specific indefinites can always serve as antecedent for upcoming anaphoric expressions. In terms of Karttunen, specific indefinites establish **permanent** discourse referents throughout the rest of the discourse. As a result, it is not interesting to discuss the accessibility constraint of specific indefinites. For the rest of the thesis, we will only consider the non-specific interpretation of indefinites.

[3] The specificity as we discussed so far is called **scopal specificity** in the literature, there are other types of specificity as well, such as **epistemic specificity**, **partitive specificity**. However, this is beyond the interest of this thesis, and we shall only consider the scopal specificity here. For more information, please refer to Farkas (1994, 2002); Von Heusinger (2002).

5.2.2 Predicted Constraints on Accessibility

Karttunen suggested that an indefinite NP introduces a new discourse referent, which is available to be taken up by subsequent anaphoric expressions. However, Karttunen also noticed that a discourse referent has life-span. He has come up with the generalization that if a referent is established in the scope of a logical connective, its life-span is restrained within that scope.

The dynamic systems presented so far can successfully model some particular constraints, i.e., negation, disjunction, implication, universal quantification (they have been discussed intensively in section 5.1). More specifically, referents introduced under the scope of negation can not be taken up outside the scope; referents introduced in one disjunct can not be accessed either from the other disjunct, or from the following sentences; referents introduced in the antecedent of a conditional can only be picked up by anaphors in the consequent of that conditional, but not by the ones in subsequent contexts. For demonstrations, see examples (73), (74), (75), (76), (77), and (78).

In this section, we will look into two more context environments, which affect the accessibility of discourse referent. Both conditions can find an account in our current dynamic theories. For the illustration, we will use the framework TTDL.

Modal Verbs

Due to the semantics of modals, the propositions under their domains are usually yet unreal or untrue at the utterance time. Namely they are not asserted to be true. Hence if an indefinite is contained in the complement clause of some modal verb, e.g., *must*, *can*, *shall*, etc., its discourse referent can not be anaphorically linked to expressions in subsequent discourse. For instance:

(85) a. You must write $\underline{\text{a letter}_i}$ to your parents. *They are expecting $\underline{\text{the letter}_i}$.
b. Bill can make $\underline{\text{a kite}_i}$. *$\underline{\text{The kite}_i}$ has a long string. Karttunen (1969)

In the above discourses, neither of the anaphoric expressions: *the letter* and *the kite*, can refer back to the corresponding indefinite NP. That is because both indefinites are located in the complement clauses governed by model auxiliaries, i.e., *must* in (85-a), *can* in (85-b). Take (85-b) for example, we can account for its failure of anaphora by proposing the following lexical entry for *can*:

$$[\![can]\!] = \lambda c.\mathbf{can}\ c \tag{5.5}$$

Like negation, **can** is an operator of type $o \to o$. Then by applying the standard entries for the remaining linguistic elements, the semantic representation for the first part of (85-b) can be compositionally achieved through β-reduction:

$$\begin{aligned}[\![(85\text{-b})\text{-}1]\!] &= [\![can]\!]([\![make]\!]([\![a]\!][\![kite]\!])[\![Bill]\!]) \\ &\twoheadrightarrow_\beta \mathbf{can}\ (\exists x.(\mathbf{kite}\ x \land \mathbf{make\ bill}\ x))\end{aligned}$$

In TTDL, we can obtain the dynamic entry for *can* (formula 5.5) with respect to definition 4.4.4:

$$\begin{aligned}\overline{[\![can]\!]} &= \overline{\lambda c.\mathbf{can}\ c} \\ &= \lambda c.\overline{\mathbf{can}}\ c \\ &= \lambda c.\mathbb{D}_{o\to o}(\lambda e.\mathbf{can})\ c \\ &\twoheadrightarrow_\beta \lambda ce\phi.\mathbf{can}(c\ e\ \mathbf{stop}) \wedge \phi e\end{aligned} \tag{5.6}$$

In the above computation, we omit some steps, in particular the ones where the two functions $\mathbb{D}$ and $\mathbb{S}$ are involved. Interested readers may refer to section 4.4.3 or Appendix A.1 for more stepwise illustrations. From formula 5.6, one may draw that the modal verb *can* functions like dynamic negation such that the empty continuation **stop** is passed to the proposition under its scope. Now we can compute the dynamic representation of the first sentence in (85-b) straightforwardly:

$$\begin{aligned}\overline{[\![\text{(85-b)-1}]\!]} &= \overline{[\![can]\!]}(\overline{[\![make]\!]([\![a]\!][\![kite]\!])[\![Bill]\!]}) \\ &\twoheadrightarrow_\beta (\lambda ce\phi.\mathbf{can}(c\ e\ \mathbf{stop}) \wedge \phi e)(\overline{[\![make]\!]([\![a]\!][\![kite]\!])[\![Bill]\!]}) \\ &\twoheadrightarrow_\beta \lambda e\phi.\mathbf{can}\ (\exists x.(\mathbf{kite}\ x \wedge \mathbf{make\ bill}\ x)) \wedge \phi e\end{aligned}$$

According to the above result, the bound variable x is not updated into the current left context, hence no subsequent anaphors can be resolved as it. Other modals, such as *must* and *shall*, can be analyzed in a completely similar way.

Propositional Attitude Verbs

Besides the above modal verbs, a group of propositional attitude verbs also have a similar effect in restraining discourse referent within their domains, e.g., *want*, *hope*, *try*, *promise*, *believe*, *think*, *doubt*, etc. Typically, these verbs take either infinitive complements or *that* complements. Indefinites inside their complement clauses are not allowed to be anaphorically related to NPs outside them. For instance:

(86) a. John wants to catch a fish_i. *Do you see the fish_i from here?
b. Mary expects to have a baby_i. *The baby_i's name is Sue.
c. I doubt that Mary has a car_i. *Bill has seen it_i. Karttunen (1969)

Since we limit our attention to readings where indefinites are interpreted non-specifically, none of the highlighted expressions in (86): *a fish*, *a baby* and *a car*, can serve as antecedent for subsequent anaphors. The reason is similar as in example (85): the indefinites are embedded in the scope of some other operators. Again, this could be accounted for with dynamic theories, in particular TTDL, as long as we propose a proper lexical entry for the propositional attitude verb. Take *want* for instance. Syntactically, it takes a VP (infinitive clause) and returns a VP. Its semantic representation is as follows:

$$[\![want]\!] = \lambda VS.S(\lambda x.\mathbf{want}\ x\ \lambda y.(V(\lambda P.P)y)) \tag{5.7}$$

where the predicate **want** is of type $\iota \to (\iota \to o) \to o$. Using the standard entries for the other linguistic expressions, the semantics of first sentence in (86-a) will be computed as follows:

$$\begin{aligned}\llbracket (86\text{-a})\text{-}1 \rrbracket &= \llbracket \mathit{want} \rrbracket (\llbracket \mathit{catch} \rrbracket (\llbracket \mathit{a} \rrbracket \llbracket \mathit{fish} \rrbracket)) \llbracket \mathit{John} \rrbracket \\ &\twoheadrightarrow_\beta \mathbf{want}\ \mathbf{john}\ \lambda y.(\exists x.(\mathbf{fish}\ x \wedge \mathbf{catch}\ y\ x))\end{aligned}$$

Then the dynamic entry for *want* can be obtained by applying corresponding rules in definition 4.4.4 to formula 5.7:

$$\begin{aligned}\overline{\llbracket \mathit{want} \rrbracket} &= \overline{\lambda V S.(\lambda x.\mathbf{want}\ x\ \lambda y.(V(\lambda P.P)y))} \\ &= \lambda V S.(\lambda x.\overline{\mathbf{want}}\ x\ \lambda y.(V(\lambda P.P)y)) \\ &= \lambda V S.(\lambda x.\mathbb{D}_{\iota\to(\iota\to o)\to o}(\lambda e.\mathbf{want})\ x\ \lambda y.(V(\lambda P.P)y)) \\ &\twoheadrightarrow_\beta \lambda V S.S(\lambda x e\phi.\mathbf{want}\ x\ \lambda y.(V(\lambda P.P)\ y\ e\ \mathbf{stop})) \wedge \phi e\end{aligned} \tag{5.8}$$

Again, like in formula 5.6, we leave out some steps in the above computation. Interested readers may refer to section 4.4.3 or Appendix A.1 for more stepwise illustrations. From formula 5.8, we can draw that the propositional attitude verb *want* is analogous to dynamic negation and modal verbs on the aspect that the empty continuation **stop** is passed to the proposition it governs. Finally, the dynamic representation of (81) is achieved in a relatively straightforward way:

$$\begin{aligned}\overline{\llbracket (86\text{-a})\text{-}1 \rrbracket} &= \overline{\llbracket \mathit{want} \rrbracket (\llbracket \mathit{catch} \rrbracket (\llbracket \mathit{a} \rrbracket \llbracket \mathit{fish} \rrbracket)) \llbracket \mathit{John} \rrbracket} \\ &\twoheadrightarrow_\beta \lambda e\phi.\mathbf{want}\ \mathbf{john}\ \lambda y.(\exists x.(\mathbf{fish}\ x \wedge \mathbf{catch}\ y\ x)) \wedge \phi e\end{aligned}$$

According to the above formula, the variable x, which is introduced by the indefinite *a fish*, is not updated into the current left context. Hence it is not accessible to subsequent anaphoric expressions. The same treatment can be applied to other propositional attitude verbs which take infinitive complements.

For verbs which take *that* complements, the structure of the lexical entry ought to be slightly modified. For instance, let's look at *doubt*. It takes a sentence (*that* clause) and yields a VP. We thus assign it the following semantic entry:

$$\llbracket \mathit{doubt} \rrbracket = \lambda c S.S(\lambda x.\mathbf{doubt}\ x\ c) \tag{5.9}$$

where the predicate **doubt** is of type $\iota \to o \to o$. To achieve the dynamic counterpart of the entry, we apply rules in definition 4.4.4:

$$\begin{aligned}\overline{\llbracket \mathit{doubt} \rrbracket} &= \overline{\lambda c S.S(\lambda x.\mathbf{doubt}\ x\ c)} \\ &= \lambda c S.S(\lambda x.\overline{\mathbf{doubt}}\ x\ c) \\ &= \lambda c S.S(\lambda x.\mathbb{D}_{\iota\to o\to o}(\lambda e.\mathbf{doubt})\ x\ c) \\ &\twoheadrightarrow_\beta \lambda c S.S(\lambda x e\phi.\mathbf{doubt}\ x\ (c\ e\ \mathbf{stop}) \wedge \phi e)\end{aligned} \tag{5.10}$$

The above entry explains the impossibility of anaphora in (86-c): the empty continuation *stop* is passed to the embedded proposition. Analogously, other *that* complement-taking verbs, which cease the accessibility of referents within their scope, can be treated in the same way.

In the next subsection, we shall present some exceptions, which fail to find an account in standard dynamic theories. Before that, let's take a look at a couple of interesting examples, which involves the so-called short-term discourse referents, as named by Karttunen:

(87) a. You must write a letter$_i$ to your parents and mail the letter$_i$ right away. *They are expecting the letter$_i$.
b. John wants to catch a fish$_i$ and eat it$_i$ for supper. *Do you see the fish$_i$ over there? Karttunen (1969)

As discussed earlier, indefinites appearing in complements of modal verbs can not antecede any subsequent anaphor. This correctly rules out the second occurrence of *the letter* in (87-a) and the definite NP *the fish* in (87-b), which is exactly the case as in example (85-a) and (86-a). However, one may have noticed that the first occurrence of *the letter* in (87-a) the pronoun *it* in (87-b) are perfectly admitted. Some analogous example, where negation is concerned with, is presented below:

(88) I don't believe that Mary had a baby$_i$ and named her$_i$ Sue. *The baby$_i$ has mumps. Karttunen (1969)

Similar as in (87), the two complements in the first sentence of (88), namely *Mary had a baby* and *Mary named her Sue*, form a conjoined complement clause, within which anaphora is allowed. So is there any problem with our analysis so far? The answer is no. Actually, in example (87) or (88), the complement clause that contains the acceptable anaphor, and the one that contains its antecedent, together form a conjoined complement clause. It is this particular constituent structure that ensures the felicity of the first anaphor in the discourse. These examples can be accounted for without difficulty. We conjoin the sub-clauses before passing the whole complement conjunction to the scope-bearing expression, such as modals, negation, etc., so the referent from the indefinite antecedent is associated to the anaphor before being blocked. In other words, the life-span of the referent lasts as long as the scope of the dominating expression does not terminate. Take (87-b) as an illustration, we propose the following lexical entry for *and*, which conjoins two VPs[4]:

$$[\![and]\!] = \lambda V_1 V_2 S.S(\lambda x.(V_1(\lambda P.P)x) \wedge (V_2(\lambda P.P)x))$$

According to definition 4.4.4, the dynamic counterpart of $[\![and]\!]$ in TTDL can be obtained as follows:

$$\begin{aligned}\overline{[\![and]\!]} &= \overline{\lambda V_1 V_2 S.S(\lambda x.(V_1(\lambda P.P)x) \wedge (V_2(\lambda P.P)x))} \\ &= \lambda V_1 V_2 S.S(\lambda x.(V_1(\lambda P.P)x) \overline{\wedge} (V_2(\lambda P.P)x)) \\ &\twoheadrightarrow_\beta \lambda V_1 V_2 S.S(\lambda x e \phi.(V_1(\lambda P.P)x e(\lambda e'.V_2(\lambda P.P)x e' \phi)))\end{aligned}$$

Then by employing the standard lexical entries for the other expressions (the entry for *want* is provided in formula 5.7), we can compute the semantic representation of the

[4]The problem of coordination is a hard one in natural language semantics: the coordinator *and* can be used to combine almost all sorts of syntactic categories, such as sentences, NPs, VPs, adjective, adverbs, etc. We do not want to dive into the detail of the problem. The solution we provide here is just a naive attempt, but it will be sufficient for our current purpose.

first sentence in (87-b):

$$\overline{[\![(87\text{-b})\text{-}1]\!]} = \overline{[\![\mathit{want}]\!]([\![\mathit{and}]\!]([\![\mathit{catch}]\!]([\![a]\!][\![\mathit{fish}]\!]))([\![\mathit{eat}]\!][\![\mathit{it}]\!]))[\![\mathit{John}]\!]}$$
$$\twoheadrightarrow_\beta \lambda e\phi.\textbf{want john}\ \exists x.(\textbf{fish}\ x \wedge \textbf{catch john}\ x \wedge \textbf{eat john}\ \mathsf{sel}(x :: e)) \wedge \phi e$$

As indicated in the above formula, although referent x is not updated into the current left context for following utterances, it can nevertheless be retrieved by the anaphoric pronoun *it*, which is within the same domain as the indefinite. As a result, the short term anaphoras, such as the ones in examples (87) and (88), are perfectly accepted under the predictions of dynamic frameworks.

5.3 Exceptions to the Accessibility Constraints

Although Karttunen's generalization is applicable to the accessibility of many discourse anaphoras, it is not without its problems. In Karttunen (1969), the author remarks a number of examples where the life-span of a discourse referent is longer than expected. These exceptions can not be accounted for with the setup of the dynamic frameworks presented so far. The perspective of this section is to investigate these exceptional cases. First we will present some pseudo-exceptions. Then three real exceptions, i.e., the double negation problems, the disjunction problem and the modal subordination, will be studied. They will be the topic of the next two chapters.

5.3.1 Ascriptive and Generic Indefinite

Typically in English, it is the function of indefinite to refer to entities which the speaker assumes that the addressee does not know, namely the **hearer-new** entities (in contrast to the ones which have already been mentioned, called the **hearer-old** entities) Prince (1992). However, it is not the case that every indefinite NP in use picks out a novel entity. More specifically, indefinite NPs do not specify any individual when they are either in **ascriptive** or **generic** usage, for instance:

(89) a. Bill is not <u>a linguist</u>. Karttunen (1969)
  b. <u>A lion</u> is a mighty hunter. Karttunen (1969)
  c. <u>A donkey</u> is an animal. Le Pore and Garson (1983)
  d. <u>A blue-eyed bear</u> is (always) intelligent. Heim (2011)

In example (89), none of the sentences is concerned with the accessibility constraint (e.g., negation, quantifications, complement-taking verbs). Thus, the indefinite NPs ought to introduce permanent discourse referents. However, it is unacceptable to continue the above sentences with anaphoric expressions, which refer back to the highlighted indefinites. In (89-a), *a linguist* functions as predicative complement (the ascriptive usage), it provides information on the individual introduced by *Bill*, rather than establishing a referent on its own. In (89-b), (89-c), and (89-d), the indefinite NPs are used generically. They are used to describe lions, donkeys, or blue-eyed bears in general, instead of any particular lion, donkey, or blue-eyed bear. These generic indefinite do not have the potential to introduce referent either. As a result, since the ascriptive or generic indefinites do not establish referent at any rate, they can by no means participate in anaphoric relations.

Logically, an ascriptive or generic indefinite is not translated into existential quantification[5]. This specific usage of indefinite is typically not taken into consideration when studying anaphora. So examples such as (89) are not really exceptions to Karttunen's generalization. For the rest of the thesis, we will ignore the ascriptive and generic indefinites, and concentrate on those which establish discourse referents (the ones that can be readily translated into existential quantifications).

Implicative Verbs

The implicative verbs in English typically include *manage*, *remember*, *venture*, *dare*, etc. They typically take infinitive complements, and share the following property:

> *If not negated, imply the truth of the proposition represented by their complement sentence.* Karttunen (1969)

In other words, the proposition in the complement clause will be true if the main sentence is affirmative; otherwise, if the main sentence is negated, the complement proposition will not hold. In the former case, the discourse referent introduced by an indefinite occurring within the scope of the implicative is still accessible for future reference, as in example (90). While in the latter case, similar anaphoric relations will not be acceptable, as shown in (91).

(90) a. John managed to find $\underline{\text{an apartment}}_i$. $\underline{\text{The apartment}}_i$ has a balcony.
b. Bill ventured to ask $\underline{\text{a question}}_i$. The lecturer answered $\underline{\text{it}}_i$. Karttunen (1969)

(91) a. John didn't manage to find $\underline{\text{an apartment}}_i$. *$\underline{\text{The apartment}}_i$ has a balcony.
b. Bill didn't dare to ask $\underline{\text{a question}}_i$. *The lecturer answered $\underline{\text{it}}_i$. Karttunen (1969)

The dynamic theories presented so far, i.e., DRT, DPL and TTDL, provide an account for (91): the indefinites are located under the scope of negation, hence their accessibility is blocked. However, they fail to explain the anaphoric links in (90): the indefinites outlive the scope of the higher order verbs, such as *manage* and *venture*. So is implicative verb an exception to the accessibility constraints? To answer this question, we need a closer look at the property of implicative verbs.

Since implicative verbs imply the truth of the embedded complements, anyone who asserts (92-a) must commit that (92-b) is true:

(92) a. John managed to solve the problem.
b. John solved the problem. Karttunen (1971)

When the main verb is an implicative verb, e.g., *manage*, and it is affirmative, then there exists a clear implication between the proposition expressed by the main sentence and the one expressed by the complement. In other words, if the main sentence is true, the complement necessarily has to be true. Assume the semantic representations for (92-a) and (92-b) are $[\![(92\text{-a})]\!]$ and $[\![(92\text{-b})]\!]$, respectively, then the relation between the two sentences can be expressed as, where symbol $\rightarrow$ is ordinary logical implication:

$$[\![(92\text{-a})]\!] \rightarrow [\![(92\text{-b})]\!] \tag{5.11}$$

[5] To explore their semantics, ad hoc treatment is needed. However, this falls out of the domain of this thesis, we shall not go into detail.

It is interesting to see what is going on if the main sentence involves negation. Looking at the following pair of examples:

(93) a. John didn't manage to solve the problem.
b. John didn't solve the problem. Karttunen (1971)

It appears that anyone who utters (93-a) grants the truth of (93-b). Namely, if the main verb of a sentence is an implicative verb, e.g., *manage*, and it is negative, then the whole sentence implies the negation of its complement. Let's use $[\![(93\text{-a})]\!]$ and $[\![(93\text{-b})]\!]$ to denote the semantic representations of (93-a) and (93-b), then analogous to formula 5.11, we can draw the following relation:

$$[\![(93\text{-a})]\!] \rightarrow [\![(93\text{-b})]\!] \tag{5.12}$$

Further more, since (93-a) is the negation of (92-a), in parallel, (93-b) is the negation of (92-b), formula 5.12 can be rewritten as follows, where symbol $\neg$ denotes ordinary logical negation:

$$\neg[\![(92\text{-a})]\!] \rightarrow \neg[\![(92\text{-b})]\!] \tag{5.13}$$

Then putting formula 5.11 and 5.13 together, we arrive at the conclusion that sentences (92-a) and (92-b) are logically equivalent, namely $[\![(92\text{-a})]\!] \leftrightarrow [\![(92\text{-b})]\!]$. This result is somehow unintuitive, because that would mean implicative verbs such as *manage* do not contribute to the meaning of the sentence. Of course this is not the case: (92-a) presupposes that John at least tried to solve the problem, while (92-a) does not. However, for the current research on discourse anaphora, it is sufficient to simplify (92-a) and (92-b) as semantically identical. As a result, to account for the felicitous anaphora in example (90-a), we propose the following lexical entry for the implicative verb *manage*:

$$[\![manage]\!] = \lambda VS.VS \tag{5.14}$$

Basically, the entry $[\![manage]\!]$ takes a VP (the infinitive complement) and a NP (the subject), then returns the result of applying the VP to the NP. With the standard entries for the remaining linguistic expressions, we can compute the semantics of the first sentence in (90-a) as follows:

$$\begin{aligned}
[\![(90\text{-a})\text{-}1]\!] &= [\![manage]\!]([\![find]\!]([\![a]\!][\![apartment]\!]))[\![John]\!] \\
&= (\lambda VS.VS)([\![find]\!]([\![a]\!][\![apartment]\!]))[\![John]\!] \\
&\twoheadrightarrow_\beta [\![find]\!]([\![a]\!][\![apartment]\!])[\![John]\!] \\
&\twoheadrightarrow_\beta \exists x.(\mathbf{apartment}\ x \wedge \mathbf{find\ john}\ x)
\end{aligned}$$

Thus, $[\![(90\text{-a})\text{-}1]\!]$ indeed represents the semantics of the complement *John found an apartment*. The dynamic translation of $[\![manage]\!]$ in TTDL is trivial because only variables are concerned. As for the dynamic translation of the whole sentence, it can be conducted step by step with respect to definition 4.4.4:

$$\begin{aligned}
\overline{[\![(90\text{-a})\text{-}1]\!]} &= \overline{[\![\mathit{manage}]\!]([\![\mathit{find}]\!]([\![a]\!][\![\mathit{apartment}]\!]))[\![\mathit{John}]\!]} \\
&= (\overline{\lambda VS.VS})\overline{([\![\mathit{find}]\!]([\![a]\!][\![\mathit{apartment}]\!]))[\![\mathit{John}]\!]} \\
&= (\lambda VS.VS)\overline{([\![\mathit{find}]\!]([\![a]\!][\![\mathit{apartment}]\!]))[\![\mathit{John}]\!]} \\
&\twoheadrightarrow_{\beta} \overline{[\![\mathit{find}]\!]([\![a]\!][\![\mathit{apartment}]\!])[\![\mathit{John}]\!]} \\
&\twoheadrightarrow_{\beta} \lambda e\phi.\exists x.(\mathbf{apartment}\ x \wedge \mathbf{find\ john}\ x \wedge \phi(x :: e))
\end{aligned}$$

The variable x, which is from the indefinite NP *an apartment*, is updated into the current left context of (90-a)-1. It is thus accessible to subsequent anaphoric expressions. This gives a correct prediction for the acceptability of anaphora in (90-a).

Remark that formula 5.14 is a simplification on the semantics of *manage*. As explained above, it is not necessary to take the presuppositions from implicative verbs into consideration in order to account for the accessibility of discourse anaphoras. We will leave it as a task for pragmatics. Other implicative verbs, e.g., *remember*, *venture*, *dare*, etc., can thus be treated in exactly the same way. Namely, the entry 5.14 will be universal among implicatives.

Besides the implicative verbs discussed above, there is another group of complement-taking verbs, which are inherently accompanied with implications. However, these verbs seem to incorporate negation. More specifically, they imply the negation of their complements if the main sentence is affirmative. Otherwise, the propositions expressed by complements will be implied. We call them the negative implicative verbs, typical examples include *forget*, *fail*, *neglect*, *avoid*, etc. A negative implicative often has a positive counterpart, e.g., *forget* and *remember*, *fail* and *manage*. However, it is not a generalization for all negative implicatives: there are no obvious positive implicative verbs corresponding to *neglect* and *avoid*.

Concerning the accessibility of discourse referent, the negative implicative verbs have an opposite effect compared with the (positive) implicatives. Namely, if the main verb of a sentence is a negative implicative, and the sentence is affirmative, then the indefinite NP embedded in the complement clause shall not be picked up by subsequent anaphors. For instance:

(94) a. John forgot to write a term paper$_i$. *He cannot show it$_i$ to the teacher.
  b. John fails to find an answer$_i$. *It$_i$ was wrong. Karttunen (1969)

Looking at the above discourses, the anaphoric pronoun *it* fails to refer to either *a term paper* in (94-a) or *an answer* in (94-b). Can this be accounted for in dynamic frameworks presented so far? The answer is yes. Analogous to (positive) implicatives, we may simply define the semantics of negative implicative verbs in a way such that they contribute to the sentences by negating the complement clause. Hence we may assign the following lexical entry for *fail*:

$$[\![\mathit{fail}]\!] = \lambda VS.\neg(VS) \tag{5.15}$$

Similar to formula 5.14, the above entry takes a VP (the infinitive complement) and a NP (the subject) as input. While instead of the application, its negation is returned. Turn to the first sentence in (94-b), its semantics under standard predicate logic can be compositionally obtained as follows:

$$
\begin{aligned}
[\![(94\text{-b})\text{-}1]\!] &= [\![\mathit{fail}]\!]([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!]))[\![\mathit{John}]\!] \\
&= (\lambda VS.\neg(VS))([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!]))[\![\mathit{John}]\!] \\
&\twoheadrightarrow_\beta \neg([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!])[\![\mathit{John}]\!]) \\
&\twoheadrightarrow_\beta \neg\exists x.(\mathbf{apartment}\ x \wedge \mathbf{find\ john}\ x)
\end{aligned}
$$

Hence the indefinite in the complement is guaranteed a narrower scope than negation. And it will not be accessible outside the sentence in dynamic semantics. In what follows, we provide the TTDL representation of (94-b)-1 with respect to definition 4.4.4:

$$
\begin{aligned}
\overline{[\![(94\text{-b})\text{-}1]\!]} &= \overline{[\![\mathit{fail}]\!]([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!]))[\![\mathit{John}]\!]} \\
&= (\overline{\lambda VS.\neg(VS)})\overline{([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!]))[\![\mathit{John}]\!]} \\
&= (\lambda VS.\overline{\neg}(VS))\overline{([\![\mathit{find}]\!]([\![a]\!][\![\mathit{answer}]\!]))[\![\mathit{John}]\!]} \\
&\twoheadrightarrow_\beta \overline{\neg}\overline{([\![\mathit{find}]\!]([\![a]\!][\![\mathit{apartment}]\!])[\![\mathit{John}]\!])} \\
&\twoheadrightarrow_\beta \lambda e\phi.(\neg\exists x.(\mathbf{answer}\ x \wedge \mathbf{find\ john}\ x)) \wedge \phi e
\end{aligned}
$$

The current left context is empty, hence no variables are available for upcoming reference. A similar analysis can be applied to *forget*, then the acceptability of anaphora in (94-a) can also be accounted for. Note that as observed by Karttunen, negative implicative verbs support the cancellation of a double negation. However the current setup in dynamic semantics will not give an account on that. We will discuss that problem in more detail in section 5.3.3.

In this subsection, we have shown that examples (90), (91) and (94) are well covered by the current dynamic theories, provided the lexical entries 5.14 and 5.15. Hence implicative verbs (both positive and negative) are not real exceptions to the accessibility constraints.

5.3.2 Factive Verbs

Besides the implicative verbs, there is another special group of complement-taking verbs. These verbs are called factive verbs, they have the characteristic of presupposing the embedded propositions Karttunen (1969, 1971). Namely, if the main verb of a sentence is factive, the proposition represented by the complement is presupposed to be true. Typical examples include *know*, *realize*, *regret*, etc.

Due to their peculiar semantic property, an indefinite NP in the complement of a factive verb can perfectly serve as antecedent for subsequent anaphors. For instance:

(95) John knew that Mary had a car$_i$, but he had never seen it$_i$. Karttunen (1969)

Assume *know* is a scope-bearing operator. Although the indefinite *a car* is under the scope of *know*, it can still be understood as anaphorically linked to the pronoun *it* in the second sentence. Putting examples (95) and (90) together, it seems that factive verbs extend the scope of indefinite NPs in a similar way as implicative verbs. So do they share a common account? Unfortunately, the answer is no. Because in contrast to implicative verbs, factive verbs presuppose their complements, and presupposition survives under negation. That is to say, even if the main sentence is negated, the truth of the embedded proposition is still ensured. Hence negation does not affect the accessibility of indefinite, which locates in the complement of a factive verb. Take the following discourse as an example:

(96) Bill didn't realize that he had a dime$_i$. It$_i$ was in his pocket. Karttunen (1969)

By reading the first sentence in (96), one may draw that Bill indeed had a dime. Thus the upcoming anaphor *it* can be successfully resolved with the dime Bill owned. It is obviously not the same case if the main verb is implicative, see example (91). Thus a similar entry like formula 5.14 fails to apply to factive verbs.

Let's try to assign factive verbs an analogous entry as modals (formula 5.5) and propositional attitude verbs (formula 5.7, 5.9). Take *know* for example:

$$[\![know]\!] = \lambda cS.S(\lambda x.\mathbf{know}\ x\ c) \tag{5.16}$$

where the predicate **know** is of type $\iota \to o \to o$. Clearly the above entry does not suffice to reflect the semantics of *know*. In our opinion, the potential to trigger presupposition concerns the lexical semantics of the predicate. Hence, to account for the semantics of factive verbs, a promising solution is to devise a pair of meaning postulates Carnap (1952):

$$\forall x \forall c.(\mathbf{know}\ x\ c \leftrightarrow (c \wedge \mathbf{be\_aware\_of}\ x\ c)) \tag{5.17}$$

$$\forall x \forall c.(\neg\mathbf{know}\ x\ c \leftrightarrow (c \wedge \neg\mathbf{be\_aware\_of}\ x\ c)) \tag{5.18}$$

Same as **know**, the semantic type of the predicate **be_aware_of** is also $\iota \to o \to o$. The above two meaning postulates together convey the presupposition of *know*: 5.17 denotes that somebody knowing a proposition is identical to the proposition is true and the person is aware of it; 5.18 denotes that somebody not knowing a proposition is identical to the proposition is true and the person is not aware of it. Thus, a copy of the embedded proposition is accommodated outside the scope of **know**, no matter whether the sentence is affirmative or negative. In this way, subsequent anaphors become possible, and they will be linked to the indefinites in the accommodated proposition.

Take the first sentence of (95) as an example, its semantics under standard predicate logic can be computed in the following compositional way:

$$\begin{aligned}
[\![(95)\text{-}1]\!] &= [\![know]\!]([\![have]\!]([\![a]\!][\![car]\!])[\![Mary]\!])[\![John]\!] \\
&= (\lambda cS.S(\lambda x.\mathbf{know}\ x\ c))([\![have]\!]([\![a]\!][\![car]\!])[\![Mary]\!])[\![John]\!] \\
&\twoheadrightarrow_\beta (\lambda S.S(\lambda x.\mathbf{know}\ x\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{mary}\ x)))[\![John]\!] \\
&\twoheadrightarrow_\beta \mathbf{know}\ \mathbf{john}\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{mary}\ x)
\end{aligned}$$

This is where we normally end for other propositional attitude verbs. But since *know* is factive, we may continue the computation by applying the meaning postulate 5.17:

$$\begin{aligned}
[\![(95)\text{-}1]\!] &\twoheadrightarrow_\beta \mathbf{know}\ \mathbf{john}\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{mary}\ x) \\
&= \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{mary}\ x) \wedge (\mathbf{be_aware_of}\ \mathbf{john}\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{mary}\ x))
\end{aligned}$$

Finally, the dynamic translation of the above representation under TTDL shall be straightforward with respect to definition 4.4.4:

$$\begin{aligned}\overline{[\![(95)\text{-}1]\!]} &= \overline{[\![know]\!]([\![have]\!]([\![a]\!][\![car]\!])[\![Mary]\!])[\![John]\!]}\\ &= \overline{\exists x.(\mathbf{car}\ x \wedge \mathbf{have\ mary}\ x) \wedge (\mathbf{be\_aware\_of\ john}\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have\ mary}\ x))}\\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{car}\ x \wedge \mathbf{have\ mary}\ x \wedge\\ &\qquad\qquad (\mathbf{be\_aware\_of\ john}\ \exists x.(\mathbf{car}\ x \wedge \mathbf{have\ mary}\ x)) \wedge \phi(x :: e))\end{aligned}$$

The variable x is updated into the current left context. This explains why the anaphora in (95) is possible. With the above analysis, a similar analysis can be carried out for example (96), where we just have to apply the meaning postulate corresponding to the negative case.

As a result, although implicative verbs allow embedded indefinites to be accessed from outside (in both affirmative and negative situations), they are merely pseudo-exceptions: meaning postulates render a proper account on them.

5.3.3 Double Negation

According to the generalization of Karttunen, negation, which is a typical barrier to anaphoras, does not allow discourse referent to outlive its scope. The negative marker may be in the standard forms, e.g., *not*, *no*, *it is not the case that* ..., etc., see examples (73) and (74) for demonstration. Also, it may come from the lexical semantics of the predicates, in particular, negative implicatives (e.g., *forge*, *fail*, *avoid*), as shown in example (94).

In all the dynamic frameworks presented so far, negation is treated as a plug for anaphoric binding. More specifically, it is an operator that blocks discourse referents once and forever. Following that, a double negation is standardly treated as applying (single) negation twice. The second negation will impose the referent-blocking effect twice, rather than to dismiss the one from the first negation. The consequence is that referents will be doubly blocked, hence are not accessible in future context. Let's illustrate this with the following triplet:

(97) a. John brought an umbrella.
b. John didn't bring an umbrella.
c. It is not true that John didn't bring an umbrella.

Example (97-a) is an affirmative sentence, example (97-b) is derived by negating it. As to (97-c), it is the negation of (97-b), namely the double-negated version of (97-a). The dynamic representation of the three sentences under TTDL can be achieved compositionally as follows:

$$\begin{aligned}\overline{[\![(97\text{-a})]\!]} &= \overline{[\![bring]\!]([\![a]\!][\![umbrella]\!])[\![John]\!]}\\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x \wedge \phi(x :: e))\end{aligned} \tag{5.19}$$

$$\begin{aligned}\overline{[\![(97\text{-b})]\!]} &= \overline{[\![not]\!]([\![bring]\!]([\![a]\!][\![umbrella]\!])[\![John]\!])}\\ &= \neg\overline{[\![(97\text{-a})]\!]}\\ &\twoheadrightarrow_\beta (\lambda A e\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)\\ &\qquad\qquad (\lambda e\phi.\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x \wedge \phi(x :: e)))\\ &\twoheadrightarrow_\beta \lambda e\phi.\neg\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x) \wedge \phi e\end{aligned}$$

$$\begin{aligned}\overline{[\![(97\text{-c})]\!]} &= \overline{[\![not]\!]([\![not]\!]([\![bring]\!]([\![a]\!][\![umbrella]\!])[\![John]\!]))}\\ &= \neg\overline{[\![(97\text{-b})]\!]}\\ &\twoheadrightarrow_\beta (\lambda A e\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)\\ &\qquad\qquad (\lambda e\phi.\neg\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x) \wedge \phi e)\\ &\twoheadrightarrow_\beta \lambda e\phi.\neg\neg\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x) \wedge \phi e\\ &= \lambda e\phi.\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x) \wedge \phi e\end{aligned} \tag{5.20}$$

The last step of computation in formula 5.20 is possible because double negation can be eliminated unconditionally in standard FOL. From formula 5.19 and 5.20, we can infer that $\overline{[\![(97\text{-a})]\!]}$ and $\overline{[\![(97\text{-c})]\!]}$ are truth-conditionally (statically) equivalent. Namely their semantics are identical when interpreted against the empty continuation. More specifically, the result is $\exists x.(\mathbf{umbrella}\ x \wedge \mathbf{bring\ john}\ x)$. However at the same time, the semantics of the two sentences differ from each other on the dynamic aspect. As predicted by TTDL, example (97-a) will allow subsequent reference to the indefinite *an umbrella* because variable x is updated into the left context, while it is not the case for (97-c). Hence the law of double negation, which exists unconditionally in classical logics such as PL and FOL, does not hold in TTDL. Essentially, the other two dynamic theories: DRT and DPL, render the same analysis. This is not surprising because as we mentioned earlier, negation is treated in a similar way in all three frameworks.

Now let's turn to double negation in natural language. Generally speaking, double negation is a universal linguistic construction, where the negative effect of each negation will be erased:

> *All the languages seem to have a common law, that is, two negative makes a positive.* Jespersen (1933)

Moreover, it has also been observed that if an indefinite NP occurs within the scope of double negation, it may antecede anaphoric expressions in the upcoming context. That is to say, the accessibility of discourse referents under double negation is similar to that in affirmative sentences. Concrete linguistic examples are as follows:

(98) a. It is not true that John didn't bring <u>an umbrella</u>$_i$. <u>It</u>$_i$ was purple and <u>it</u>$_i$ stood in the hallway. Krahmer and Muskens (1995)
b. It is not true that there was <u>no lion</u>$_i$ in the cage. I saw <u>it</u>$_i$ sleeping and heard <u>it</u>$_i$ snoring. Kaup and Lüdtke (2008)

(10) a. John did not fail to find <u>an answer</u>$_i$. <u>The answer</u>$_i$ was even right.
b. John did not remember not to bring <u>an umbrella</u>$_i$, although we had no room for <u>it</u>$_i$. Karttunen (1969)

All discourses of (98) and (10) involve double negation, where the negative marker is either explicitly established (i.e., *not*, *no*), or it comes from the predicate (i.e., *fail*). One may see that in both pairs of examples, discourse referents will not be blocked by double negation: all the anaphoric expressions in (98) and (10) are interpreted as depending on the indefinites contained in preceding double-negated sentences. Since double negation will be treated in the way of formula 5.20, examples such as (98) and (10) fail to find a correct account in TTDL. It is also the case in DRT and DPL. All three dynamic theories will predict that the anaphoras in (98) and (10) are impossible, while this is not as it should be.

As a result, double negation is indeed an exception for the accessibility constraints of dynamic semantics. We call it **the double negation problem**. In order to cover the linguistic data presented above, we need to remedy dynamic frameworks by allowing double negation elimination. This will be the topic of Chapter 6. By doing so, we assume that double negation behaves as if there was no negation at all. Of course this hypothesis is questionable, because the meaning of an affirmative sentence shall not completely be the same as its double negated counterpart. Otherwise, there is no need to distinguish the two linguistic constructions. But the hypothesis seems valid if we are only concerned with truth conditions and referent accessibility. As to the difference between affirmatives and double negatives, we leave it as a pragmatic issue and will not enter it in this thesis.

A last remark, there are some linguistic examples which do not exactly support double negation cancellation, for instance:

(99) a. Some of the students$_i$ passed the examination. They$_i$ must have studied hard.

b. Not all the students$_i$ failed the examination. ?They$_i$ must have studied hard. Hintikka (2002)

The logical formulas (in FOL) which correspond to the first sentences of (99-a) and (99-b) are respectively as follows:

$$[\![(99\text{-a})\text{-}1]\!] = \exists x.(\mathbf{student}\ x \wedge \mathbf{pass}\ x)$$
$$[\![(99\text{-b})\text{-}1]\!] = \neg(\forall x.(\mathbf{student}\ x \rightarrow \neg(\mathbf{pass}\ x)))$$

Clearly, two negations are contained in the latter representation $[\![(99\text{-b})\text{-}1]\!]$. These two negations are not stacking over one another, hence they disqualify to cancel each other out directly. However, by employing the De Morgan's Laws see formula 3.1 and 3.5, we can reduce $[\![(99\text{-b})\text{-}1]\!]$ to the form(s) where the law of double negation can be applied:

$$\begin{aligned}
[\![(99\text{-b})\text{-}1]\!] &= \neg(\forall x.(\mathbf{student}\ x \rightarrow \neg(\mathbf{pass}\ x))) \\
&= \neg(\neg(\exists x.\neg(\mathbf{student}\ x \rightarrow \neg(\mathbf{pass}\ x)))) \\
&= \neg(\neg(\exists x.\neg(\neg(\mathbf{student}\ x \wedge \neg(\neg(\mathbf{pass}\ x)))))) \\
&= \exists x.(\mathbf{student}\ x \wedge \mathbf{pass}\ x)
\end{aligned}$$

Hence, the first sentences (99-a)-1 and (99-b)-1 are truth conditionally equivalent: formula $[\![(99\text{-a})\text{-}1]\!]$ and $[\![(99\text{-b})\text{-}1]\!]$ can be mutually transformed. However, the same second sentence: *they must have studied hard*, where an anaphoric pronoun is involved, is felicitous in (99-a) while somehow awkward, if not completely unacceptable, in (99-b). This gives rise to a doubt on the law of double negation. If double negation cancellation was always meaning-preserving in English, then (99-a)-1 and (99-b)-1 would be interchangeable in all situations. However, just as we presented, it does not always seem to be the case. So what is the problem with example (99)? Is the law of double negation subject to any specific condition? We shall come back to it in the next chapter.

5.3.4 Disjunction

As it has been shown in section 5.1, the dynamic frameworks presented in the previous chapter predict that no anaphoric links are possible between the two parts of a disjunction or from outside. Namely, on the one hand, referents of neither disjunct are accessible from the other; on the other hand, all referents introduced in a disjunction are not available for subsequent references. This characterization of disjunction's impact on anaphora is supported by the oddity of examples (75) and (76).

However, it has been noticed that in a disjunction, if the first disjunct is negated and it contains an indefinite NP, then it may well serve as antecedent for anaphoric expressions in the second disjunct. We call this **the disjunction problem**. It can be best illustrated with the following examples, where (11) is notoriously known as the bathroom example.

(100) Either Jones does not own $\underline{\text{a car}}_i$ or he hides $\underline{\text{it}}_i$. Kamp and Reyle (1993)

(11) Either there's $\underline{\text{no bathroom}}_i$ in the house, or $\underline{\text{it}}_i$'s in a funny place. Roberts (1989) (motivated by Barbara Partee)

The anaphoric links in both above sentences are perfectly acceptable. Can this be explained in dynamic theories? Let's take (100) as an illustration. We abbreviate the first disjunct *Jones does not own a car* as (100)-1, the second disjunct *Jones hides it* as (100)-2[6]. Then their dynamic representations under TTDL can be achieved as usual with respect to definition 4.4.4:

$$\begin{aligned}\overline{[\![(100)\text{-}1]\!]} &= \overline{[\![not]\!]([\![own]\!]([\![a]\!][\![car]\!])[\![Jones]\!])}\\ &= \overline{\neg}(\overline{[\![own]\!]([\![a]\!][\![car]\!])[\![Jones]\!]})\\ &\twoheadrightarrow_\beta \lambda e\phi.\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x) \wedge \phi e\end{aligned}$$

$$\begin{aligned}\overline{[\![(100)\text{-}2]\!]} &= \overline{([\![hide]\!][\![it]\!][\![Jones]\!])}\\ &\twoheadrightarrow_\beta \lambda e\phi.\mathbf{hide\ jones}\ (\mathsf{sel}\ e) \wedge \phi e\end{aligned}$$

As for the semantics of the whole sentence (100), we compose (100)-1 and (100)-2 with dynamic disjunction (the detailed entry of dynamic disjunction can be found in formula 5.3):

[6]The original sentence (100) is concerned with two anaphoric pronouns in the second disjunct. We "manually" resolve *he* as *Jones*, because at this moment we are mainly dealing with *it*. This simplification will make the illustration easier to understand.

$$\begin{aligned}
\overline{[\![(100)]\!]} &= \overline{[\![(100)\text{-}1]\!] \vee [\![(100)\text{-}2]\!]} \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\overline{[\![(100)\text{-}1]\!]}\ e\ \mathbf{stop}) \wedge \neg(\overline{[\![(100)\text{-}2]\!]}\ e\ \mathbf{stop})) \wedge \phi e \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\neg(\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x))) \wedge \neg(\mathbf{hide\ jones}\ (\mathsf{sel}\ e))) \wedge \phi e \\
&= \lambda e\phi.\neg(\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x) \wedge \neg(\mathbf{hide\ jones}\ (\mathsf{sel}\ e))) \wedge \phi e \\
&= \lambda e\phi.(\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x) \vee \mathbf{hide\ jones}\ (\mathsf{sel}\ e)) \wedge \phi e
\end{aligned}$$

The variable x, which is associated with the indefinite *a car*, is not updated into the current left context. Hence no anaphoric expressions outside the disjunction can refer to it, which is indeed the case. However, this also makes the choice operator sel to select variable from an empty list. Thus the anaphor *it* in the second disjunct will not be properly resolved. An analogous analysis can be applied to (11), as well. Meanwhile, the same result will be obtained by the other two dynamic frameworks. Namely, DRT and DPL also predict that the anaphoric links in (100) and (11) ought to be unacceptable.

Let's take a closer look at examples (100) and (11). In each sentence, the indefinites is under the scope of two constraints which block accessibility, i.e., negation and disjunction. However, similar to the examples where double negation is involved, e.g., (98), (10), under the effect of two constraints, the referent accessibility seems to be "unlocked" rather than being "twice blocked". Hence, disjunctions such as (100) and (11), where the first part is negative, indeed pose a challenge to dynamic frameworks.

In fact, there have been some attempts to resolve the disjunction problem with the already presented dynamic frameworks. One potential solution was proposed by Kamp and Reyle (1993). The basic idea is as follows: since disjunction is often used to give alternative possibilities, a sentence of the form "*A or B*" can generally be paraphrased as "*A or else B*" or "*A or otherwise B*". The particular word, *else* or *otherwise*, is used to designate the alternative cases, where the first disjunct A does not hold. As a result, we may spell out the alternative cases by explicitly adding the negation of the first disjunct to the second disjunct. The general rule of transformation can be described as follows:

$$A\ or\ B \Longrightarrow A\ or\ (not\ A\ and\ B) \tag{5.21}$$

When both disjuncts are affirmative, e.g., (75) and (76), the above transformation 5.21 does not change the referent accessibility predicted by the standard dynamic frameworks. Namely, anaphoric links within or outside the disjunction are still impossible. However, if the first disjunct is negative, e.g., (100) and (11), rule 5.21 does make a difference. Again, let's use example (100) as an illustration. According to the above analysis, (100) can be rephrased as any of the following sentences:

(101) a. Either Jones does not own a car$_i$ or else he hides it$_i$.
b. Either Jones does not own a car$_i$ or otherwise he hides it$_i$.
c. Either Jones does not own a car or Jones owns a car$_i$ and he hides it$_i$. Kamp and Reyle (1993)

In particular, focusing on (101-c), we label *Jones does not own a car* as (101-c)-1, *Jones owns a car* as (101-c)-2, *he hides it* as (101-c)-3. Then the dynamic interpretations of the three component sentences in TTDL are as follows:

$$\overline{[\![(101\text{-c})\text{-}1]\!]} = \overline{[\![(100)\text{-}1]\!]}$$
$$\twoheadrightarrow_\beta \lambda e\phi.\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ john}\ x) \wedge \phi e$$

$$\overline{[\![(101\text{-c})\text{-}2]\!]} = \overline{[\![own]\!]([\![a]\!][\![car]\!])[\![Jones]\!]}$$
$$\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ john}\ x \wedge \phi(x :: e))$$

$$\overline{[\![(101\text{-c})\text{-}3]\!]} = \overline{[\![(100)\text{-}2]\!]}$$
$$\twoheadrightarrow_\beta \lambda e\phi.\mathbf{hide\ jones}\ (\mathsf{sel}\ e) \wedge \phi e$$

To obtain the dynamic representation of the whole sentence (101-c), we disjunct $\overline{[\![(101\text{-c})\text{-}1]\!]}$ with the conjunction of $\overline{[\![(101\text{-c})\text{-}2]\!]}$ and $\overline{[\![(101\text{-c})\text{-}3]\!]}$, namely:

$$\begin{aligned}\overline{[\![(101\text{-c})]\!]} &= \overline{[\![(101\text{-c})\text{-}1]\!] \vee ([\![(101\text{-c})\text{-}2]\!] \wedge [\![(101\text{-c})\text{-}3]\!])}\\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\overline{[\![(101\text{-c})\text{-}1]\!]}\ e\ \mathbf{stop}) \wedge \neg(\overline{[\![(101\text{-c})\text{-}2]\!] \wedge [\![(101\text{-c})\text{-}3]\!]}\ e\ \mathbf{stop})) \wedge \phi e\\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x))\\
&\qquad\qquad \wedge \neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x \wedge \mathbf{hide\ jones}\ \mathsf{sel}(x :: e))) \wedge \phi e\\
&\twoheadrightarrow_\beta \lambda e\phi.(\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x)\\
&\qquad\qquad \vee \exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x \wedge \mathbf{hide\ jones}\ \mathsf{sel}(x :: e))) \wedge \phi e\end{aligned}$$

Due to the transformation rule 5.21, the negation of the first disjunct forms a conjunction with the second disjunct. This makes the indefinites in the first part of the disjunction (which is negative) accessible from the second disjunct. Hence, the choice operator may select variable x in the left context. This means that the anaphoric pronoun *it* can be successfully resolved as *a car*.

The above treatment, which is based on the paraphrasing rule 5.21, seems to give a valid account to the disjunction problem. However, it has been criticized on several aspects. First of all, as indicated in rule 5.21, the material to be updated in the second disjunct is the negation of the first disjunct. In the above analysis, we took for granted that (101-c)-2 is the negation of (101-c)-1, because double negation is often dropped in natural language. But as discussed in section 5.3.3, it is not the case in dynamic semantics. Strictly speaking, the negation of (101-c)-1 ought to be *it is not the case that Jones does not own a car*, and its dynamic representation in TTDL is $\overline{\neg[\![(101\text{-c})\text{-}1]\!]}$ rather than $\overline{[\![(101\text{-c})\text{-}2]\!]}$. Then the semantics of (101-c) should be re-computed as follows:

$$\begin{aligned}\overline{[\![(101\text{-c})]\!]} &= \overline{[\![(101\text{-c})\text{-}1]\!] \vee (\neg[\![(101\text{-c})\text{-}1]\!] \wedge [\![(101\text{-c})\text{-}3]\!])}\\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\overline{[\![(101\text{-c})\text{-}1]\!]}\ e\ \mathbf{stop}) \wedge \neg(\overline{\neg[\![(101\text{-c})\text{-}1]\!] \wedge [\![(101\text{-c})\text{-}3]\!]}\ e\ \mathbf{stop})) \wedge \phi e\\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x))\\
&\qquad\qquad \wedge \neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x \wedge \mathbf{hide\ jones}\ (\mathsf{sel}\ e))) \wedge \phi e\\
&\twoheadrightarrow_\beta \lambda e\phi.(\neg\exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x)\\
&\qquad\qquad \vee \exists x.(\mathbf{car}\ x \wedge \mathbf{own\ jones}\ x \wedge \mathbf{hide\ jones}\ (\mathsf{sel}\ e))) \wedge \phi e\end{aligned}$$

There is no variable in the left context for the selection function sel to choose from. Hence, even if rule 5.21 is employed, dynamic semantics still fails to resolve the disjunction problem. Because the negation of the first negative disjunct is concerned with double negation, which is itself an exception for dynamic theories. Unless we had a solution for the double negation problem, or the above analysis does not really resolve the disjunction problem.

In addition, the proposal in Kamp and Reyle (1993) suffers from another severe criticism: rule 5.21 is truth-conditionally problematic. Let's forget about the dynamic meaning for the moment. Assume there is a situation, where Jones owns two cars, and he hides only one of them, the other is right in front of his house. Then most people would agree that sentence (100) is false in this situation. However, as regards to (101-c), which is obtained from rule 5.21, it will be simply true because the second part of the disjunction: *Jones owns a car and he hides it*, is true: there does exist a car such that Jones hides it. As pointed out in Roberts (1989), the paraphrase reflecting the semantics of (100) should rather be:

(102) Either Jones does not own a car or if Jones owns $\underline{\text{a car}}_i$ then he hides $\underline{\text{it}}_i$.

The idea is that the negation of the first disjunct, which refers to the alternative case, should be accommodated to provide an antecedent to the second disjunct (in forming an implication). Put it more succinct, the paraphrasing rule for disjunction should be updated as:

$$A \textit{ or } B \Longrightarrow A \textit{ or } (\textit{if not } A \textit{ then } B) \tag{5.22}$$

Further suggested by Krahmer and Muskens (1995), rule 5.22 is equivalent to the following more consice one:

$$A \textit{ or } B \Longrightarrow \textit{if not } A \textit{ then } B \tag{5.23}$$

Namely, the original disjunction (100) is semantically equivalent not only to (102), but also to (103):

(103) If Jones owns $\underline{\text{a car}}_i$ then he hides $\underline{\text{it}}_i$.

For both paraphrases (102) and (103), the current dynamic frameworks can well account for the anaphoric links. As discussed in section 5.1, the referent of the antecedent in implication is accessible from the consequence. However, a careful reflection would reveal that we have again implicitly erased a double negation when generating (102) and (103). If we do it step by step, the negation of the first disjunct is *it is not the case that Jones does not own a car*, while not (101-c)-2, although it is often the case that they are identified as semantically equivalent. Note that the same identification does not hold in dynamic frameworks, because as we explained in the previous subsection, we will construct different DRSs for the two sentences, see the above illustrations for example (97). Hence, without the law of double negations, the remedial rules from Roberts (1989) and Krahmer and Muskens (1995) still won't help in solving the disjunction problem.

As a summary, we admit the validity of the transformation rules 5.22 and 5.23 for paraphrasing disjunction. However, unless the double negation elimination could be achieved, the anaphoric exceptions presented in examples (100) and (11) still can not be properly accounted for in dynamic frameworks. Consequently, we may generalize the exceptions described in this subsection and the previous subsection as the same sort. A single account should be able to deal with both the double negation and the disjunction problem.

5.3.5 Modal Subordination

As presented in section 5.2.2, a non-specific indefinite occurring within the scope of a modal can not antecede any anaphor in subsequent text, see examples (85) and (86). Even though it is involved in an anaphoric relation sometimes, the corresponding anaphoric expression is required to be located under the same limited domain of modality, as in example (87). The referent-blocking potential of modals can be captured by dynamic frameworks, as long as appropriate lexical entries (e.g., the one for *and*) are devised in TTDL. Accordingly, the problematic anaphora in the following discourse is ruled out:

(104) $\underline{\text{A thief}_i}$ might break into the house. *$\underline{\text{He}_i}$ takes the silver.

Example (104) is similar to (85) and (86) We assign *might* the following semantic entry, which is structurally parallel to the one for *can* (formula 5.5):

$$[\![might]\!] = \lambda c.\mathbf{might}\ c \tag{5.24}$$

Its dynamic counterpart is obtained straightforwardly (for a detailed computation, see formula 5.6):

$$\begin{aligned}\overline{[\![might]\!]} &= \overline{\lambda c.\mathbf{might}\ c} \\ &\twoheadrightarrow_\beta \lambda ce\phi.\mathbf{might}(c\ e\ \mathbf{stop}) \wedge \phi e\end{aligned} \tag{5.25}$$

The variable introduced by the indefinite NP *a thief* will not be updated in the left context:

$$\begin{aligned}\overline{[\![(104)\text{-}1]\!]} &= \overline{[\![might]\!]([\![break\_into\_the\_house]\!]([\![a]\!][\![thief]\!]))} \\ &\twoheadrightarrow_\beta (\lambda ce\phi.\mathbf{might}(c\ e\ \mathbf{stop}) \wedge \phi e)\overline{([\![break\_into\_the\_house]\!]([\![a]\!][\![thief]\!]))} \\ &\twoheadrightarrow_\beta \lambda e\phi.\mathbf{might}\ (\exists x.(\mathbf{thief}\ x \wedge \mathbf{break\_into\_the\_house}\ x)) \wedge \phi e\end{aligned} \tag{5.26}$$

Thus according to TTDL, the pronoun *he* in the second part of (104) can not be anaphorically related to *a thief*. Let's take a look at another example:

(105) If John bought $\underline{\text{a book}_i}$, he'll be home reading $\underline{\text{it}_i}$ by now. *$\underline{\text{It}_i}$'s a murder mystery. Roberts (1989)

That is because the first part of example (105) can be treated in the same way as implication. Since the referent in the antecedent is accessible from the consequence, the first anaphoric pronoun *it* in (105) is acceptable. If we ignore the tense in (105), the semantic representation of the first sentence can be computed as follows:

$$\begin{aligned}\overline{[\![(105)\text{-}1]\!]} = {} & \overline{([\![buy]\!]([\![a]\!][\![book]\!])[\![John]\!])} \overline{\Rightarrow} \overline{([\![be\_home\_reading]\!][\![it]\!][\![he]\!])} \\ & \twoheadrightarrow_\beta \lambda e\phi.\exists x.((\mathbf{book}\ x \wedge \mathbf{buy}\ x\ \mathbf{john}) \\ & \qquad \rightarrow \mathbf{be\_home\_reading}\ \mathsf{sel}_{he}(x :: e)\ \mathsf{sel}_{it}(x :: e)) \wedge \phi e\end{aligned} \tag{5.27}$$

As for the second occurrence of *it*, it is infelicitous because the pronoun is out of the scope of the implication, which is externally static. In the above formula, variable x is not updated into the current left context. So far the linguistic data seem to fit the prediction of standard dynamic theories. However, if modalities of a certain kind are involved in subsequent sentences, things will turn out to be pointing in an opposite direction. For instance, by adding a necessity modal into the second sentence of (105), we obtain another discourse (12), where both anaphoras become possible:

(12) If John bought a book$_i$, he'll be home reading it$_i$ by now. It$_i$'ll be a murder mystery. Roberts (1989)

It must be the modality in the second sentence that plays a central role in making the anaphora felicitous. Because the only difference between (105) and (12) is that the upcoming text in the formal is in factual mood, while in the latter it is in hypothetical mood. It is the same case if we add an appropriate modal to the second part of (104):

(13) A thief$_i$ might break into the house. He$_i$ would take the silver. Roberts (1989)

Similarly, the anaphoric pronoun *he*, which is infelicitous in (104), becomes admitted in (13). Discourses (12) and (13) share the property that the modality in subsequent sentences is interpreted in a context 'subordinate' to that created by the first modal. Or in other words, subsequent sentences are interpreted as being conditional on the scenario introduced in the first sentence. As to examples of this sort, standard dynamic frameworks fail to give an explanation. Because no discourse referent can survive outside the scope of modal operators. For illustration, see formula 5.26 and 5.27, where the current left context is empty. This phenomenon, where the anaphoric potential of non-specific indefinites extend beyond the limits predicted by standard dynamic frameworks, is known as **modal subordination** Roberts (1987, 1989). And modal subordination is a real exception to the accessibility constraints in dynamic frameworks, which is different from the double negation problem or the disjunction problem.

As a summary, in this chapter, we first reviewed the accessibility constraints in various dynamic frameworks, and examined the linguistic examples that they are designed to account for. After that we briefly reviewed discourse referent and specificity, and studied case by case the accessibility behavior of discourse referent under various linguistic environments. Finally we presented a list of anomalies which can not be correctly predicted by dynamic semantics, in particular, the double negation problem, the disjunction problem, and the modal subordination, among which the first two are reduced to a single one. In the next two chapters, we will focus on the two exceptions, we will study them in detail and solutions for each case will be proposed respectively.

Chapter 6

Double Negation

Contents

Following the preceding chapter, we arrive at the following two agreements on the relation between negation and anaphora:

- Single negation blocks discourse referents introduced within its scope;
- Double negation re-opens the accessibility of discourse referents within its scope.

The first principle has been successfully modeled in dynamic frameworks through different forms, such as the structural configurations in DRT, the dynamicity in DPL, and the left context in TTDL. For more discussions, please refer back to section 5.1. However, as explained in section 5.3.3, the second principle falls out of the prediction of dynamic frameworks. In this chapter, we will focus on the double negation exception, and propose a solution under the framework of TTDL. The structure of this chapter is as follows: we will first review a variant of DRT aiming at the same problem. It was proposed in Krahmer and Muskens (1995) and is called Double Negation DRT (DN-DRT). After that we will give the technical details of our proposal, which we name Double Negation TTDL (DN-TTDL).

6.1 Double Negation Adapted in DRT

In this section, we shall first present the formal framework of the Double Negation DRT, the illustrations will be provided afterwards.

6.1.1 Formal Framework

As its name implies, Double Negation DRT (DN-DRT) Krahmer and Muskens (1995) is an extension of DRT. It is proposed specifically for the problem of double negation. In DN-DRT, the negation of a DRS, in notation[1] $\sim K$, is treated as an independent DRS, rather than a DRS-condition. This is the only aspect on which the syntax of DN-DRT differs from the one of the standard DRT language Kamp and Reyle (1993). In the following presentation, we will ignore some preliminary definitions, such as the one for alphabet, terms (variable or constant) $\mathcal{T}$, universe R (set of discourse referents), conditions Con, etc., see definition 4.2.1 and 4.2.2 in section 4.2 for more details.

Definition 6.1.1. The syntax of the Double Negation DRT (DN-DRT), including DN-DRS and DN-DRS-condition, are mutually defined on each other as follows:

- A DN-DRS K has one of the following possible forms:
  1. Standard form: $\langle R_K, Con_K \rangle$, where $R_K = \{x_1, ..., x_n\}$, a set of discourse referents, is the **universe** of K, $Con_K = \{\phi_1, ..., \phi_n\}$ is the set of **DN-DRS-conditions** of K;
  2. Union form: $K_1; K_2$, if K_1 and K_2 are DRSs, symbol ";" denotes the merge of two DRSs;
  3. Negation form: $\sim K_1$, if K_1 is a DRS.
- A DN-DRS-condition ϕ has one of the following possible forms:
  1. Atomic condition: $\mathbf{P}t_1, ..., t_n$, if $\mathbf{P} \in \mathcal{P}$, and $t_1, ..., t_n \in \mathcal{T}$, n is the arity of $\mathbf{P}$;
  2. Link condition: $x \doteq t$, if $x \in \mathcal{X}$, and $t \in \mathcal{T}$, and $x \neq t$;
  3. Complex condition: $K_1 \vee K_2$, $K_1 \rightarrow K_2$, if K_1 and K_2 are DRSs.

For notations, we follow the convention as in standard DRT, see notation 4.2.1. As one may see from the above definition 6.1.1, the negation $\sim K$ is a DRS in DN-DRT. Besides, the union form $K_1; K_2$ is also a DRS. Recall that in standard DRT, we proposed the merge operation $\oplus$ (definition 4.2.3) to compose two DRSs. Every two DRSs can be reduced into a single one because $\oplus$ straightforwardly conjoins the referents of the two DRSs, as well as their conditions. However, it is not the case in DN-DRT. For instance, assume there are two DN-DRSs K_1 and K_2, if both of them are in the standard form as indicated in definition 6.1.1, the operator $\oplus$ will work as usual; while if any of K_1 and K_2 is of the negation form, $\oplus$ will fail to apply to them, because the notion of discourse referents and conditions are not defined for $\sim K$. As a result, we have to distinguish the merge operation in standard DRT (the operator $\oplus$) and DN-DRT (the operator ";"). That is also why the union form is among the definition of DN-DRS (in case $K_1; K_2$ can not be reduced).

Now let's have a look at its semantics. For the model, we stick to the one of FOL, see definition 3.1.13. The assignment function and relevant notation are also the same as in FOL, see definition 3.1.14 and notation 3.1.5. In addition, the notation $h[X]g$ is used in the same way as in FOL, see definition 3.1.15.

[1]In order to distinguish the negation of DN-DRT from the negation in standard DRT, the authors of Krahmer and Muskens (1995) employ the symbol "$\sim$" instead of "$\neg$", here we will stick to their notation.

As to the interpretation of DN-DRSs under a model M, it is different from standard DRT: every DN-DRS K is associated with two extensions, **the positive extension** $[\![K]\!]^{+M}_{DN\text{-}DRT}$ (also called the extension) and **the negative extension** $[\![K]\!]^{-M}_{DN\text{-}DRT}$ (also called the anti-extension), their detailed interpretations are defined inductively on the semantics of K. Generally speaking, the positive extension is used when the sentence is affirmative, the negative extension is used when the sentence is negative. Hence among all the conditions, atoms, links, and complex conditions of the form $K_1 \rightarrow K_2$ are interpreted in the same way as before, since no negations are involved. While a disjunction condition $K_1 \vee K_2$ is transformed into $\sim K_1 \rightarrow K_2$, where a negation is added to the first disjunct. So its interpretation deviates from the standard version such that it makes use of the negative extension. Detailed semantics of DN-DRT is presented below.

Definition 6.1.2. Let M be a model. The semantics of the Double Negation DRT, namely the interpretation of a DN-DRS K and a DN-DRS-condition ϕ, in notation $[\![K]\!]^{M}_{DN\text{-}DRT}$ or $[\![\phi]\!]^{M}_{DN\text{-}DRT}$, is defined as follows:

1. $[\![\mathbf{P}t_1, ..., t_n]\!]^{M}_{DN\text{-}DRT} = \{g \mid \langle [\![t_1]\!]^{M,g}_{DN\text{-}DRT}, ..., [\![t_n]\!]^{M,g}_{DN\text{-}DRT} \rangle \in I(\mathbf{P})\}$;
2. $[\![x \doteq t]\!]^{M}_{DN\text{-}DRT} = \{g \mid g(x) = [\![t]\!]^{M,g}_{DN\text{-}DRT}\}$;
3. $[\![K_1 \vee K_2]\!]^{M}_{DN\text{-}DRT} = \{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{-M}_{DN\text{-}DRT} \rightarrow \exists h : \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\}$;
4. $[\![K_1 \rightarrow K_2]\!]^{M}_{DN\text{-}DRT} = \{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \exists h : \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\}$;
5. $[\![\langle R_K, Con_K\rangle]\!]^{+M}_{DN\text{-}DRT} = \{\langle g, h\rangle \mid h[R_K]g \text{ and } \forall \phi \in Con_K : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}$;
6. $[\![\langle R_K, Con_K\rangle]\!]^{-M}_{DN\text{-}DRT} = \{\langle g, g\rangle \mid \neg\exists h : h[R_K]g \text{ and } \forall \phi \in Con_K : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}$;
7. $[\![K_1; K_2]\!]^{+M}_{DN\text{-}DRT} = \{\langle g, h\rangle \mid \exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\}$;
8. $[\![K_1; K_2]\!]^{-M}_{DN\text{-}DRT} = \{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\}$;
9. $[\![\sim K_1]\!]^{+M}_{DN\text{-}DRT} = [\![K_1]\!]^{-M}_{DN\text{-}DRT}$;
10. $[\![\sim K_1]\!]^{-M}_{DN\text{-}DRT} = [\![K_1]\!]^{+M}_{DN\text{-}DRT}$.

Now we define the notion of equivalence in DN-DRT.

Definition 6.1.3. Let M be a model, ϕ and ψ two DN-DRS-conditions, K_1 and K_2 two DN-DRSs. We say that:

- ϕ is **equivalent** to ψ iff $[\![\phi]\!]^{M}_{DN\text{-}DRT} = [\![\psi]\!]^{M}_{DN\text{-}DRT}$;
- K_1 is **equivalent** to K_2 iff $[\![K_1]\!]^{+M}_{DN\text{-}DRT} = [\![K_2]\!]^{+M}_{DN\text{-}DRT}$ and $[\![K_1]\!]^{-M}_{DN\text{-}DRT} = [\![K_2]\!]^{-M}_{DN\text{-}DRT}$.

One crucial point we can draw from the above semantics is that the rule of **double negation elimination** holds in DN-DRT, namely a DN-DRS K is equivalent to its double negated form: $\sim\sim K$. This can be revealed simply by comparing the extensions of K and $\sim\sim K$, which always coincide. More specifically, based on rules 9 and 10 in definition 6.1.2, we have the following relations:

- $[\![\sim\sim K]\!]^{+M}_{DN\text{-}DRT} = [\![\sim K]\!]^{-M}_{DN\text{-}DRT} = [\![K]\!]^{+M}_{DN\text{-}DRT}$
- $[\![\sim\sim K]\!]^{-M}_{DN\text{-}DRT} = [\![\sim K]\!]^{+M}_{DN\text{-}DRT} = [\![K]\!]^{-M}_{DN\text{-}DRT}$

Further more, in order to investigate the accessibility of discourse referent, the authors of Krahmer and Muskens (1995) have defined two concepts for a DN-DRS: **active discourse referents** (ADR) and **passive discourse referents** (PDR).

Definition 6.1.4. The active discourse referents (ADR) and passive discourse referents (PDR) of a given DN-DRS K are defined as follows:

1. $ADR(\langle R_K, Con_K \rangle) = R_K$;

2. $PDR(\langle R_K, Con_K \rangle) = \emptyset$;

3. $ADR(K_1; K_2) = ADR(K_1) \cup ADR(K_2)$;

4. $PDR(K_1; K_2) = \emptyset$;

5. $ADR(\sim K_1) = PDR(K_1)$;

6. $PDR(\sim K_1) = ADR(K_1)$.

According to the above definition, both $ADR(K)$ and $PDR(K)$ designate a set of referents which other DRSs or conditions might access. The difference between the two is that $ADR(K)$ is the set when the positive extension of K is applied; $PDR(K)$ is the set when the negative extension of K is applied.

We define the relation of subordination (both positive and negative) between two DN-DRSs as follows.

Definition 6.1.5. Let K_1 and K_2 be DN-DRSs. We say K_1 **positively subordinates** K_2, in notation $K_1 >^+ K_2$, iff either:

1. There exists some DN-DRS K_3 such that it is in the union form $K_1; K_2$;

2. $K_1 \to K_2$ is a condition of some DN-DRS;

3. $K_1 = \langle R_{K_1}, Con_{K_1} \rangle$ is in the standard form, and

   (a) $K_2 \vee K_3 \in Con_{K_1}$ or $K_3 \vee K_2 \in Con_{K_1}$;

   (b) $K_2 \to K_3 \in Con_{K_1}$ or $K_3 \to K_2 \in Con_{K_1}$;

   where K_3 is a DN-DRS;

4. There exists some DN-DRS K_3 such that $K_1 >^+ K_3$, and $K_3 >^+ K_2$.

We say K_1 **negatively subordinates** K_2, in notation $K_1 >^- K_2$, iff either:

1. $K_1 \vee K_2$ is a condition of some DN-DRS;

2. $K_1 >^- K_3$, and $K_3 >^+ K_2$, where K_3 is a DN-DRS.

In addition, a DN-DRS may occur in another DN-DRS, so does a DN-DRS-condition. The notion of occurrence is thus defined as follows.

Definition 6.1.6. Let K_1 and K_2 be DN-DRSs, ϕ a DN-DRS-condition. We say K_1 **occurs in** K_2 iff:

1. K_1 is equivalent to K_2;

2. There exists some DN-DRS K_3 such that K_3 occurs in K_2 and K_1 occurs in K_3;

3. $K_2 = \langle R_{K_2}, Con_{K_2} \rangle$ is in the standard form, and

   (a) $K_1 \vee K_3 \in Con_{K_2}$ or $K_3 \vee K_1 \in Con_{K_2}$;

   (b) $K_1 \rightarrow K_3 \in Con_{K_2}$ or $K_3 \rightarrow K_1 \in Con_{K_2}$;

   where K_3 is a DN-DRS;

4. $K_2 = {\sim}K_3$ is in the negation form, and K_1 occurs in K_3.

We say ϕ **occurs in** K_2 iff there exists some DN-DRS $K_3 = \langle R_{K_3}, Con_{K_3} \rangle$ such that K_3 occurs in K_2, and $\phi \in Con_{K_3}$.

Based on the above concepts, we can define the accessibility in the DN-DRT language.

Definition 6.1.7. The accessibility function ACC either takes two DN-DRSs K_1 and K_2 such that K_1 occurs in K_2, or a condition ϕ and a DN-DRS K_2 such that ϕ occurs in K_2, it returns the set of accessible discourse referents from K_1 or ϕ in K_2. ACC is defined as follows:

- $ACC(K_1, K_2)$ is the smallest set such that:
  1. $\forall K_p$ occurring in $K_2 : K_p >^+ K_1 \rightarrow ADR(K_p) \subseteq ACC(K_1, K_2)$, and
  2. $\forall K_n$ occurring in $K_2 : K_n >^- K_1 \rightarrow PDR(K_p) \subseteq ACC(K_1, K_2)$.
- $ACC(\phi, K_2) = ACC(K_3, K_2) \cup R_{K_3}$, where $K_3 = \langle R_{K_3}, Con_{K_3} \rangle$ occurs in K_2 and $\phi \in Con_{K_3}$.

The notion of free variable and proper DRS is defined as follows:

Definition 6.1.8. Let K be a DN-DRS, ϕ an atomic condition or a link condition occurring in K, x a discourse referent in ϕ. We say x is **free** in K iff $x \notin ACC(\phi, K)$.

Let ψ be a DN-DRS-condition, x a discourse referent in ψ. We say x is **free** in ψ iff x is free in $\langle \emptyset, \psi \rangle$.

According to definition 6.1.2, the interpretation of a DN-DRS-condition is a set of assignment functions. If a condition contains free referents, then its interpretation will depend on the values assigned to those referents. In other words, bound referents (referents that are not free) of a condition will not affect its interpretation.

As mentioned in the syntax of DN-DRT (definition 6.1.1), not only the negation form ${\sim}K$, but also the union form $K_1; K_2$ is treated as a DRS. One side effect of this is that we might not be able to reduce the size of a complex DRS, like what we usually do in standard DRT. In fact, the merge operator ";" in DN-DRT works in a similar way as the conventional $\oplus$ under some conditions. For practical reason, we will prove the following lemma.

Lemma 6.1.1 (Merging Lemma)**.** Let K_1 and K_2 be two standard DN-DRSs, namely $K_1 = \langle R_{K_1}, Con_{K_1} \rangle$ and $K_2 = \langle R_{K_2}, Con_{K_2} \rangle$. If no referent in R_{K_2} is free in any condition of Con_{K_1}, then $K_1; K_2$ is equivalent to $\langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2} \rangle$.

Proof. According to definition 6.1.3, two DN-DRSs are equivalent if their extensions (both positive and negative) coincide. The proof will be divided into two parts, we will first check the two positive extensions, then go to the negative extensions.

1. Positive Extensions

$\boxed{\text{For } K_1; K_2}$ According to definition 6.1.2, its positive extension is as follows:

$$\begin{aligned}
&[\![K_1; K_2]\!]^{+M}_{DN\text{-}DRT} \\
=&\{\langle g, h\rangle \mid \exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{\langle g, h\rangle \mid \exists f : \langle g, f\rangle \in [\![\langle R_{K_1}, Con_{K_1}\rangle]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{\langle g, h\rangle \mid \exists f : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and} \\
&\qquad h[R_{K_2}]f \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned}$$

According to the hypothesis of the lemma, $\forall x \in R_{K_2}$, $\forall\phi \in Con_{K_1}$, x is not free in ϕ. This means, assignments on referents in R_{K_2} will not affect the interpretations of conditions in Con_{K_1}. Since h and f differ at most with respect to values on elements of R_{K_2}, they will work exactly in the same way for interpreting Con_{K_1}. Put it formally, assume x is a variable in ϕ such that $x \notin FV(\phi)$, h, f are assignments such that $h[\{x\}]f$, if $f \in [\![\phi]\!]$, then $h \in [\![\phi]\!]$. As a result, we replace $\forall\phi \in Con_{K_1} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT}$ with $\forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}$ in the previous step:

$$\begin{aligned}
&[\![K_1; K_2]\!]^{+M}_{DN\text{-}DRT} \\
=&\{\langle g, h\rangle \mid \exists f : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and} \\
&\qquad h[R_{K_2}]f \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned}$$

Based on definition 3.1.15, we can infer $h[R_{K_1} \cup R_{K_2}]g$ from $f[R_{K_1}]g$ and $h[R_{K_2}]f$. As a result, we reduce the previous step as follows:

$$\begin{aligned}
&[\![K_1; K_2]\!]^{+M}_{DN\text{-}DRT} \\
=&\{\langle g, h\rangle \mid \exists f : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and} \\
&\qquad h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, h\rangle \mid h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and} \\
&\qquad \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.1}$$

$\boxed{\text{For } \langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle}$ According to definition 6.1.2, its positive extension is as follows:

$$\begin{aligned}
&[\![\langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT} \\
=&\{\langle g, h\rangle \mid h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} \cup Con_{K_2} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, h\rangle \mid h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and} \\
&\qquad \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.2}$$

Compare formulas 6.1 and 6.2, we obtain the following relation:

$$[\![K_1;K_2]\!]^{+M}_{DN\text{-}DRT} = [\![\langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}$$

2. Negative Extensions

To check the two negative extensions, we carry out the same operation.

$\boxed{\text{For } K_1;K_2}$ According to definition 6.1.2, its negative extension is as follows:

$$\begin{aligned}
&[\![K_1;K_2]\!]^{-M}_{DN\text{-}DRT}\\
=&\{\langle g,g\rangle \mid \neg\exists f\exists h : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f,h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\}\\
=&\{\langle g,g\rangle \mid \neg\exists f\exists h : \langle g,f\rangle \in [\![\langle R_{K_1}, Con_{K_1}\rangle]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f,h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\}\\
=&\{\langle g,g\rangle \mid \neg\exists f\exists h : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and}\\
&\qquad h[R_{K_2}]f \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned}$$

Based on the hypothesis of the lemma: $\forall x \in R_{K_2}$, $\forall\phi \in Con_{K_1}$, x is not free in ϕ, we replace $\forall\phi \in Con_{K_1} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT}$ with $\forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}$ in the above formula. For more discussion, see the first part of the proof.

$$\begin{aligned}
&[\![K_1;K_2]\!]^{-M}_{DN\text{-}DRT}\\
=&\{\langle g,g\rangle \mid \neg\exists f\exists h : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and}\\
&\qquad h[R_{K_2}]f \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned}$$

Similarly, based on definition 3.1.15, we can infer $h[R_{K_1} \cup R_{K_2}]g$ from $f[R_{K_1}]g$ and $h[R_{K_2}]f$. As a result, we reduce the previous step as follows:

$$\begin{aligned}
&[\![K_1;K_2]\!]^{-M}_{DN\text{-}DRT}\\
=&\{\langle g,g\rangle \mid \neg\exists f\exists h : f[R_{K_1}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and}\\
&\qquad h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,g\rangle \mid \neg\exists h : h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and}\\
&\qquad \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.3}$$

$\boxed{\text{For } \langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle}$ According to definition 6.1.2, its negative extension is as follows:

$$\begin{aligned}
&[\![\langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT}\\
=&\{\langle g,g\rangle \mid \neg\exists h : h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} \cup Con_{K_2} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,g\rangle \mid \neg\exists h : h[R_{K_1} \cup R_{K_2}]g \text{ and } \forall\phi \in Con_{K_1} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT} \text{ and}\\
&\qquad \forall\psi \in Con_{K_2} : h \in [\![\psi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.4}$$

Compare formulas 6.3 and 6.4, we obtain the following relation:

$$[\![K_1; K_2]\!]^{-M}_{DN\text{-}DRT} = [\![\langle R_{K_1} \cup R_{K_2}, Con_{K_1} \cup Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT}$$

As a result, provided that $K_1 = \langle R_{K_1}, Con_{K_1}\rangle$, $K_2 = \langle R_{K_2}, Con_{K_2}\rangle$, and no referent in R_{K_2} is free in any condition of Con_{K_1}, definition 6.1.3 concludes that K_1 is equivalent to K_2. □

Syntactically, DN-DRT differs from the standard DRT language by defining negation as a DRS rather than a condition. A consequence is that when we merge two DN-DRSs, if any of them is of the negation form $\sim K$, the union can not be reduced to a DN-DRS of the standard form, and one might end up with a rather long representational structure (lemma 6.1.1 only applies to DRSs of the standard form). In order to reduce the size of representation, also to have a more explicit comparison between DN-DRT and the standard DRT, Krahmer and Muskens (1995) reintroduces the original negation $\neg$ in the vocabulary of DN-DRT, and defines in terms of the new negation $\sim$ as follows:

$$\neg K \triangleq K \rightarrow \sim\langle \emptyset, \emptyset\rangle \tag{6.5}$$

As indicated in formula 6.5, different from $\sim K$, $\neg K$ is still a condition, like in standard DRT (implications are conditions). The above definition leads to the following semantics of $\neg$ under DN-DRT, which can be obtained based on definition 6.1.2:

$$[\![\neg K]\!]^{M}_{DN\text{-}DRT} = \{g \mid \neg\exists h : \langle g, h\rangle \in [\![K]\!]^{+M}_{DN\text{-}DRT}\} \tag{6.6}$$

Some useful lemma concerning single negation is provided in the original reference Krahmer and Muskens (1995). We will add the corresponding proof.

Lemma 6.1.2 (Single Negation Lemma)**.** Let K_1 and $\langle R_{K_2}, Con_{K_2}\rangle$ be two DN-DRSs, the following propositions hold:

1. $K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle$ is equivalent with $K_1 \rightarrow \langle \emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle$
2. $\sim\langle R_{K_2}, Con_{K_2}\rangle \rightarrow K_1$ is equivalent with $\langle \emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle \rightarrow K_1$
3. $K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle$ is equivalent with $K_1; \langle \emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle$
4. $\sim\langle R_{K_2}, Con_{K_2}\rangle; K_1$ is equivalent with $\langle \emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle; K_1$

Proof. The proof of lemma 6.1.2 is straightforward. The first two equivalences are about DN-DRS-conditions, the last two are about DN-DRSs. We will only focus on the first and the third proposition. As to the second and the fourth, they can be proved in a similar way.

(a) According to definition 6.1.3, two DN-DRS-conditions are equivalent if their extensions coincide. Hence we will check the extensions of $K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle$ and $K_1 \rightarrow \langle \emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle$, then make a comparison between them.

$\boxed{\text{For } K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle}$ According to definition 6.1.2, its extension is as follows:

$$\begin{aligned}
&[\![K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \exists h : \langle f, h\rangle \in [\![\sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \exists h : \langle f, h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \\
&\qquad \exists h : \langle f, h\rangle \in \{\langle i, i\rangle \mid \neg\exists j : j[R_{K_2}]i \text{ and } \forall\phi \in Con_{K_2} : j \in [\![\phi]\!]^M_{DN\text{-}DRT}\}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \\
&\qquad \langle f, f\rangle \in \{\langle i, i\rangle \mid \neg\exists j : j[R_{K_2}]i \text{ and } \forall\phi \in Con_{K_2} : j \in [\![\phi]\!]^M_{DN\text{-}DRT}\}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \neg\exists h : h[R_{K2}]f \text{ and } \forall\phi \in Con_{K_2} : h \in [\![\phi]\!]^M_{DN\text{-}DRT}\}
\end{aligned} \tag{6.7}$$

$\boxed{\text{For } K_1 \rightarrow \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle}$ According to definition 6.1.2, its extension is as follows:

$$\begin{aligned}
&[\![K_1 \rightarrow \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^M_{DN\text{-}DRT} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \exists h : \langle f, h\rangle \in [\![\langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \exists h : h[\emptyset]f \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow f \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT}\}
\end{aligned} \tag{6.8}$$

Let's focus on the sub-part $[\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT}$, based on formula 6.6 and definition 6.1.2, its interpretation is as follows:

$$\begin{aligned}
&[\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT} \\
=&\{g \mid \neg\exists h : \langle g, h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{g \mid \neg\exists h : h[R_{K_2}]g \text{ and } \forall\phi \in Con_{K_2} : h \in [\![\phi]\!]^M_{DN\text{-}DRT}\}
\end{aligned} \tag{6.9}$$

We substitute $[\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT}$ in formula 6.8 with the result of formula 6.9:

$$\begin{aligned}
&[\![K_1 \rightarrow \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^M_{DN\text{-}DRT} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow f \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT}\} \\
=&\{g \mid \forall f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \rightarrow \neg\exists h : h[R_{K_2}]f \text{ and } \forall\phi \in Con_{K_2} : h \in [\![\phi]\!]^M_{DN\text{-}DRT}\}
\end{aligned} \tag{6.10}$$

Compare formula 6.7 and 6.10, we obtain the following relation:

$$[\![K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^M_{DN\text{-}DRT} = [\![K_1 \rightarrow \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^M_{DN\text{-}DRT}$$

As a result, definition 6.1.3 concludes that condition $K_1 \rightarrow \sim\langle R_{K_2}, Con_{K_2}\rangle$ and condition $K_1 \rightarrow \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle$ are equivalent.

(b) According to definition 6.1.3, two DN-DRSs are equivalent if their extensions (both positive and negative) coincide. We will first look at the two positive extensions, then check the negative extensions.

1. Positive Extensions

$\boxed{\text{For } K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle}$ According to definition 6.1.2, its positive extension is as follows:

$$\begin{aligned}
&[\![K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f,h\rangle \in [\![\sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f,h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f = h \text{ and } \neg\exists j : j[R_{K_2}]h \text{ and}\\
&\qquad \forall\phi \in Con_{K_2} : j \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \langle g,h\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \neg\exists f : f[R_{K_2}]h \text{ and } \forall\phi \in Con_{K_2} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.11}$$

$\boxed{\text{For } K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle}$ According to definition 6.1.2, its positive extension is as follows:

$$\begin{aligned}
&[\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f,h\rangle \in [\![\langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f[\emptyset]h \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \exists f : \langle g,f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f = h \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \langle g,h\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.12}$$

We substitute $[\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}$ in formula 6.12 with the result of formula 6.9:

$$\begin{aligned}
&[\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}\\
=&\{\langle g,h\rangle \mid \langle g,h\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\}\\
=&\{\langle g,h\rangle \mid \langle g,h\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \neg\exists f : f[R_{K_2}]h \text{ and } \forall\phi \in Con_{K_2} : f \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.13}$$

Compare formula 6.11 and 6.13, we obtain the following relation:

$$[\![K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT} = [\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}$$

2. Negative Extensions

$\boxed{\text{For } K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle}$ According to definition 6.1.2, its negative extension is as follows:

$$\begin{aligned}
&[\![K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![K_2]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![\sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![\langle R_{K_2}, Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f = h \text{ and } \neg\exists j : j[R_{K_2}]h \text{ and} \\
&\qquad \forall\phi \in Con_{K_2} : j \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \neg\exists h : h[R_{K_2}]f \text{ and} \\
&\qquad \forall\phi \in Con_{K_2} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.14}$$

$\boxed{\text{For } K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle}$ According to definition 6.1.2, its negative extension is as follows:

$$\begin{aligned}
&[\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{-M}_{DN\text{-}DRT} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \langle f, h\rangle \in [\![\langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{+M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } h[\emptyset]f \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f \exists h : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } h = f \text{ and } h \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.15}$$

We substitute $[\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}$ in formula 6.15 with the result of formula 6.9:

$$\begin{aligned}
&[\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{-M}_{DN\text{-}DRT} \\
=&\{\langle g, g\rangle \mid \neg\exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } f \in [\![\neg\langle R_{K_2}, Con_{K_2}\rangle]\!]^{M}_{DN\text{-}DRT}\} \\
=&\{\langle g, g\rangle \mid \neg\exists f : \langle g, f\rangle \in [\![K_1]\!]^{+M}_{DN\text{-}DRT} \text{ and } \neg\exists h : h[R_{K_2}]f \text{ and} \\
&\qquad \forall\phi \in Con_{K_2} : h \in [\![\phi]\!]^{M}_{DN\text{-}DRT}\}
\end{aligned} \tag{6.16}$$

Compare formula 6.14 and 6.16, we obtain the following relation:

$$[\![K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle]\!]^{-M}_{DN\text{-}DRT} = [\![K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle]\!]^{-M}_{DN\text{-}DRT}$$

As a result, definition 6.1.3 concludes that the DN-DRS $K_1; \sim\langle R_{K_2}, Con_{K_2}\rangle$ is equivalent to the DN-DRS $K_1; \langle\emptyset, \neg\langle R_{K_2}, Con_{K_2}\rangle\rangle$.

We will leave out the proof for the remaining propositions, which can be conducted in an analogous way. □

With the above configurations, DN-DRT is able to explain examples of anaphora, where double negation or disjunction is concerned with, such as (98), (10), (100) and (11) in section 5.3. We will provide an illustration in the next subsection.

6.1.2 Illustration

For a simple illustration of DN-DRT, we use discourse (98-a) as an example, which is repeated as follows:

(98-a) It is not true that John didn't bring $\underline{\text{an umbrella}}_i$. $\underline{\text{It}}_i$ was purple and $\underline{\text{it}}_i$ stood in the hallway. Krahmer and Muskens (1995)

First of all, let's look back at a triplet of examples in the previous chapter, sentences in which are affirmative, negative, and double-negative, respectively:

(97) a. John brought an umbrella.
b. John didn't bring an umbrella.
c. It is not true that John didn't bring an umbrella.

In DN-DRT, the representation for each sentence in (97) is listed as follows:

$$K_{(97\text{-a})}: \begin{array}{|c|}\hline x \\ \hline \textbf{umbrella}\ x \\ \textbf{bring john}\ x \\ \hline\end{array} \qquad K_{(97\text{-b})}: \sim \begin{array}{|c|}\hline x \\ \hline \textbf{umbrella}\ x \\ \textbf{bring john}\ x \\ \hline\end{array}$$

$$K_{(97\text{-c})}: \sim\sim \begin{array}{|c|}\hline x \\ \hline \textbf{umbrella}\ x \\ \textbf{bring john}\ x \\ \hline\end{array}$$

We have already shown that a double negation in DN-DRT can be cancelled, namely a DN-DRS K is equivalent to its double negated form $\sim\sim K$. Hence $K_{(97\text{-a})}$ is equivalent to $K_{(97\text{-c})}$, they can be used interchangeably in discourse incrementation. The linguistic intuition behind this is that sentence (97-a) means the same as sentence (97-c) in discourses such as (98-a), which correctly corresponds to what we expect. For the DN-DRS of the second sentence in (98-a), it can be constructed as:

$$K_{(98\text{-a})\text{-}2}: \begin{array}{|c|}\hline y \\ \hline \textbf{purple}\ y \\ \textbf{stand\_in\_the\_hallway}\ y \\ \hline\end{array}$$

In order to achieve the semantics of the whole discourse (98-a), we simply update the DN-DRS of the second sentence to the one of the first sentence. The corresponding representational structure is constructed as follows:

$$K_{(98\text{-a})}: \begin{array}{|c|}\hline x \\ \hline \textbf{umbrella}\ x \\ \textbf{bring john}\ x \\ \hline\end{array} ; \begin{array}{|c|}\hline y \\ \hline \textbf{purple}\ y \\ \textbf{stand\_in\_the\_hallway}\ y \\ \hline\end{array}$$

According to definition 6.1.5 on subordination, we can infer that $K_{(98\text{-a})\text{-}1} >^{+} K_{(98\text{-a})\text{-}2}$. Then based on definition 6.1.7, the accessible referents from $K_{(98\text{-a})\text{-}2}$ in $K_{(98\text{-a})}$, namely $ACC(K_{(98\text{-a})\text{-}2}, K_{(98\text{-a})})$ is the smallest set subsuming $ADR(K_{(98\text{-a})\text{-}1})$, which is simply $\{x\}$ according to definition 6.1.4. As a result, the pronoun *it* can be resolved by identifying y with x, and the double negation problem is solved.

In addition, the above DN-DRSs satisfy the condition of the Merging lemma 6.1.1: both $K_{(98\text{-a})\text{-}1}$ and $K_{(98\text{-a})\text{-}2}$ are in the standard form, variable y is not free in $K_{(98\text{-a})\text{-}1}$. Hence we will end up with a more succinct representation for discourse (98-a), where the link condition $y \doteq x$ is added:

$K_{(98\text{-a})}$:

$x,\ y$
umbrella x
bring john x
purple y
stand_in_the_hallway y
$y \doteq x$

Note that the above representation is also a valid DRS in standard DRT, which corresponds to the following discourse:

(106) John brought an umbrella$_i$. It$_i$ was purple and it$_i$ stood in the hallway.

For more examples, please refer to the original reference Krahmer and Muskens (1995). As a summary, DN-DRT treats the double negation exception by proposing two extensions for a DRS, which corresponds to the affirmative and negative polarity of the sentence in question. In the next section, we will extend TTDL to cover the same problem, while in a more compositional and standard manner.

6.2 Adaptation of TTDL to Double Negation

Like other classical dynamic frameworks such as DRT and DPL, TTDL fails to explain why double negation re-allows anaphora, where the antecedent is within its scope. In this section, we provide an adaptation of TTDL, rendering it the potential to handle the double negation exception. The framework is called Double Negation TTDL (DN-TTDL).

As shown in the previous section, the interpretation of each DN-DRS is associated with two extensions, one is positive, the other is negative. This modification complicates the semantics of the formal system, see definition 4.2.4 and 6.1.2 for a comparison with the semantics of the standard DRT. To avoid that, we propose to handle the same problem on the syntactic level.

The basic idea of DN-TTDL is as follows. When we translate a natural language expression into logical language, we encapsulate two propositions as an ordered pair. Among the two representations, one corresponds to the representation that the expression will obtain in the affirmative case, the other corresponds to the representation that it will obtain in the negative case. The purpose of keeping two representations is that we can treat negation as a syntactic flip-flop device, which switches the order of elements in the pair. If a second negation occurs in the sentence, it re-switches the order again, which "cancels" the effects of the first negation. Likewise, one can infer the situation where more negations are involved. In such way, negation modifies the logical form in a reversible manner, rather than "damaging" it.

This property distinguishes DN-TTDL from DRT, DPL, and TTDL, where the semantic representation only contains the positive case. Further more, in these frameworks, negation is an irreversible operation: it permanently blocks or deletes the discourse referents within its scope. Hence a referent will be twice blocked under a double negation, this is why these standard dynamic theories claim that anaphoras are not permitted under double negations.

For the rest of this chapter, we will focus on presenting DN-TTDL. We start with its technical details, such as syntax, semantics, typing information, etc., then we will examine a number of examples, which are concerned with double negations and can be successfully handled in DN-TTDL.

6.2.1 Formal Framework

As an extension of TTDL, the formal framework of DN-TTDL is also the same as in the simply typed λ-calculus, which has been presented in section 3.2. For the syntax of DN-TTDL, please refer to section 3.2.1. Like TTDL, DN-TTDL is also a typed system: every term in DN-TTDL is associated with a unique type. Again, we will not repeat the notions which the language of types are concerned with, such as types, typing assumption, typing context, typing rule, etc., for more discussions, please refer to section 3.2.2.

The signature for DN-TTDL is exactly the same as the one for TTDL, see definition 4.4.1. We do not need extra atomic types or constants in DN-TTDL. As explained at the beginning of this section, DN-TTDL extends TTDL by encapsulating two propositions in a single representation (a pair). Since each of the two propositions conforms to the syntax of TTDL independently, the two frameworks are able to share the same types, constants, etc.

Now we shall present how sentences and discourses are evaluated in DN-TTDL. As discussed in section 4.4.2, sentences and discourses are interpreted in TTDL as dynamic propositions (of type Ω), which are functions from left (of type γ) and right (of type $\gamma \to o$) contexts to truth values (of type o), see formula 4.31. In DN-TTDL, we interpret discourses in exactly the same way as in TTDL. Thus, its semantic type remains Ω, namely (assume d is the syntactic category of discourses):

$$[\![d]\!]_{DN\text{-}TTDL} = \Omega \tag{6.17}$$

While the way that sentences are interpreted in DN-TTDL will be recast as follows (let s be the syntactic category of sentences):

$$[\![s]\!]_{DN\text{-}TTDL} = \Omega \times \Omega \tag{6.18}$$

As shown in formula 6.18, in DN-TTDL, a sentence is a pair of dynamic propositions, one corresponds to the affirmative interpretation, the other to the negative one. Hereinafter, we will use Ω^{dn} as an abbreviation for $\Omega \times \Omega$, namely:

$$\Omega^{dn} \triangleq \Omega \times \Omega \tag{6.19}$$

The reason that we consider sentences and discourses as different semantic objects is because they diverge in various semantic aspects, particularly on the issue of negation: it is more often the case that we negate a sentence, instead of negating a whole discourse.

Hence there is no need to keep the negation of a discourse in the meaning representation. Note that TTDL does not make use of the product type, which, however, is of vital importance in DN-TTDL. So all the rules that are concerned with product in simply typed λ-calculus, which are left out in TTDL, should be taken into consideration for DN-TTDL. Below we define the swap function, based on the two projection operators π_1 and π_2:

$$\mathbf{swap} \triangleq \lambda A.\langle \pi_2 A, \pi_1 A \rangle \tag{6.20}$$

The function **swap** takes an expression of the product type, returns another product-type expression. The difference between the input and output pairs is that the order of elements is switched. Hence we may infer an additional typing rule, which reflects the functionality of **swap**:

$$\frac{\Gamma \vdash M : \tau \times \sigma}{\Gamma \vdash \mathbf{swap}\ M : \sigma \times \tau}$$

In addition, we propose a lemma on the swap function, followed by its proof.

Lemma 6.2.1 (Swap Lemma)**.** Let M be a term that is in the form of a pair, namely $M = \langle M_1, M_2 \rangle$, then two consecutive applications of the **swap** function is the identity function:

$$\mathbf{swap}(\mathbf{swap}\ M) = M \tag{6.21}$$

Proof. The function **swap** is defined in formula 6.20. Then the left hand side of formula 6.21: $\mathbf{swap}(\mathbf{swap}\ M)$, is transformed as follows:

$$\begin{aligned}
\mathbf{swap}(\mathbf{swap}\ M) &= (\lambda A.\langle \pi_2 A, \pi_1 A \rangle)((\lambda A.\langle \pi_2 A, \pi_1 A \rangle)M) \\
&\to_\beta (\lambda A.\langle \pi_2 A, \pi_1 A \rangle)((\lambda A.\langle \pi_2 A, \pi_1 A \rangle)\langle M_1, M_2 \rangle) \\
&\to_\beta (\lambda A.\langle \pi_2 A, \pi_1 A \rangle)(\langle \pi_2 \langle M_1, M_2 \rangle, \pi_1 \langle M_1, M_2 \rangle \rangle) \\
&\twoheadrightarrow_\beta (\lambda A.\langle \pi_2 A, \pi_1 A \rangle)\langle M_2, M_1 \rangle \\
&\to_\beta \langle \pi_2 \langle M_2, M_1 \rangle, \pi_1 \langle M_2, M_1 \rangle \rangle \\
&\twoheadrightarrow_\beta \langle M_1, M_2 \rangle \\
&= M
\end{aligned} \tag{6.22}$$

As a result, $\mathbf{swap}(\mathbf{swap}\ M)$ can be β-reduced to M, the two terms are equivalent. □

In DN-TTDL, a sentence is interpreted as a pair of propositions. To negate a sentence, we just need to switch the order of the propositions in the pair. Then based upon the function **swap**, we propose the dynamic negation under DN-TTDL:

$$\neg^d_{DN\text{-}TTDL} \triangleq \lambda A.\mathbf{swap}\ A \tag{6.23}$$

As indicated in formula 6.23, $\neg^d_{DN\text{-}TTDL}$ takes a dynamic sentence, and returns its negated counterpart, which differs from the input form merely on the order of elements in the pair. For a comparison, see the definition for $\neg^d_{TTDL}$ in formula 4.37, which is the dynamic negation in TTDL: $\neg^d_{TTDL}$ directly passes the empty right context **stop** to the proposition under its scope, while it is not the case for $\neg^d_{DN\text{-}TTDL}$.

One may wonder how the discourse incrementation can be achieved with the current way of interpreting sentence and discourse? In fact, when we update a sentence into a preceding discourse, one of the two propositions in the pair will be picked out. Technically, we set the first position in the pair as the **positive position**, the second as the **negative position**. And the representation which denotes the current semantics of the sentence will take the positive position; correspondingly, the alternative semantics, which will be stipulated when a negation is added, will take the negative position.

In discourse incrementation, we always project the proposition corresponding to the current interpretation, namely the one at the positive position[2]. For instance, assume there is an affirmative simple sentence S, its standard TTDL representation $[\![S]\!]_{TTDL}$ takes the positive position, its dynamic negation $\bar{\neg}[\![S]\!]_{TTDL}$ takes the negative position. If S is not negated, $[\![S]\!]_{TTDL}$ will be retrieved; if S is negated once, the order in the representation is switched, and the negative interpretation $\bar{\neg}[\![S]\!]_{TTDL}$ will be projected; if S is negated twice, the order of the pair is switched once again because of the **swap** function, then $[\![S]\!]_{TTDL}$ is to be retrieved, as if no negation occurs. One can simply infer the case for multiple negations. In this way, discourse referents within the scope of DN-TTDL negation are "hidden", or "deactivated" temporarily, rather than being blocked permanently, as it is the case in DRT, DPL, and TTDL. Hence, same as in standard propositional logic and predicate logic, the law of double negation holds unconditionally in DN-TTDL, as indicated in the following lemma.

Lemma 6.2.2 (Double Negation Lemma)**.** Let M be a term of type Ω^{dn}, then the following equivalence holds:

$$\neg^d_{DN\text{-}TTDL}(\neg^d_{DN\text{-}TTDL}M) = M$$

Namely, two consecutive occurrences of the negation $\neg^d_{DN\text{-}TTDL}$ can be eliminated.

Proof. The definition of the negation $\neg^d_{DN\text{-}TTDL}$ can be retrieved in formula 6.23. The left hand side of the equivalence in the above lemma: $\neg^d_{DN\text{-}TTDL}(\neg^d_{DN\text{-}TTDL}\ M)$, is computed as follows:

$$\begin{aligned} \neg^d_{DN\text{-}TTDL}(\neg^d_{DN\text{-}TTDL}\ M) &= (\lambda A.\mathbf{swap}\ A)((\lambda A.\mathbf{swap}\ A)M) \\ &\to_\beta = (\lambda A.\mathbf{swap}\ A)(\mathbf{swap}\ M) \\ &\to_\beta \mathbf{swap}(\mathbf{swap}\ M) \end{aligned} \tag{6.24}$$

According to lemma 6.2.1, two consecutive applications of the swap function is the identity function, so formula 6.24 is reduced further as follows:

$$\begin{aligned} \neg^d_{DN\text{-}TTDL}(\neg^d_{DN\text{-}TTDL}\ M) &\twoheadrightarrow_\beta \mathbf{swap}(\mathbf{swap}\ M) \\ &= M \end{aligned}$$

As a result, a double negation consisting of $\neg^d_{DN\text{-}TTDL}$ can be eliminated. □

In section 5.3.3, we have exemplified that the above rule does not hold in TTDL. This

[2] Of course one can set the first element as negative, the second element as positive, this is just a personal flavor. The important thing is that one should pick up the representation at the positive position.

can be formally shown as follows: assume M is a dynamic proposition of type Ω, then by applying $\neg^d_{TTDL}$ (formula 4.37) twice to M, we obtain the following result:

$$\begin{aligned}
\neg^d_{TTDL}(\neg^d_{TTDL}\ M) &= (\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)((\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)M) \\
&\to_\beta (\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)(\lambda e\phi.\neg(M\ e\ \mathbf{stop}) \wedge \phi e) \\
&\to_\beta \lambda e\phi.\neg((\lambda e'\phi'.\neg(M\ e'\ \mathbf{stop}) \wedge \phi' e')\ e\ \mathbf{stop}) \wedge \phi e \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\neg(M\ e\ \mathbf{stop}) \wedge \top) \wedge \phi e \\
&= \lambda e\phi.(M\ e\ \mathbf{stop}) \wedge \phi e
\end{aligned} \tag{6.25}$$

As shown in formula 6.25, although the two terms M and $\neg^d_{TTDL}(\neg^d_{TTDL}\ M)$ are truth-conditionally/statically equivalent (they share the same semantics when interpreted against the empty continuation), their dynamic semantics may differ. Assume the left context of M is not empty, then M is able to take subsequent anaphoric expressions while $\neg^d_{TTDL}(\neg^d_{TTDL}\ M)$ can not. So the law of double negation does not apply to $\neg^d_{TTDL}$ in all conditions.

In order to update a sentence to a preceding discourse, we can not use the previous composition rule (formula 4.32) or update function (formula 4.33) as proposed in TTDL, because sentences and discourses are treated as different semantic objects in DN-TTDL. Further more, there is one additional operation involved in DN-TTDL: projecting the positive representation out of a pair. Because of that, we propose the following update function for DN-TTDL, which is defined in terms of the update function in TTDL:

$$\begin{aligned}
\mathbf{update}_{DN\text{-}TTDL} &\triangleq \lambda DS.(\mathbf{update}_{TTDL}\ D\ (\pi_1 S)) \\
&\twoheadrightarrow_\beta \lambda DSe\phi.De(\lambda e'.(\pi_1 S)e'\phi)
\end{aligned} \tag{6.26}$$

The function $\mathbf{update}_{DN\text{-}TTDL}$ is of type $\Omega \to \Omega^{dn} \to \Omega$, it takes a discourse and a sentence as input, which are of type Ω and Ω^{dn}, respectively, and yields an updated discourse. Basically, $\mathbf{update}_{DN\text{-}TTDL}$ does the same job as $\mathbf{update}_{TTDL}$, except for that the former picks up the positive representation from the sentence. Since we set the first position as positive, we employ the projection operator π_1 in order to select the first element.

The dynamic conjunction in DN-TTDL, which is used to compose two sentences, is defined in terms of the dynamic conjunction and the dynamic negation in TTDL (see formula 4.34 and 4.37) as follows:

$$\begin{aligned}
\wedge^d_{DN\text{-}TTDL} &\triangleq \lambda AB.\langle(\pi_1 A) \wedge^d_{TTDL} (\pi_1 B), \\
&\qquad \neg^d_{TTDL}((\pi_1 A) \wedge^d_{TTDL} (\pi_1 B))\rangle \\
&\twoheadrightarrow_\beta \lambda AB.\langle\lambda e\phi.(\pi_1 A)e(\lambda e'.(\pi_1 B)e'\phi), \\
&\qquad \lambda e\phi.\neg((\pi_1 A)e(\lambda e'.(\pi_1 B)\ e'\ \mathbf{stop})) \wedge \phi e\rangle
\end{aligned} \tag{6.27}$$

As shown above, $\wedge^d_{DN\text{-}TTDL}$ takes two sentences A and B, which are both of type Ω^{dn}, and returns a composed sentence. In the resulting proposition pair, the first element is the TTDL conjunction of the first projections in A and B. As we can see, the second element in the pair is the TTDL negation of the first element, which blocks the accessibility of referents within its scope: after β-reduction, an empty continuation is passed and the current left context is not updated.

Similarly, we propose the dynamic existential quantifier in DN-TTDL in terms of the dynamic negation and the dynamic existential quantifier in TTDL (see formula 4.37 and 4.38) as follows:

$$\begin{aligned}\exists^d_{DN\text{-}TTDL} &\triangleq \lambda P.\langle \exists^d_{TTDL}(\lambda x.\pi_1(Px)),\\ &\qquad\quad \neg^d_{TTDL}(\exists^d_{TTDL}(\lambda x.\pi_1(Px)))\rangle\\ &\twoheadrightarrow_\beta \lambda P.\langle \lambda e\phi.\exists(\lambda x.(\pi_1(Px))(x :: e)\phi),\\ &\qquad\quad \lambda e\phi.\neg(\exists(\lambda x.(\pi_1(Px))(x :: e)\ \mathbf{stop})) \wedge \phi e\rangle\end{aligned} \tag{6.28}$$

The existential quantifier takes a dynamic property $\mathbf{P}$, which is of type $\iota \to \Omega^{dn}$, and returns a dynamic proposition of type Ω^{dn}. Its first element is obtained by applying the TTDL existential quantifier to the first projection of (Px); its second element is the TTDL negation of the first one. After β-reduction, the bound variable x is updated in the left context of the positive proposition, while its accessibility is blocked in the negative one: the empty right context **stop** is passed to it.

Like TTDL, there is also a systematic way in DN-TTDL to translate standard lexical entries to their dynamic counterparts. We present it below. To distinguish the dynamic translations in TTDL and DN-TTDL, we introduce a specific notation, different form the one in notation 4.4.1:

Notation 6.2.1. We use the double-bar notation, for instance, $\overline{\overline{\tau}}$ or $\overline{\overline{M}}$, to denote the **dynamic translation** of a type τ or a λ-term M in DN-TTDL.

The dynamic translation of types in DN-TTDL is slightly different from the one in TTDL, because we have assigned a different interpretation to sentences: two dynamic propositions are encapsulated in a single representation. Apart from that, the other types are dynamically translated in a similar way as in definition 4.4.2.

Definition 6.2.1. The **dynamic translation of a type** $\tau \in T$: $\overline{\overline{\tau}}$, is defined inductively as follows:

1. $\overline{\overline{\iota}} = \iota$;
2. $\overline{\overline{o}} = \Omega^{dn}$;
3. $\overline{\overline{\sigma \to \tau}} = \overline{\overline{\sigma}} \to \overline{\overline{\tau}}$, where $\tau, \sigma \in T$.

As indicated in definition 6.2.1, the dynamic proposition in DN-TTDL is of type Ω^{dn}, rather than Ω. Same as in TTDL, we introduce two functions: $\mathbb{D}^{dn}$ and $\mathbb{S}^{dn}$, before presenting the detailed dynamic translation of λ-terms[3]. We will define them in terms of the TTDL negation and the two previous functions: $\mathbb{D}$ and $\mathbb{S}$, see formula 4.37 and definition 4.4.3.

Definition 6.2.2. The **dynamization function** $\mathbb{D}^{dn}_\tau$, which takes an input λ-term A of type $(\gamma \to \tau)$, returns an output λ-term A' of type $\overline{\overline{\tau}}$; the **staticization function** $\mathbb{S}^{dn}_\tau$, which takes an input λ-term A' of type $\overline{\overline{\tau}}$, returns an output λ-term A of type $(\gamma \to \tau)$.

[3] The superscripts of $\mathbb{D}^{dn}$ and $\mathbb{S}^{dn}$ are used to distinguish them from the two previous functions $\mathbb{D}$ and $\mathbb{S}$ in TTDL.

- $\mathbb{D}^{dn}_{\tau}$ is defined inductively on type τ as follows:

  1. $\mathbb{D}^{dn}_{\iota} A = \mathbb{D}_{\iota} A$;
  2. $\mathbb{D}^{dn}_{o} A = \langle \mathbb{D}_o A, \neg^d_{TTDL}(\mathbb{D}_o A)\rangle$;
  3. $\mathbb{D}^{dn}_{\alpha\to\beta} A = \lambda x.\mathbb{D}^{dn}_{\beta}(\lambda e.Ae(\mathbb{S}^{dn}_{\alpha} xe))$.

- $\mathbb{S}^{dn}_{\tau}$ is defined inductively on type τ as follows:

  1. $\mathbb{S}^{dn}_{\iota} A' = \mathbb{S}_{\iota} A'$;
  2. $\mathbb{S}^{dn}_{o} A' = \lambda e.\mathbb{S}_o(\pi_1 A')e$;
  3. $\mathbb{S}^{dn}_{\alpha\to\beta} A' = \lambda e.(\lambda x.\mathbb{S}^{dn}_{\beta}(A'(\mathbb{D}^{dn}_{\alpha}(\lambda e'.x)))e)$.

The above definition is almost the same as definition 4.4.3, except for $\mathbb{D}^{dn}_o$ and $\mathbb{S}^{dn}_o$. In particular, $\mathbb{D}^{dn}_o$ yields a pair, the first element of which is the standard dynamization, the second element is its dynamic negation; $\mathbb{S}^{dn}_o$ makes use of the projection operator π_1, and returns the standard staticization. For more remarks on $\mathbb{D}^{dn}$ and $\mathbb{S}^{dn}$, especially their general cases, please refer back to section 4.4.2. The dynamic translation of λ-terms in DN-TTDL is also similar to that in TTDL, compare the following definition with definition 4.4.4:

Definition 6.2.3. The **double negation dynamic translation of a λ-term** M (of type τ): $\overline{\overline{M}}$, which is another λ-term of type $\overline{\overline{\tau}}$, is defined as follows:

1. $\overline{\overline{x}} = x$, if $x \in \mathcal{X}$;
2. $\overline{\overline{\mathbf{a}}} = \mathbb{D}^{dn}_{\tau}(\lambda e.\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}_{NL}$ and $\mathbf{a} : \tau$;
3. $\overline{\overline{\wedge}} = \wedge^d_{DN\text{-}TTDL}$, see formula 6.27;
4. $\overline{\overline{\neg}} = \neg^d_{DN\text{-}TTDL}$, see formula 6.23;
5. $\overline{\overline{\exists}} = \exists^d_{DN\text{-}TTDL}$, see formula 6.28;
6. $\overline{\overline{(MN)}} = (\overline{\overline{M}}\,\overline{\overline{N}})$;
7. $\overline{\overline{(\lambda x.M)}} = (\lambda x.\overline{\overline{M}})$.

Same as in TTDL, since the derived operators $\vee$ (disjunction), $\to$ (implication), and $\forall$ (universal quantifier) are defined in terms of primitive logical constants (see formula 3.1, 3.2, and 3.5), their dynamic translations can be deduced by applying the corresponding rules in definition 6.2.3. Take implication for instance:

$$\begin{aligned}
\overline{\overline{A \to B}} &= \overline{\overline{\neg(A \wedge \neg B)}} \\
&= \overline{\overline{\neg}}(\overline{\overline{A}}\overline{\overline{\wedge}}(\overline{\overline{\neg}}\overline{\overline{B}})) \\
&= \neg^d_{DN\text{-}TTDL}(\overline{\overline{A}} \wedge^d_{DN\text{-}TTDL} (\neg^d_{DN\text{-}TTDL}\overline{\overline{B}})) \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.\neg((\pi_1\overline{\overline{A}})e(\lambda e'.(\pi_2\overline{\overline{B}})\ e'\ \mathbf{stop})) \wedge \phi e, \\
&\qquad \lambda e\phi.(\pi_1\overline{\overline{A}})e(\lambda e'.(\pi_2\overline{\overline{B}})e\phi)\rangle
\end{aligned} \tag{6.29}$$

We will ignore the explicit computations for disjunction and universal quantifier, which can be trivially conducted with β-reductions. Finally, as for the semantics of DN-TTDL, it follows from TTDL, which is also the same as in FOL: all logical constants (i.e., $\wedge$, $\neg$ and $\exists$) receive their standard interpretations. In the next subsection, we will provide some logical properties of DN-TTDL, which gives us a deeper understanding of the framework.

6.2.2 From TTDL to DN-TTDL

This subsection is devoted to some additional formal details of DN-TTDL. We will first expose some logical facts, then a comparison between DN-TTDL and TTDL will be made.

First of all, as shown in lemma 6.2.2, the law of double negation does hold in DN-TTDL. In other words, a double negation made up of $\overline{\overline{\neg}}$ can be cancelled. Based on that, we can obtain a bunch of equivalences. Assume ϕ and ψ are DN-TTDL terms of type Ω^{dn}, then:

$$\phi \overline{\overline{\wedge}} \psi = \overline{\overline{\neg}}(\phi \overline{\overline{\Rightarrow}}(\overline{\overline{\neg}}\psi)) = \overline{\overline{\neg}}((\overline{\overline{\neg}}\phi)\overline{\overline{\vee}}(\overline{\overline{\neg}}\psi)) \tag{6.30}$$

$$\overline{\overline{\exists}}(\lambda x.\overline{\overline{\neg}}\phi) = \overline{\overline{\neg}}\overline{\overline{\forall}}(\lambda x.\phi) \tag{6.31}$$

$$(\overline{\overline{\neg}}(\overline{\overline{\neg}}\phi))\overline{\overline{\wedge}}\psi = \phi \overline{\overline{\wedge}} \psi \tag{6.32}$$

$$(\overline{\overline{\neg}}\phi)\overline{\overline{\vee}}\psi = \overline{\overline{\neg}}(\phi \overline{\overline{\wedge}}(\overline{\overline{\neg}}\psi)) \tag{6.33}$$

The above relations do not hold in TTDL, because double negations consisting of $\overline{\neg}$ can not be removed, as shown in formula 6.25. If we look at the formulas in detail, 6.32 can be employed to account for the felicitous anaphoric links in examples such as (98) and (10); 6.33 gives an explanation for the acceptability of anaphoras in bathroom examples, such as (100) and (11). We will present the detailed illustrations with corresponding linguistic examples in the next subsection.

Since DN-TTDL is an extension of TTDL, one may wonder whether the former system can deal with the set of examples, which are successfully handled by the latter. The answer is yes. For the rest of this subsection, we will focus on providing a formal account on it. In order to characterize the relation between TTDL and the extension DN-TTDL, we propose the following three notions, which deserve a simultaneous recursive definition.

Definition 6.2.4. Given signature Σ_0 (definition 3.2.13). The set of formulas $\mathbb{F}_{\Sigma_0}$, the set of positive formulas $\mathbb{F}^+_{\Sigma_0}$, and the set of negative formulas $\mathbb{F}^-_{\Sigma_0}$, are defined mutually on one another by induction:

1. $M \in \mathbb{F}_{\Sigma_0}$, whenever $M \in \mathbb{F}^+_{\Sigma_0}$ or $M \in \mathbb{F}^-_{\Sigma_0}$;
2. $\mathbf{P}t_1...t_n \in \mathbb{F}^+_{\Sigma_0}$, whenever $\mathbf{P} \in \mathcal{C}_{NL}$, $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and $\mathbf{P} : \underbrace{\iota \to ... \to \iota}_{n} \to o$, $t_1, ..., t_n : \iota$;
3. $M_1 \wedge M_2 \in \mathbb{F}^+_{\Sigma_0}$, whenever $M_1, M_2 \in \mathbb{F}_{\Sigma_0}$;
4. $\exists(\lambda x.M_1) \in \mathbb{F}^+_{\Sigma_0}$, whenever $x \in \mathcal{X}$, $M_1 \in \mathbb{F}_{\Sigma_0}$;
5. $\neg M_1 \in \mathbb{F}^-_{\Sigma_0}$, whenever $M_1 \in \mathbb{F}^+_{\Sigma_0}$.

Please note that the set of formulas $\mathbb{F}_{\Sigma_0}$ in definition 6.2.4 is a subset of all possible formulas that can be constructed from Σ_0: those containing multi-negation (two or more negations stacking over one another) are not included in $\mathbb{F}_{\Sigma_0}$. We restrict ourselves to $\mathbb{F}_{\Sigma_0}$ for the moment, and we will check whether TTDL and DN-TTDL make the same prediction to discourses where no multi-negation occurs. Before that, we provide the following property for first-order predicate terms, which will be helpful in future proofs.

Lemma 6.2.3. Given signature Σ_0 (definition 3.2.13), let M_n be a λ-term of type $\underbrace{\iota \to \ldots \to \iota}_{n} \to o$, then:

$$\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{n} \to o}(\lambda e.M_n) = \lambda x_1 \ldots x_n.\mathbb{D}_o(\lambda e.M_n x_1 \ldots x_n) \tag{6.34}$$

$$\mathbb{D}^{dn}_{\underbrace{\iota \to \ldots \to \iota}_{n} \to o}(\lambda e.M_n) = \lambda x_1 \ldots x_n.\mathbb{D}^{dn}_o(\lambda e.M_n x_1 \ldots x_n) \tag{6.35}$$

Proof. We prove the lemma by induction on the semantic type of M_n, namely the value of n.

- Let $n = 0$, then $M_0 : o$. It is obvious that $\mathbb{D}_o(\lambda e.M_0)$ precisely corresponds to the form $\lambda x_1 \ldots x_n.\mathbb{D}_o(\lambda e.M_n x_1 \ldots x_n)$.

- Let $n = 1$, then $M_1 : \iota \to o$. According to definition 4.4.3:

$$\begin{aligned}
\mathbb{D}_{\iota \to o}(\lambda e.M_1) &= \lambda x_1.\mathbb{D}_o(\lambda e.(\lambda e'.M_1)e(\mathbb{S}_\iota x_1 e)) \\
&= \lambda x_1.\mathbb{D}_o(\lambda e.(\lambda e'.M_1)e x_1) \\
&\to_\beta \lambda x_1.\mathbb{D}_o(\lambda e.M_1 x_1)
\end{aligned}$$

- Let $n = i$, x_1 a variable, then $M_i : \underbrace{\iota \to \ldots \to \iota}_{i} \to o$, $M_i x_1 : \underbrace{\iota \to \ldots \to \iota}_{i-1} \to o$. By induction hypothesis:

$$\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{i-1} \to o}(\lambda e.M_i x_1) = \lambda x_2 \ldots x_i.\mathbb{D}_o(\lambda e.M_i x_1 x_2 \ldots x_i) \tag{6.36}$$

  Now let's turn to the dynamic translation of M_i, according to definition 4.4.3 and formula 6.36:

$$\begin{aligned}
\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{i} \to o}(\lambda e.M_i) &= \lambda x_1.\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{i-1} \to o}(\lambda e.(\lambda e'.M_i)e(\mathbb{S}_\iota x_1 e)) \\
&= \lambda x_1.\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{i-1} \to o}(\lambda e.(\lambda e'.M_i)e x_1) \\
&\to_\beta \lambda x_1.\mathbb{D}_{\underbrace{\iota \to \ldots \to \iota}_{i-1} \to o}(\lambda e.M_i x_1) \\
&= \lambda x_1 x_2 \ldots x_i e\phi.(M_i x_1 x_2 \ldots x_i \wedge \phi e)
\end{aligned} \tag{6.37}$$

Since $\mathbb{D}_\tau^{dn}$ is defined in the similar way as $\mathbb{D}_\tau$, in particular, $\mathbb{D}_{\alpha\to\beta}$ and $\mathbb{D}^{dn}_{\alpha\to\beta}$ are identical (see definition 4.4.3 and 6.2.2), the equivalence 6.35 can be obtained with exactly the same process. We will thus leave out the detailed proof for it. □

Then let's take a look at the following lemma, which describes the relation between TTDL and DN-TTDL on representations of $\mathbb{F}_{\Sigma_0}$.

Lemma 6.2.4. Given signature Σ_0 (definition 3.2.13). Let $M \in \mathbb{F}_{\Sigma_0}$ be a formula. Its dynamic translation under TTDL $\overline{M}$ and its dynamic translation under DN-TTDL $\overline{\overline{M}}$ bear the following relation:

1. If $M \in \mathbb{F}^+_{\Sigma_0}$, then:

$$\overline{\overline{M}} = \langle \overline{M}, \overline{\neg}\overline{M} \rangle \tag{6.38}$$

2. Else if $M = \neg M' \in \mathbb{F}^-_{\Sigma_0}$, where $M' \in \mathbb{F}^+_{\Sigma_0}$, then:

$$\overline{\overline{M}} = \langle \overline{\neg}\overline{M'}, \overline{M'} \rangle \tag{6.39}$$

Proof. We prove the lemma by induction on the form of M.

1. Let $M = \mathbf{P}t_1...t_n \in \mathbb{F}^+_{\Sigma_0}$, where $\mathbf{P} \in \mathcal{C}_{NL}$, $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$

   (a) First we examine $\overline{M}$ and $\overline{\neg}\overline{M}$. Since $\mathbf{P}$ is of type $\underbrace{\iota \to ... \to \iota}_{n} \to o$, by lemma 6.2.3:

$$\mathbb{D}_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}_o(\lambda e.\mathbf{P}x_1...x_n)$$

   Since $\mathbf{P} \in \mathcal{C}_{NL}$, according to definition 4.4.4:

$$\overline{\mathbf{P}} = \mathbb{D}_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}_o(\lambda e.\mathbf{P}x_1...x_n) \tag{6.40}$$

   Since $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and all of them are of type ι, according to definition 4.4.4 and 4.4.3:

$$\overline{t_i} = t_i, \text{ for all } i \in \{1, ..., n\} \tag{6.41}$$

   Finally, according to definition 4.4.4, formulas 6.40 and 6.41:

$$\begin{aligned} \overline{M} &= \overline{\mathbf{P}t_1...t_n} \\ &= \overline{\mathbf{P}}\overline{t_1}...\overline{t_n} \\ &= (\lambda x_1...x_n.\mathbb{D}_o(\lambda e.\mathbf{P}x_1...x_n))t_1...t_n \\ &\twoheadrightarrow_\beta \mathbb{D}_o(\lambda e.\mathbf{P}t_1...t_n) \end{aligned} \tag{6.42}$$

   As for $\overline{\neg}\overline{M}$, according to formula 6.42, it is straightforward that:

$$\overline{\neg}\overline{M} = \overline{\neg}\mathbb{D}_o(\lambda e.\mathbf{P}t_1...t_n) \tag{6.43}$$

(b) Then let's have a look at $\overline{\overline{M}}$, since $\mathbf{P}$ is of type $\underbrace{\iota \to ... \to \iota}_{n} \to o$, by lemma 6.2.3:

$$\mathbb{D}^{dn}_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}^{dn}_o(\lambda e.\mathbf{P}x_1...x_n)$$

Since $\mathbf{P} \in \mathcal{C}_{NL}$, according to definition 6.2.3:

$$\overline{\overline{\mathbf{P}}} = \mathbb{D}^{dn}_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}^{dn}_o(\lambda e.\mathbf{P}x_1...x_n) \tag{6.44}$$

Since $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and all of them are of type ι, according to definitions 6.2.3 and 6.2.2:

$$\overline{\overline{t_i}} = t_i, \text{ for all } i \in \{1, ..., n\} \tag{6.45}$$

Finally, according to definitions 6.2.3, 6.2.2, formulas 6.44 and 6.45:

$$\begin{aligned}
\overline{\overline{M}} &= \overline{\overline{\mathbf{P}t_1...t_n}} \\
&= \overline{\overline{\mathbf{P}}}\;\overline{\overline{t_1}}...\overline{\overline{t_n}} \\
&= \lambda x_1...x_n.\mathbb{D}^{dn}_o(\lambda e.\mathbf{P}x_1...x_n)t_1...t_n \\
&\twoheadrightarrow_\beta \mathbb{D}^{dn}_o(\lambda e.\mathbf{P}t_1...t_n) \\
&= \langle \mathbb{D}_o(\lambda e.\mathbf{P}t_1...t_n), \bar{\neg}\mathbb{D}_o(\lambda e.\mathbf{P}t_1...t_n)\rangle
\end{aligned} \tag{6.46}$$

As we can see, compare formulas 6.46, 6.42 and 6.43, we can draw:

$$\overline{\overline{M}} = \langle \overline{M}, \bar{\neg}\overline{M}\rangle$$

So when $M = \mathbf{P}t_1...t_n$, formula 6.38 is satisfied.

2. Let $M = M_1 \wedge M_2 \in \mathbb{F}^+_{\Sigma_0}$, where $M_1, M_2 \in \mathbb{F}_{\Sigma_0}$

(a) First we examine $\overline{M}$ and $\bar{\neg}\overline{M}$, according to definition 4.4.4:

$$\overline{M} = \overline{M_1 \wedge M_2} = \overline{M_1} \mathbin{\bar{\wedge}} \overline{M_2} \tag{6.47}$$

$$\bar{\neg}\overline{M} = \bar{\neg}\overline{M_1 \wedge M_2} = \bar{\neg}(\overline{M_1} \mathbin{\bar{\wedge}} \overline{M_2}) \tag{6.48}$$

(b) Then let's turn to $\overline{\overline{M}}$, according to definition 6.2.3 and formula 6.27:

$$
\begin{aligned}
\overline{\overline{M}} &= \overline{\overline{M_1 \wedge M_2}} \\
&= \overline{\overline{M_1}}\ \overline{\overline{\wedge}}\ \overline{\overline{M_2}} \\
&= \overline{\overline{M_1}} \wedge^{d}_{DN\text{-}TTDL} \overline{\overline{M_2}} \\
&= \lambda AB.\langle(\pi_1 A)\barwedge(\pi_1 B), \overline{\neg}((\pi_1 A)\barwedge(\pi_1 B))\rangle\ \overline{\overline{M_1}}\ \overline{\overline{M_2}} \\
&\twoheadrightarrow_{\beta} \langle(\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}), \overline{\neg}((\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}))\rangle
\end{aligned}
\tag{6.49}
$$

i. When $M_1, M_2 \in \mathbb{F}^{+}_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_i}} = \langle\overline{M_i}, \overline{\neg}\overline{M_i}\rangle$, where $i \in \{1, 2\}$. We continue formula 6.49 as follows:

$$
\begin{aligned}
\overline{\overline{M}} &= \langle(\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}), \overline{\neg}((\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}))\rangle \\
&\twoheadrightarrow_{\beta} \langle\overline{M_1} \barwedge \overline{M_2}, \overline{\neg}(\overline{M_1} \barwedge \overline{M_2})\rangle
\end{aligned}
\tag{6.50}
$$

As we can see, compare formulas 6.50, 6.47 and 6.48, we can draw that

$$\overline{\overline{M}} = \langle\overline{M}, \overline{\neg}\overline{M}\rangle$$

ii. When $M_1 \in \mathbb{F}^{+}_{\Sigma_0}$ and $M_2 = \neg M_2' \in \mathbb{F}^{-}_{\Sigma_0}$, where $M_2' \in \mathbb{F}^{+}_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_1}} = \langle\overline{M_1}, \overline{\neg}\overline{M_1}\rangle$, $\overline{\overline{M_2}} = \langle\overline{\neg}\overline{M_2'}, \overline{M_2'}\rangle$.
According to definition 4.4.4: $\overline{M_2} = \overline{\neg}\overline{M_2'}$, then formulas 6.47 and 6.48 can be further expanded as follows:

$$\overline{M} = \overline{M_1 \wedge M_2} = \overline{M_1} \barwedge \overline{M_2} = \overline{M_1} \barwedge \overline{\neg}\overline{M_2'} \tag{6.51}$$

$$\overline{\neg}\overline{M} = \overline{\neg}\overline{M_1 \wedge M_2} = \overline{\neg}(\overline{M_1} \barwedge \overline{M_2}) = \overline{\neg}(\overline{M_1} \barwedge \overline{\neg}\overline{M_2'}) \tag{6.52}$$

We continue formula 6.49 as follows:

$$
\begin{aligned}
\overline{\overline{M}} &= \langle(\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}), \overline{\neg}((\pi_1\overline{\overline{M_1}})\barwedge(\pi_1\overline{\overline{M_2}}))\rangle \\
&\twoheadrightarrow_{\beta} \langle\overline{M_1} \barwedge \overline{\neg}\overline{M_2'}, \overline{\neg}(\overline{M_1} \barwedge \overline{\neg}\overline{M_2'})\rangle
\end{aligned}
\tag{6.53}
$$

As we can see, compare formulas 6.53, 6.51 and 6.52, we can draw that

$$\overline{\overline{M}} = \langle\overline{M}, \overline{\neg}\overline{M}\rangle$$

iii. When $M_1 = \neg M_1' \in \mathbb{F}^{-}_{\Sigma_0}$ and $M_2 \in \mathbb{F}^{+}_{\Sigma_0}$, where $M_1' \in \mathbb{F}^{+}_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_1}} = \langle\overline{\neg}\overline{M_1'}, \overline{M_1'}\rangle$, $\overline{\overline{M_2}} = \langle\overline{M_2}, \overline{\neg}\overline{M_2}\rangle$.
According to definition 4.4.4: $\overline{M_1} = \overline{\neg}\overline{M_1'}$, then formulas 6.47 and 6.48 can

be further expanded as follows:

$$\overline{M} = \overline{M_1 \wedge M_2} = \overline{M_1} \barwedge \overline{M_2} = \neg\overline{M_1'} \barwedge \overline{M_2} \tag{6.54}$$

$$\neg\overline{M} = \neg\overline{M_1 \wedge M_2} = \neg(\overline{M_1} \barwedge \overline{M_2}) = \neg(\neg\overline{M_1'} \barwedge \overline{M_2}) \tag{6.55}$$

We continue formula 6.49 as follows:

$$\begin{aligned} \overline{\overline{M}} &= \langle (\pi_1 \overline{\overline{M_1}}) \barwedge (\pi_1 \overline{\overline{M_2}}), \neg((\pi_1 \overline{\overline{M_1}}) \barwedge (\pi_1 \overline{\overline{M_2}})) \rangle \\ &\twoheadrightarrow_\beta \langle \neg\overline{M_1'} \barwedge \overline{M_2}, \neg(\neg\overline{M_1'} \barwedge \overline{M_2}) \rangle \end{aligned} \tag{6.56}$$

As we can see, compare formulas 6.56, 6.54 and 6.55, we can draw that

$$\overline{\overline{M}} = \langle \overline{M}, \neg\overline{M} \rangle$$

iv. When $M_1 = \neg M_1' \in \mathbb{F}^-_{\Sigma_0}$ and $M_2 = \neg M_2' \in \mathbb{F}^-_{\Sigma_0}$, where $M_1', M_2' \in \mathbb{F}^+_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_i}} = \langle \neg\overline{M_i'}, \overline{M_i'} \rangle$, where $i \in \{1, 2\}$.
According to definition 4.4.4: $\overline{M_i} = \neg\overline{M_i'}$, where $i \in \{1, 2\}$, then formulas 6.47 and 6.48 can be further expanded as follows:

$$\overline{M} = \overline{M_1 \wedge M_2} = \overline{M_1} \barwedge \overline{M_2} = \neg\overline{M_1'} \barwedge \neg\overline{M_2'} \tag{6.57}$$

$$\neg\overline{M} = \neg\overline{M_1 \wedge M_2} = \neg(\overline{M_1} \barwedge \overline{M_2}) = \neg(\neg\overline{M_1'} \barwedge \neg\overline{M_2'}) \tag{6.58}$$

We continue formula 6.49 as follows:

$$\begin{aligned} \overline{\overline{M}} &= \langle (\pi_1 \overline{\overline{M_1}}) \barwedge (\pi_1 \overline{\overline{M_2}}), \neg((\pi_1 \overline{\overline{M_1}}) \barwedge (\pi_1 \overline{\overline{M_2}})) \rangle \\ &\twoheadrightarrow_\beta \langle \neg\overline{M_1'} \barwedge \neg\overline{M_2'}, \neg(\neg\overline{M_1'} \barwedge \neg\overline{M_2'}) \rangle \end{aligned} \tag{6.59}$$

As we can see, compare formulas 6.59, 6.57 and 6.58, we can draw that

$$\overline{\overline{M}} = \langle \overline{M}, \neg\overline{M} \rangle$$

So when $M = M_1 \wedge M_2$, formula 6.38 is satisfied.

3. Let $M = \exists(\lambda x.M_1) \in \mathbb{F}^+_{\Sigma_0}$, where $x \in \mathcal{X}$, $M_1 \in \mathbb{F}_{\Sigma_0}$

   (a) First we examine $\overline{M}$ and $\neg\overline{M}$, according to definition 4.4.4:

$$\overline{M} = \overline{\exists(\lambda x.M_1)} = \overline{\exists}(\lambda x.\overline{M_1}) = \overline{\exists}(\lambda x.\overline{M_1}) \tag{6.60}$$

$$\neg\overline{M} = \neg\overline{\exists(\lambda x.M_1)} = \neg\overline{\exists}(\lambda x.\overline{M_1}) = \neg\overline{\exists}(\lambda x.\overline{M_1}) \tag{6.61}$$

(b) Then let's turn to $\overline{\overline{M}}$, according to definition 6.2.3 and formula 6.28:

$$\begin{aligned}\overline{\overline{M}} &= \overline{\overline{\exists(\lambda x.M_1)}} = \overline{\overline{\exists}}(\lambda x.\overline{\overline{M_1}}) = \exists^d_{DN\text{-}TTDL}(\lambda x.\overline{\overline{M_1}})\\ &= \lambda P.\langle\overline{\exists}(\lambda x.\pi_1(Px)), \neg\overline{\exists}(\lambda x.\pi_1(Px))\rangle(\lambda x.\overline{\overline{M_1}})\\ &\twoheadrightarrow_\beta \langle\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}}), \neg\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}})\rangle\end{aligned} \tag{6.62}$$

i. When $M_1 \in \mathbb{F}^+_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_1}} = \langle\overline{M_1}, \neg\overline{M_1}\rangle$. We continue formula 6.62 as follows:

$$\begin{aligned}\overline{\overline{M}} &= \langle\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}}), \neg\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}})\rangle\\ &\twoheadrightarrow_\beta \langle\overline{\exists}(\lambda x.\overline{M_1}), \neg\overline{\exists}(\lambda x.\overline{M_1})\rangle\end{aligned} \tag{6.63}$$

As we can see, compare formulas 6.63, 6.60 and 6.61, we can draw that

$$\overline{\overline{M}} = \langle\overline{M}, \neg\overline{M}\rangle$$

ii. When $M_1 = \neg M_1' \in \mathbb{F}^-_{\Sigma_0}$, where $M_1' \in \mathbb{F}^+_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_1}} = \langle\neg\overline{M_1'}, \overline{M_1'}\rangle$.
According to definition 4.4.4: $\overline{M_1} = \neg\overline{M_1'}$, then formulas 6.60 and 6.61 can be further expanded as follows:

$$\overline{M} = \overline{\exists(\lambda x.M_1)} = \overline{\exists}(\lambda x.\overline{M_1}) = \overline{\exists}(\lambda x.\overline{M_1}) = \overline{\exists}(\lambda x.\neg\overline{M_1'}) \tag{6.64}$$

$$\neg\overline{M} = \neg\overline{\exists(\lambda x.M_1)} = \neg\overline{\exists}(\lambda x.\overline{M_1}) = \neg\overline{\exists}(\lambda x.\overline{M_1}) = \neg\overline{\exists}(\lambda x.\neg\overline{M_1'}) \tag{6.65}$$

We continue formula 6.62 as follows:

$$\begin{aligned}\overline{\overline{M}} &= \langle\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}}), \neg\overline{\exists}(\lambda x.\pi_1\overline{\overline{M_1}})\rangle\\ &\twoheadrightarrow_\beta \langle\overline{\exists}(\lambda x.\neg\overline{M_1'}), \neg\overline{\exists}(\lambda x.\neg\overline{M_1'})\rangle\end{aligned} \tag{6.66}$$

As we can see, compare formulas 6.66, 6.64 and 6.65, we can draw that

$$\overline{\overline{M}} = \langle\overline{M}, \neg\overline{M}\rangle$$

So when $M = \exists(\lambda x.M_1)$, formula 6.38 is satisfied.

4. Let $M = \neg M_1 \in \mathbb{F}^-_{\Sigma_0}$, where $M_1 \in \mathbb{F}^+_{\Sigma_0}$. By induction hypothesis, $\overline{\overline{M_1}} = \langle\overline{M_1}, \neg\overline{M_1}\rangle$.

According to definition 6.2.3 and formula 6.23:

$$\begin{aligned}
\overline{\overline{M}} &= \overline{\overline{\neg M_1}} \\
&= \overline{\neg}\overline{\overline{M_1}} \\
&= \neg^{d}_{DN\text{-}TTDL}\langle \overline{M_1}, \overline{\neg}\overline{M_1} \rangle \\
&= (\lambda A.\mathbf{swap}\ A)\langle \overline{M_1}, \overline{\neg}\overline{M_1} \rangle \\
&\twoheadrightarrow_\beta \langle \overline{\neg}\overline{M_1}, \overline{M_1} \rangle
\end{aligned} \tag{6.67}$$

As a result, when $M = \neg M_1$, formula 6.39 is satisfied.

As a result, for every formula M in $\mathbb{F}_{\Sigma_0}$, if $M \in \mathbb{F}^{+}_{\Sigma_0}$, relation 6.38 holds; otherwise, if $M \in \mathbb{F}^{-}_{\Sigma_0}$, relation 6.39 holds. □

Finally, the following theorem concludes that the two dynamic systems: TTDL and DN-TTDL, obtain the same result for discourses in which no multi-negation occurs. We shall stick to our previous notations, namely $[\![\ ]\!]$, $[\![\ ]\!]_{TTDL}$, and $[\![\ ]\!]_{DN\text{-}TTDL}$ are functions which return the logical representations of a linguistic expression under simply typed λ-calculus, TTDL, and DN-TTDL, respectively.

Theorem 6.2.1. Let "$S_1.S_2....S_n.$" be a discourse D_n, S_i is the i-th sentence in D_n, D_i is the discourse consisting of the first i sentences, as shown in figure 6.1:

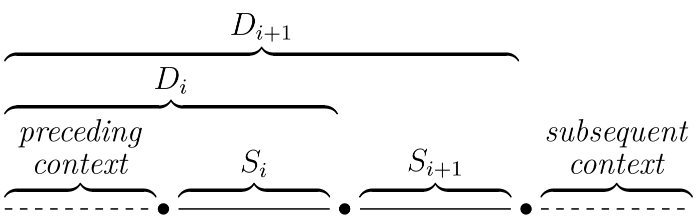


Fig. 6.1 Hierarchy of A General Discourse

Given signature Σ_0 (definition 3.2.13). For all S_i, where $0 \le i \le n$, if $[\![S_i]\!] \in \mathbb{F}_{\Sigma_0}$ is a formula (namely, S_i does not contain any multi-negation), then

$$[\![D_n]\!]_{TTDL} = [\![D_n]\!]_{DN\text{-}TTDL} \tag{6.68}$$

Proof. We prove the lemma by induction on the value of n.

- Let $n = 0$, then D_0 is the initial discourse containing zero sentence. We define the term **start** for the void discourse as follows:

$$\mathbf{start} \triangleq \lambda e\phi.\phi\ \mathsf{nil} \tag{6.69}$$

  Equivalence 6.68 holds because **start** is the logical representation for D_0 in both TTDL and DN-TTDL, namely:

$$[\![D_0]\!]_{TTDL} = [\![D_0]\!]_{DN\text{-}TTDL} = \mathbf{start} = \lambda e\phi.\phi\ \mathsf{nil} \tag{6.70}$$

- Let $n = 1$, then D_1 is the discourse containing the sentence S_1, which does not contain any multi-negation. Since $[\![S_1]\!] \in \mathbb{F}_{\Sigma_0}$, we distinguish the two cases when $[\![S_1]\!]$ is an element of $\mathbb{F}^+_{\Sigma_0}$ and when it is an element of $\mathbb{F}^-_{\Sigma_0}$.

  1. When $[\![S_1]\!] \in \mathbb{F}^+_{\Sigma_0}$, its semantic representation under TTDL is as follows:

  $$[\![S_1]\!]_{TTDL} = \overline{[\![S_1]\!]} \tag{6.71}$$

  In addition, according to lemma 6.2.4, its semantic representation under DN-TTDL is as follows:

  $$\begin{aligned} [\![S_1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![S_1]\!]}} \\ &= \langle [\![S_1]\!]_{TTDL}, \bar{\neg}[\![S_1]\!]_{TTDL} \rangle \\ &= \langle \overline{[\![S_1]\!]}, \bar{\neg}\overline{[\![S_1]\!]} \rangle \end{aligned} \tag{6.72}$$

  We first compute $[\![D_1]\!]_{TTDL}$. It is achieved by updating S_1 to D_0 with the function $\mathbf{update}_{TTDL}$ (formula 4.33):

  $$\begin{aligned} [\![D_1]\!]_{TTDL} &= \mathbf{update}_{TTDL}\ [\![D_0]\!]_{TTDL}\ [\![S_1]\!]_{TTDL} \\ &= \mathbf{update}_{TTDL}\ \mathbf{start}\ \overline{[\![S_1]\!]} \end{aligned} \tag{6.73}$$

  Then let's have a look at $[\![D_1]\!]_{DN\text{-}TTDL}$. Similarly, it is achieved by updating S_1 to D_0, while with the function $\mathbf{update}_{DN\text{-}TTDL}$ in DN-TTDL (formula 6.26). The representation of S_1 under TTDL has already been presented in formula 6.72, hence:

  $$\begin{aligned} [\![D_1]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ [\![D_0]\!]_{DN\text{-}TTDL}\ [\![S_1]\!]_{DN\text{-}TTDL} \\ &= \lambda DS.(\mathbf{update}_{TTDL}\ D\ (\pi_1 S))\ \mathbf{start}\ [\![S_1]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \mathbf{update}_{TTDL}\ \mathbf{start}\ (\pi_1 [\![S_1]\!]_{DN\text{-}TTDL}) \\ &= \mathbf{update}_{TTDL}\ \mathbf{start}\ (\pi_1 \langle \overline{[\![S_1]\!]}, \bar{\neg}\overline{[\![S_1]\!]} \rangle) \\ &\rightarrow_\beta \mathbf{update}_{TTDL}\ \mathbf{start}\ \overline{[\![S_1]\!]} \end{aligned} \tag{6.74}$$

  By comparing formulas 6.73 and 6.74, we can draw that $[\![D_1]\!]_{TTDL} = [\![D_1]\!]_{DN\text{-}TTDL}$.

  2. When $[\![S_1]\!] = \neg[\![S_1']\!] \in \mathbb{F}^-_{\Sigma_0}$, where $[\![S_1']\!] \in \mathbb{F}^+_{\Sigma_0}$, the semantic representation of S_1 under TTDL is as follows:

  $$[\![S_1]\!]_{TTDL} = \overline{[\![S_1]\!]} = \bar{\neg}\overline{[\![S_1']\!]} \tag{6.75}$$

  In addition, according to lemma 6.2.4, its semantic representation under DN-TTDL is as follows:

  $$\begin{aligned} [\![S_1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![S_1]\!]}} \\ &= \langle \bar{\neg}[\![S_1']\!]_{TTDL}, [\![S_1']\!]_{TTDL} \rangle \\ &= \langle \bar{\neg}\overline{[\![S_1']\!]}, \overline{[\![S_1']\!]} \rangle \end{aligned} \tag{6.76}$$

We first compute $[\![D_1]\!]_{TTDL}$. It is achieved by updating S_1 to D_0 with the function $\mathbf{update}_{TTDL}$ (formula 4.33):

$$\begin{aligned}[\![D_1]\!]_{TTDL} &= \mathbf{update}_{TTDL}\ [\![D_0]\!]_{TTDL}\ [\![S_1]\!]_{TTDL} \\ &= \mathbf{update}_{TTDL}\ \mathbf{start}\ \bar{\neg}\overline{[\![S'_1]\!]}\end{aligned} \tag{6.77}$$

Then let's have a look at $[\![D_1]\!]_{DN\text{-}TTDL}$. Similarly, it is achieved by updating S_1 to D_0, while with the function $\mathbf{update}_{DN\text{-}TTDL}$ in DN-TTDL (formula 6.26). The representation of S_1 under TTDL has already been presented in formula 6.76, hence:

$$\begin{aligned}[\![D_1]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ [\![D_0]\!]_{DN\text{-}TTDL}\ [\![S_1]\!]_{DN\text{-}TTDL} \\ &= \lambda DS.(\mathbf{update}_{TTDL}\ D\ (\pi_1 S))\ \mathbf{start}\ [\![S_1]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \mathbf{update}_{TTDL}\ \mathbf{start}\ (\pi_1[\![S_1]\!]_{DN\text{-}TTDL}) \\ &= \mathbf{update}_{TTDL}\ \mathbf{start}\ (\pi_1\langle\bar{\neg}\overline{[\![S'_1]\!]}, \overline{[\![S'_1]\!]}\rangle) \\ &\rightarrow_\beta \mathbf{update}_{TTDL}\ \mathbf{start}\ \bar{\neg}\overline{[\![S'_1]\!]}\end{aligned} \tag{6.78}$$

By comparing formulas 6.77 and 6.78, we can draw that $[\![D_1]\!]_{TTDL} = [\![D_1]\!]_{DN\text{-}TTDL}$.

As a result, no matter whether $[\![S_1]\!]$ belongs to $\mathbb{F}^+_{\Sigma_0}$ or $\mathbb{F}^-_{\Sigma_0}$, equivalence 6.68 holds.

- Let $n = j$, then D_j is the discourse containing the j sentences, namely S_1, S_2, ..., S_j, none of which contains any multi-negation. By induction hypothesis, we assume that:

$$[\![D_j]\!]_{TTDL} = [\![D_j]\!]_{DN\text{-}TTDL} \tag{6.79}$$

Let S_{j+1} be a sentence such that it does not contain any multi-negation. Since $[\![S_{j+1}]\!] \in \mathbb{F}_{\Sigma_0}$, we distinguish the two cases when $[\![S_{j+1}]\!]$ is an element of $\mathbb{F}^+_{\Sigma_0}$ and when it is an element of $\mathbb{F}^-_{\Sigma_0}$.

1. When $[\![S_{j+1}]\!] \in \mathbb{F}^+_{\Sigma_0}$, its semantic representation under TTDL is as follows:

$$[\![S_{j+1}]\!]_{TTDL} = \overline{[\![S_{j+1}]\!]} \tag{6.80}$$

In addition, according to lemma 6.2.4, its semantic representation under DN-TTDL is as follows:

$$\begin{aligned}[\![S_{j+1}]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![S_{j+1}]\!]}} \\ &= \langle[\![S_{j+1}]\!]_{TTDL}, \bar{\neg}[\![S_{j+1}]\!]_{TTDL}\rangle \\ &= \langle\overline{[\![S_{j+1}]\!]}, \bar{\neg}\overline{[\![S_{j+1}]\!]}\rangle\end{aligned} \tag{6.81}$$

We first compute $[\![D_{j+1}]\!]_{TTDL}$. It is achieved by updating S_{j+1} to D_j with the function $\textbf{update}_{TTDL}$ (formula 4.33):

$$\begin{aligned} [\![D_{j+1}]\!]_{TTDL} &= \textbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ [\![S_{j+1}]\!]_{TTDL} \\ &= \textbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ \overline{[\![S_{j+1}]\!]} \end{aligned} \tag{6.82}$$

Then let's have a look at $[\![D_{j+1}]\!]_{DN\text{-}TTDL}$. Similarly, it is achieved by updating S_{j+1} to D_j, while with the function $\textbf{update}_{DN\text{-}TTDL}$ in DN-TTDL (formula 6.26). The representation of S_{j+1} under TTDL has already been presented in formula 6.81, hence:

$$\begin{aligned} [\![D_{j+1}]\!]_{DN\text{-}TTDL} &= \textbf{update}_{DN\text{-}TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ [\![S_{j+1}]\!]_{DN\text{-}TTDL} \\ &= \lambda DS.(\textbf{update}_{TTDL}\ D\ (\pi_1 S))\ [\![D_j]\!]_{DN\text{-}TTDL}\ [\![S_{j+1}]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \textbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ (\pi_1 [\![S_{j+1}]\!]_{DN\text{-}TTDL}) \\ &= \textbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ (\pi_1 \langle \overline{[\![S_{j+1}]\!]}, \overline{\neg}\overline{[\![S_{j+1}]\!]} \rangle) \\ &\rightarrow_\beta \textbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ \overline{[\![S_{j+1}]\!]} \\ &\rightarrow_\beta \textbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ \overline{[\![S_{j+1}]\!]} \end{aligned} \tag{6.83}$$

By comparing formulas 6.82 and 6.83, we can draw that $[\![D_{j+1}]\!]_{TTDL} = [\![D_{j+1}]\!]_{DN\text{-}TTDL}$.

2. When $[\![S_{j+1}]\!] = \neg[\![S'_{j+1}]\!] \in \mathbb{F}^-_{\Sigma_0}$, where $[\![S'_{j+1}]\!] \in \mathbb{F}^+_{\Sigma_0}$, the semantic representation of S_{j+1} under TTDL is as follows:

$$[\![S_{j+1}]\!]_{TTDL} = \overline{[\![S_{j+1}]\!]} = \overline{\neg}\overline{[\![S'_{j+1}]\!]} \tag{6.84}$$

In addition, according to lemma 6.2.4, its semantic representation under DN-TTDL is as follows:

$$\begin{aligned} [\![S_{j+1}]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![S_{j+1}]\!]}} \\ &= \langle \overline{\neg}[\![S'_{j+1}]\!]_{TTDL}, [\![S'_{j+1}]\!]_{TTDL} \rangle \\ &= \langle \overline{\neg}\overline{[\![S'_{j+1}]\!]}, \overline{[\![S'_{j+1}]\!]} \rangle \end{aligned} \tag{6.85}$$

We first compute $[\![D_{j+1}]\!]_{TTDL}$. It is achieved by updating S_{j+1} to D_j with the function $\textbf{update}_{TTDL}$ (formula 4.33):

$$\begin{aligned} [\![D_{j+1}]\!]_{TTDL} &= \textbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ [\![S_{j+1}]\!]_{TTDL} \\ &= \textbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ \overline{\neg}\overline{[\![S'_{j+1}]\!]} \end{aligned} \tag{6.86}$$

Then let's have a look at $[\![D_{j+1}]\!]_{DN\text{-}TTDL}$. Similarly, it is achieved by updating S_{j+1} to D_j, while with the function $\textbf{update}_{DN\text{-}TTDL}$ in DN-TTDL (formula 6.26). The representation of S_{j+1} under TTDL has already been presented in formula 6.85, hence:

$$\begin{aligned}
[\![D_{j+1}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ [\![S_{j+1}]\!]_{DN\text{-}TTDL} \\
&= \lambda DS.(\mathbf{update}_{TTDL}\ D\ (\pi_1 S))\ [\![D_j]\!]_{DN\text{-}TTDL}\ [\![S_{j+1}]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \mathbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ (\pi_1 [\![S_{j+1}]\!]_{DN\text{-}TTDL}) \\
&= \mathbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ (\pi_1 \langle \bar{\neg} \overline{[\![S'_{j+1}]\!]}, \overline{[\![S'_{j+1}]\!]} \rangle) \\
&\to_\beta \mathbf{update}_{TTDL}\ [\![D_j]\!]_{DN\text{-}TTDL}\ \bar{\neg} \overline{[\![S'_{j+1}]\!]} \\
&\to_\beta \mathbf{update}_{TTDL}\ [\![D_j]\!]_{TTDL}\ \bar{\neg} \overline{[\![S'_{j+1}]\!]}
\end{aligned} \tag{6.87}$$

By comparing formulas 6.86 and 6.87, we can draw that $[\![D_{j+1}]\!]_{TTDL} = [\![D_{j+1}]\!]_{DN\text{-}TTDL}$.

Hence, no matter $[\![S_{j+1}]\!]$ belongs to $\mathbb{F}^+_{\Sigma_0}$ or $\mathbb{F}^-_{\Sigma_0}$, equivalence 6.68 holds.

As a result, if a discourse does not contain any multi-negation, equivalence 6.68 always holds. In other words, TTDL and DN-TTDL assign it the identical logical representation. □

To summarize, lemma 6.2.4 reveals the relation between TTDL and DN-TTDL on single sentences, theorem 6.2.1 further links the two systems on the discourse level. In the next subsection, we will introduce the notion of implicit double negation. Then for the rest of this chapter, we shall illustrate the performance of DN-TTDL with concrete linguistic examples.

6.2.3 Implicit Double Negation

Typically, double negation denotes the phenomenon where two negations stack over one another. Examples include (98) and (10): the negative markers, e.g., *not*, *no*, *fail*, etc., occur consecutively. We call these double negations **explicit**. Besides, as far as we are concerned, examples like (99-b), as well as the bathroom example, i.e., (100) and (11), also involve double negation. We call those latter double negations **implicit** because they are not constructed on the surface level, as in (98) and (10). Rather, they are counted as double negation because of the way in which disjunction, implication, and universal quantifier are defined: the derived logical connectives are negations themselves. More examples of this sort are as follows:

(107) It is not the case that either there's <u>no bathroom</u>$_i$ in the house, or <u>it</u>$_i$'s in a funny place. *<u>It</u>$_i$ is well-furnished.

(108) It is not the case that if <u>a farmer</u>$_i$ owns <u>a donkey</u>$_j$, <u>he</u>$_i$ beats <u>it</u>$_j$. *<u>He</u>$_i$ hates <u>it</u>$_j$.

(109) It is not the case that <u>every farmer</u>$_i$ who owns <u>a donkey</u>$_j$ beats <u>it</u>$_j$. *<u>He</u>$_i$ hates <u>it</u>$_j$.

Although DN-TTDL can account for the double negation exception (also the disjunction exception), which standard dynamic frameworks fail to cover, it is not completely satisfactory. Take (107) for instance, where the first sentence is the negation of the classical bathroom example. The pronoun *it* in the second disjunct is correctly predicted as usual. However, with the current setup, DN-TTDL will also permit the second pronoun, which occurs in a subsequent separate sentence, as shown in its logical representation:

$$\begin{aligned}[\![(107)\text{-}1]\!]_{DN\text{-}TTDL} = \overline{\overline{\neg}}[\![(11)]\!]_{DN\text{-}TTDL} \\ \twoheadrightarrow_{\beta} \langle \lambda e\phi.(\exists x.(\textbf{bathroom}\ x \wedge \neg(\textbf{in\_funny\_place}\ (\mathsf{sel}(x :: e))) \wedge \phi(x :: e))), \\ \lambda e\phi.(\forall x.(\textbf{bathroom}\ x \to \textbf{in\_funny\_place}\ (\mathsf{sel}(x :: e))) \wedge \phi e)\rangle\end{aligned}$$

As can be inferred from the update function (formula 6.26), the first projection of $[\![(107)\text{-}1]\!]_{DN\text{-}TTDL}$ will be employed in the discourse incrementation. However, since the variable x is updated in its left context, the second pronoun *it* in (107) will be undesirably resolved. This problem results from the way that disjunction is dynamized. Assume the DN-TTDL representation for *there's a bathroom in the house* is M_1, the one for *it's in a funny place* is M_2, then the semantics of the bathroom example can be represented in the following formula:

$$\begin{aligned}[\![(11)]\!]_{DN\text{-}TTDL} &= (\overline{\overline{\neg}}M_1)\overline{\overline{\vee}}M_2 \\ &= \overline{\overline{\neg}}((\overline{\overline{\neg}}(\overline{\overline{\neg}}M_1))\overline{\overline{\wedge}}(\overline{\overline{\neg}}M_2)) \\ &= \overline{\overline{\neg}}(M_1\overline{\overline{\wedge}}(\overline{\overline{\neg}}M_2))\end{aligned}$$

As indicated by the above formula, the referents introduced in neither disjunct are accessible from subsequent anaphoric expressions. That is because the whole representation is a negation, it blocks accessibility of variables within its scope. In terms of DPL, $\overline{\overline{\neg}}$ is externally static. Meanwhile, the negated version of the bathroom example is translated as follows:

$$\begin{aligned}[\![(107)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{\neg}}[\![(11)]\!]_{DN\text{-}TTDL} \\ &= \overline{\overline{\neg}}(\overline{\overline{\neg}}(M_1\overline{\overline{\wedge}}(\overline{\overline{\neg}}M_2))) \\ &= M_1\overline{\overline{\wedge}}(\overline{\overline{\neg}}M_2)\end{aligned}$$

Because the law of double negation hold in DN-TTDL (lemma 6.2.2), when a negation is applied to (11), the representation becomes a conjunction, which is externally dynamic in terms of DPL. This is why DN-TTDL wrongly allows the inter-sentential anaphora in (107). A similar annoying predication in DN-TTDL is concerned with implication and universal quantifier as well: the anaphors in the second sentences of (108) and (109) should be blocked. However, they are well justified in DN-TTDL.

With the current configuration of DN-TTDL, we fail to give a reasonable account. The reason is as follows. Operators such as disjunction, implication, and universal quantifier are externally static: they block the discourse referents within their scope. This is achieved through the outermost negation in their definitions (see formula 3.1, 3.2 and 3.5, each of which induces some extra negations). However, when we apply an additional negation to these operators in DN-TTDL, the dynamic properties of the whole construction will be modified because double negation is erased unconditionally in DN-TTDL. That is to say, a negated externally static operator will become externally dynamic. This scheme seems to work well with explicit double negations, while it is undesired for implicit ones. From examples (107), (108) and (109), we may draw that disjunction, implication and universal quantifier should always be externally static, no matter they are negated or not. This gives us the intuition that among all double negations, only the explicit ones can be eliminated.

As a result, in order to cope with DN-TTDL's failure in implicit double negations, we abandon the conventional definition of the derived connectives (e.g., formula 6.29). Rather, we propose to redefine the dynamic disjunction, implication and universal quantifications in DN-TTDL, enforcing them to be externally static. Firstly, we introduce the following operator. It takes a TTDL proposition, and blocks the accessibility of referents in its left context:

$$\mathbf{closure}_{TTDL} \triangleq \lambda Ae\phi.(A\ e\ \mathbf{stop}) \wedge \phi e \tag{6.88}$$

Assume M is an input proposition, if the left context of M is empty, then $\mathbf{closure}_{TTDL}M$ is equivalent to M. Otherwise, the two terms are merely truth-conditionally (statically) equivalent: they differ in their dynamic meanings (the potential to address subsequent anaphors). Based on $\mathbf{closure}_{TTDL}$, we propose the counterpart operator in DN-TTDL, which takes a DN-TTDL sentence and blocks referents in both of its projections:

$$\mathbf{closure}_{DN\text{-}TTDL} \triangleq \lambda A.\langle \mathbf{closure}_{TTDL}(\pi_1 A), \mathbf{closure}_{TTDL}(\pi_2 A)\rangle \tag{6.89}$$

With the above term, we propose a set of new definitions for dynamic connectives (conjunction, negation, and existential quantifier are defined as before):

$$\overline{\overline{\nabla}} \triangleq \lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}A\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}B)) \tag{6.90}$$

$$\overline{\overline{\rightarrow}} \triangleq \lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(A\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}B)) \tag{6.91}$$

$$\overline{\overline{\forall}} \triangleq \lambda P.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\exists}}(\lambda x.\overline{\overline{\neg}}(Px)))) \tag{6.92}$$

The above definitions will not affect the potential of DN-TTDL on inter-sentential anaphoras, donkey sentences and the bathroom example, which will be properly treated as before. The only change it brings about is that no anaphoric expression can access variables introduced within a disjunction, an implication, or a universally quantified phrase. In other words, the three derived connectives are always externally static, no matter the sentence is negated or not. We will see this in more detail in the illustration. Please note that with the new definitions, some logical facts presented in section 6.2.2, such as formulas 6.30 and 6.31 will not hold any more.

6.2.4 Illustration

In the current subsection, we present the applications of the double negation adaptation of TTDL, namely how DN-TTDL deals with the exceptional examples by which we are motivated. We shall start by translating the lexical entries, then proceed to a list of examples, including inter-sentential anaphora, donkey sentence, bathroom example, double negation, etc. Finally, we will look into some interesting examples where implicit double negation is involved.

Lexical Entries

As introduced in the previous section 6.2.1, there exists a systematic way to translate standard semantic lexical entries into the dynamic counterparts in DN-TTDL. We will again take transitive verb (e.g., *beat*) as an example and conduct its translation step by step. The following process is similar to what we have shown in section 4.4.3.

1. The standard entry for *beat*:

$$[\![beat]\!] = \lambda OS.S(\lambda x.O(\lambda y.(\mathbf{beat}\ x\ y)))$$

   It takes two NPs and yields a proposition, its type is $((\iota \to o) \to o) \to ((\iota \to o) \to o) \to o$.

2. According to definition 6.2.3:

$$\begin{aligned}\overline{\overline{[\![beat]\!]}} &= \overline{\overline{\lambda OS.S(\lambda x.O(\lambda y.(\mathbf{beat}\ x\ y)))}} \\ &= \lambda OS.\overline{\overline{S(\lambda x.O(\lambda y.(\mathbf{beat}\ x\ y)))}} \\ &= \lambda OS.S(\lambda x.O(\lambda y.(\overline{\overline{\mathbf{beat}}}\ x\ y)))\end{aligned}$$

3. The predicate constant **beat** is of type $\iota \to \iota \to o$, according to lemma 6.2.3, definitions 6.2.2 and 4.4.3:

$$\begin{aligned}\mathbb{D}^{dn}_{\iota\to\iota\to o}(\lambda e.\mathbf{beat}) &= \lambda xy.\mathbb{D}^{dn}_{o}(\lambda e.\mathbf{beat}\ x\ y) \\ &= \lambda xy.\langle \mathbb{D}_o(\lambda e.\mathbf{beat}\ x\ y), \neg^{d}_{TTDL}(\mathbb{D}_o(\lambda e.\mathbf{beat}\ x\ y))\rangle \\ &= \lambda xy.\langle \lambda e\phi.((\lambda e'.\mathbf{beat}\ x\ y)e \wedge \phi e), \\ &\qquad (\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)(\lambda e\phi.((\lambda e'.\mathbf{beat}\ x\ y)e \wedge \phi e))\rangle \\ &\twoheadrightarrow_\beta \lambda xy.\langle \lambda e\phi.(\mathbf{beat}\ x\ y \wedge \phi e), \lambda e\phi.(\neg(\mathbf{beat}\ x\ y) \wedge \phi e)\rangle\end{aligned}$$

4. As a result,

$$\begin{aligned}\overline{\overline{[\![beat]\!]}} &= \lambda OS.S(\lambda x.O(\lambda y.(\overline{\overline{\mathbf{beat}}}\ x\ y))) \\ &= \lambda OS.S(\lambda x.O(\lambda y.(\mathbb{D}^{dn}_{\iota\to\iota\to o}(\lambda e.\mathbf{beat})\ x\ y))) \\ &= \lambda OS.S(\lambda x.O(\lambda y.(\lambda x'y'.\langle \lambda e\phi.(\mathbf{beat}\ x'\ y' \wedge \phi e), \\ &\qquad \lambda e\phi.(\neg(\mathbf{beat}\ x'\ y') \wedge \phi e)\rangle)xy)) \\ &\twoheadrightarrow_\beta \lambda OS.S(\lambda x.O(\lambda y.\langle \lambda e\phi.(\mathbf{beat}\ x\ y \wedge \phi e), \\ &\qquad \lambda e\phi.(\neg(\mathbf{beat}\ x\ y) \wedge \phi e)\rangle))\end{aligned}$$

   O and S are both of the dynamized NP type, namely $(\iota \to \Omega^{dn}) \to \Omega^{dn}$, x and y are both of type ι, hence $\overline{\overline{[\![beat]\!]}}$ is of type $((\iota \to \Omega^{dn}) \to \Omega^{dn}) \to ((\iota \to \Omega^{dn}) \to \Omega^{dn}) \to \Omega^{dn}$.

The same procedure can be carried out for any other category. In appendix A.2, we provide the systematic translations of more lexical entries in DN-TTDL. In what follows, we will exemplify DN-TTDL with some linguistic examples. We will start with two classical examples: the inter-sentential anaphora and donkey anaphora, then we will see how DN-TTDL deals with double negation and disjunction exceptions as presented in section 5.3.

Inter-Sentential Anaphora

For discourse anaphora, we will focus on the following two examples:

(110) Bill has $\underline{\text{a car}}_i$. $\underline{\text{It}}_i$ is black. Karttunen (1969)

(73) Bill doesn't have $\underline{\text{a car}}_i$. *$\underline{\text{It}}_i$ is black. Karttunen (1969)

Discourse (110) has a similar structure as the previous (6), and discourse (73) has been presented before in section 5.1. As predicted by dynamic frameworks such as TTDL, the anaphora in (110) is allowed, while the one in (73) is problematic, because negation blocks accessibility of variables within its scope, e.g., the referent introduced by *a car*.

According to theorem 6.2.1, the two frameworks TTDL and DN-TTDL make the same prediction for discourses where no multi-negation occurs. In examples (110) and (73), either there is no negation, either only a single negation is concerned. Consequently, like TTDL (as shown in section 5.1), DN-TTDL should also be able to account for the anaphoric links in the two discourses. Immediately below, we will illustrate this by providing the detailed semantic representations step by step.

Firstly, based on their syntactic information, the two component sentences in (110) are mapped into corresponding logical formulas as follows:

$$\begin{aligned}\llbracket(110)\text{-}1\rrbracket_{DN\text{-}TTDL} &= \overline{\overline{\llbracket have\rrbracket(\llbracket a\rrbracket\llbracket car\rrbracket)\llbracket Bill\rrbracket}}\\ &\twoheadrightarrow_\beta \langle\lambda e\phi.(\exists x.(\mathbf{car}\ x\wedge\mathbf{have\ bill}\ x\wedge\phi(x::e))),\\ &\qquad\lambda e\phi.(\neg(\exists x.(\mathbf{car}\ x\wedge\mathbf{have\ bill}\ x))\wedge\phi e)\rangle\end{aligned}$$

$$\begin{aligned}\llbracket(110)\text{-}2\rrbracket_{DN\text{-}TTDL} &= \overline{\overline{\llbracket is\_black\rrbracket\llbracket it\rrbracket}}\\ &\twoheadrightarrow_\beta \langle\lambda e\phi.(\mathbf{black}\ (\mathsf{sel}\ e)\wedge\phi e),\lambda e\phi.(\neg(\mathbf{black}\ (\mathsf{sel}\ e))\wedge\phi e)\rangle\end{aligned}$$

The process for discourse incrementation is as follows. We first update the semantic representation of (110)-1 to the initial discourse, then update the representation of (110)-2 to the result obtained in the preceding step. Since proper name (e.g., *Bill*) is treated as a presupposition, the initial left context of (110) is not empty. We propose a variant of 6.69, where the constant **bill** is predefined in the left context:

$$\mathbf{start_b} = \lambda e\phi.\phi(\mathbf{bill}::\mathsf{nil}) \tag{6.93}$$

Thus, the semantics of the two pieces of discourse are computed stepwise as follows:

$$\begin{aligned}\llbracket D_{(110)\text{-}1}\rrbracket_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ \mathbf{start_b}\ \llbracket(110)\text{-}1\rrbracket_{DN\text{-}TTDL}\\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{car}\ x\wedge\mathbf{have\ bill}\ x\wedge\phi(x::\mathbf{bill}::\mathsf{nil}))\end{aligned}$$

$$\begin{aligned}[\![D_{(110)}]\!]_{DN\text{-}TTDL} &= \textbf{update}_{DN\text{-}TTDL}\ [\![D_{(110)\text{-}1}]\!]_{DN\text{-}TTDL}\ [\![(110)\text{-}2]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x \wedge \textbf{black}\ (\mathsf{sel}(x :: \textbf{bill} :: \mathsf{nil})) \\ &\qquad\qquad\qquad\qquad\qquad\qquad \wedge \phi(x :: \textbf{bill} :: \mathsf{nil}))\end{aligned}$$

As we can see, x, the variable which corresponds to the indefinite *a car*, is among the candidates for the choice operator sel. Assume it makes the correct decision: selecting x rather than **bill**. After passing it the empty left context nil and the empty right context **stop**, the resulting logical representation will be reduced into:

$$[\![(110)]\!]_{DN\text{-}TTDL}\ \mathsf{nil}\ \textbf{stop} \twoheadrightarrow_\beta \exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x \wedge \textbf{black}\ x)$$

As to discourse (73), where a single negation is involved in the first sentence, we can carry out a similar series of computations. Firstly, the representation of the first sentence of (73) is achieved as follows:

$$\begin{aligned}[\![(73)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![not]\!]([\![have]\!]([\![a]\!][\![car]\!])[\![Bill]\!])}} \\ &= \neg^d_{DN\text{-}TTDL}\overline{\overline{([\![have]\!]([\![a]\!][\![car]\!])[\![Bill]\!])}} \\ &\twoheadrightarrow_\beta \langle \lambda e\phi.(\neg(\exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x)) \wedge \phi e), \\ &\qquad\qquad \lambda e\phi.(\exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x \wedge \phi(x :: e)))\rangle\end{aligned}$$

Since (73)-2 is exactly the same as (110)-2, so $[\![(73)\text{-}2]\!]_{DN\text{-}TTDL} = [\![(110)\text{-}2]\!]_{DN\text{-}TTDL}$, we shall not repeat the formula here. Finally for the discourse incrementation, we update the representation of (73)-1 to $\textbf{start}_\mathbf{b}$, then update the representation of (73)-2 to the preceding result in order to obtain the logical form of the whole discourse.

$$\begin{aligned}[\![D_{(73)\text{-}1}]\!]_{DN\text{-}TTDL} &= \textbf{update}_{DN\text{-}TTDL}\ \textbf{start}_\mathbf{b}\ [\![(73)\text{-}1]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \lambda e\phi.((\neg(\exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x))) \wedge \phi(\textbf{bill} :: \mathsf{nil}))\end{aligned}$$

$$\begin{aligned}[\![D_{(73)}]\!]_{DN\text{-}TTDL} &= \textbf{update}_{DN\text{-}TTDL}\ [\![D_{(73)\text{-}1}]\!]_{DN\text{-}TTDL}\ [\![(73)\text{-}2]\!]_{DN\text{-}TTDL} \\ &\twoheadrightarrow_\beta \lambda e\phi.((\neg(\exists x.(\textbf{car}\ x \wedge \textbf{have}\ \textbf{bill}\ x))) \wedge \textbf{black}\ (\mathsf{sel}(\textbf{bill} :: \mathsf{nil})) \\ &\qquad\qquad\qquad\qquad\qquad\qquad \wedge \phi(\textbf{bill} :: \mathsf{nil}))\end{aligned}$$

Obviously, the discourse referent of the indefinite *a car*, namely x, is not among the candidates for the choice operator sel. Hence the pronoun *it* can not be resolved, which coincides with the prediction of TTDL.

Donkey Sentence

As we mentioned in section 4.1, donkey sentence is one of the main motives for the emergence of dynamic semantic frameworks. In order to investigate the performance of DN-TTDL on it, we take the conditional version (7) as an illustration:

(7) If $\underline{\text{a farmer}}_i$ owns $\underline{\text{a donkey}}_j$, $\underline{\text{he}}_i$ beats $\underline{\text{it}}_j$.

The connective between the first sentence and the second sentence is an implication, which is defined in terms of other primitive logical connectors, i.e., negation and conjunction (see formula 6.29 for the detailed representation). As can be observed from (7), neither the antecedent nor the consequent of the implication is concerned with negation, so there is no multi-negation involved in the whole implication. According to lemma 6.2.4, DN-TTDL and TTDL shall return the same result. We will conduct the computation in DN-TTDL below.

Based on the syntactic function-application information, it is straightforward to obtain the logical representations of the two sentences in (7) as follows:

$$\begin{aligned}
[\![(7)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![own]\!]([\![a]\!][\![donkey]\!])([\![a]\!][\![farmer]\!])}} \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\exists x.\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y \wedge \phi(y :: x :: e))), \\
&\qquad \lambda e\phi.(\neg\exists x.(\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y)) \wedge \phi e)\rangle
\end{aligned}$$

$$\begin{aligned}
[\![(7)\text{-}2]\!]_{DN\text{-}TTDL} &= \overline{\overline{([\![beat]\!][\![it]\!])[\![he]\!]}} \\
&\twoheadrightarrow_\beta \lambda e\phi.\langle \mathbf{beat}\ (\mathsf{sel_{he}}\ e)\ (\mathsf{sel_{it}}\ e) \wedge \phi e, \neg(\mathbf{beat}\ (\mathsf{sel_{he}}\ e)\ (\mathsf{sel_{it}}\ e)) \wedge \phi e\rangle
\end{aligned}$$

For discourse incrementation, we first compose the above two representations with dynamic implication (formula 6.91), then update the result to the initial discourse.

$$\begin{aligned}
[\![(7)]\!]_{DN\text{-}TTDL} &= [\![(7)\text{-}1]\!]_{DN\text{-}TTDL} \overline{\overline{\Rightarrow}} [\![(7)\text{-}2]\!]_{DN\text{-}TTDL} \\
&= \lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(A\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}B))[\![(7)\text{-}1]\!]_{DN\text{-}TTDL}[\![(7)\text{-}2]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}([\![(7)\text{-}1]\!]_{DN\text{-}TTDL}\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}[\![(7)\text{-}2]\!]_{DN\text{-}TTDL})) \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\forall x.(\mathbf{farmer}\ x \to \forall y.(\mathbf{donkey}\ y \to (\mathbf{own}\ x\ y \\
&\qquad \to \mathbf{beat}\ (\mathsf{sel_{he}}(y :: x :: e))\ (\mathsf{sel_{it}}(y :: x :: e))))) \wedge \phi e), \\
&\qquad \lambda e\phi.(\exists x.\mathbf{farmer}\ x \wedge \exists y.(\mathbf{donkey}\ y \wedge \mathbf{own}\ x\ y \wedge \\
&\qquad \neg(\mathbf{beat}\ (\mathsf{sel_{he}}(y :: x :: e))\ (\mathsf{sel_{it}}(y :: x :: e))) \wedge \phi e))\rangle
\end{aligned}$$

Because no proper names occur in the donkey sentence (7), we use the empty initial context **start** (formula 6.69), then:

$$\begin{aligned}
[\![D_{(7)}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ \mathbf{start}\ [\![(7)]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \lambda e\phi.(\forall x.(\mathbf{farmer}\ x \to \forall y.(\mathbf{donkey}\ y \to (\mathbf{own}\ x\ y \\
&\qquad \to \mathbf{beat}\ (\mathsf{sel_{he}}(y :: x :: \mathsf{nil}))\ (\mathsf{sel_{it}}(y :: x :: \mathsf{nil}))))) \wedge \phi\ \mathsf{nil})
\end{aligned}$$

Assume both choice operators $\mathsf{sel_{he}}$ and $\mathsf{sel_{it}}$ select the appropriate referents, namely x for the former, y for the latter. Then after passing the empty left context nil and the empty right context **stop** to the above formula, we finally obtain:

$$[\![D_{(7)}]\!]_{DN\text{-}TTDL}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_\beta \forall x.(\mathbf{farmer}\ x \to \forall y.(\mathbf{donkey}\ y \to (\mathbf{own}\ x\ y \to \mathbf{beat}\ x\ y)))$$

As a result, both paradigm examples for dynamic semantics: inter-sentential anaphora and donkey anaphora, can be successfully accounted for by DN-TTDL.

Double Negation

In this subsection, we will examine the examples which are concerned with double negation, in particular, examples (98) and (10) in the previous chapter. In order to retain a uniform vocabulary with our previous illustrations, we make up an example, which is in a parallel structure with (98) and (10):

(111) It is not the case that Bill doesn't own $\underline{\text{a car}_i}$. $\underline{\text{It}_i}$ is black.

As one may have noticed, the second sentence (111)-2 is identical to (110)-2 and (73)-2, namely:

$$[\![(111)\text{-}2]\!]_{DN\text{-}TTDL} = [\![(110)\text{-}2]\!]_{DN\text{-}TTDL} = [\![(73)\text{-}2]\!]_{DN\text{-}TTDL}$$

As a result, we only need to focus on the representation of (111)-1, which can be achieved based on the syntactic information of the sentence as follows:

$$\begin{aligned}
[\![(111)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![not]\!]([\![not]\!]([\![have]\!]([\![a]\!][\![car]\!])[\![Bill]\!]))}} \\
&= \neg^d_{DN\text{-}TTDL}(\neg^d_{DN\text{-}TTDL}\overline{\overline{([\![have]\!]([\![a]\!][\![car]\!])[\![Bill]\!]))}} \\
&= \overline{\overline{([\![have]\!]([\![a]\!][\![car]\!])[\![Bill]\!])}} \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{bill}\ x \wedge \phi(x :: e))), \\
&\qquad \lambda e\phi.(\neg(\exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{bill}\ x)) \wedge \phi e)\rangle
\end{aligned}$$

The discourse incrementation is straightforward. It is the same as what we did for inter-sentential anaphora examples above.

$$\begin{aligned}
[\![D_{(111)\text{-}1}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ \mathbf{start_b}\ [\![(111)\text{-}1]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \lambda e\phi.(\exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{bill}\ x \wedge \phi(x :: \mathbf{bill} :: \mathsf{nil})))
\end{aligned}$$

$$\begin{aligned}
[\![D_{(111)}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ [\![D_{(111)\text{-}1}]\!]_{DN\text{-}TTDL}\ [\![(111)\text{-}2]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \lambda e\phi.\exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{bill}\ x \wedge \mathbf{black}\ (\mathsf{sel}(x :: \mathbf{bill} :: \mathsf{nil})) \\
&\qquad \wedge\ \phi(x :: \mathbf{bill} :: \mathsf{nil}))
\end{aligned}$$

Assume sel makes the correct choice (picking up x), then the above formula can be further reduced by applying to the empty left context nil and the empty right context **stop**:

$$[\![D_{(111)}]\!]_{DN\text{-}TTDL}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_\beta \exists x.(\mathbf{car}\ x \wedge \mathbf{have}\ \mathbf{bill}\ x \wedge \mathbf{black}\ x)$$

As we can see, the above representation is equivalent to the one for example (110), which is exactly what we are expecting: double negation cancels one another and re-opens accessibility of discourse referent within its scope.

Disjunction (Bathroom Example)

In section 5.3.4, we have generalized disjunction as the same sort of exception as double negation. That is to say, a solution for the double negation exception should automatically account for the disjunction problem. We will verify it by looking at the logical representation of the bathroom example under DN-TTDL.

(11) Either there's <u>no bathroom</u>$_i$ in the house, or <u>it</u>$_i$'s in a funny place. Roberts (1989)

As discussed in section 6.2.3, the double negation in (11) is implicit. There is no explicit double negation construction in either disjunct of (11): the first sentence is with a single negation, the second is affirmative. Since disjunction is conventionally defined as in formula 3.2, we obtain a double negation indirectly, namely:

$$\neg\phi \vee \psi = \neg(\neg(\neg\phi) \wedge \neg\psi) \tag{6.94}$$

Since a double negation is present in sentence (11), according to theorem 6.2.1, DN-TTDL and TTDL will diverge in representing discourses containing it. We have already shown in section 5.3.4 that TTDL fails to address the felicitous anaphora in the bathroom example. Below, the semantic representation for (11) in DN-TTDL will be computed in detail. Again, we first look at the two component sentences:

$$\begin{aligned}
[\![(11)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![not]\!]([\![there_is]\!]([\![a]\!][\![bathroom]\!]))}} \\
&= \neg^{d}_{DN\text{-}TTDL}(\overline{\overline{[\![there_is]\!]([\![a]\!][\![bathroom]\!])}}) \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\neg(\exists x.\mathbf{bathroom}\ x) \wedge \phi e), \\
&\qquad\qquad \lambda e\phi.(\exists x.(\mathbf{bathroom}\ x \wedge \phi(x :: e)))\rangle
\end{aligned}$$

$$\begin{aligned}
[\![(11)\text{-}2]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![in_funny_place]\!][\![it]\!]}} \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\mathbf{in_funny_place}\ (\mathsf{sel}\ e) \wedge \phi e), \\
&\qquad\qquad \lambda e\phi.(\neg(\mathbf{in_funny_place}\ (\mathsf{sel}\ e)) \wedge \phi e)\rangle
\end{aligned}$$

Now we can compose the above two formulas with the dynamic disjunction (formula 6.90), the result will then be updated to the initial context.

$$\begin{aligned}
[\![(11)]\!]_{DN\text{-}TTDL} &= [\![(11)\text{-}1]\!]_{DN\text{-}TTDL} \overline{\overline{\vee}} [\![(11)\text{-}2]\!]_{DN\text{-}TTDL} \\
&= \lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}A \mathbin{\overline{\overline{\wedge}}} \overline{\overline{\neg}}B))[\![(11)\text{-}1]\!]_{DN\text{-}TTDL}[\![(11)\text{-}2]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}[\![(11)\text{-}1]\!]_{DN\text{-}TTDL} \mathbin{\overline{\overline{\wedge}}} \overline{\overline{\neg}}[\![(11)\text{-}2]\!]_{DN\text{-}TTDL})) \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\forall x.(\mathbf{bathroom}\ x \to \mathbf{in_funny_place}\ (\mathsf{sel}(x :: e))) \wedge \phi e), \\
&\qquad \lambda e\phi.(\exists x.(\mathbf{bathroom}\ x \wedge \neg(\mathbf{in_funny_place}\ (\mathsf{sel}(x :: e))) \wedge \phi e))\rangle
\end{aligned}$$

Since there is no presupposition in the discourse, we employ the lexical entry for the empty initial context (formula 6.69), then:

$$\begin{aligned}
[\![D_{(11)}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ \mathbf{start}\ [\![(11)]\!]_{DN\text{-}TTDL} \\
&\to_\beta \lambda e\phi.(\forall x.(\mathbf{bathroom}\ x \to \mathbf{in_funny_place}\ (\mathsf{sel}(x :: \mathsf{nil}))) \wedge \phi\ \mathsf{nil})
\end{aligned}$$

This time the selection function sel can pick x from the candidate list. Besides, no further continuations, which are outside the scope of the disjunction, can access x. If we apply it to the empty left context nil and the empty continuation **stop**, we arrive at a more succinct representation:

$$[\![D_{(11)}]\!]_{DN\text{-}TTDL}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_\beta \forall x.(\mathbf{bathroom}\ x \to \mathbf{in\_funny\_place}\ x)$$

As a result, the disjunction exception can be properly treated in DN-TTDL. At the same time, we would also like to show that same as standard dynamic frameworks, DN-TTDL is able to rule out the following "non-orthdox" bathroom example:

(112) *Either there's a bathroom$_i$ in the house, or it$_i$'s in a funny place.

Since both disjuncts of (112) are affirmative, there is no multi-negation involved in the overall disjunction. By theorem 6.2.1, DN-TTDL and TTDL will make the same predication in this case. We will briefly present its semantics in DN-TTDL below.

$$\begin{aligned}
[\![(112)\text{-}1]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![\mathit{there_is}]\!]([\![a]\!][\![\mathit{bathroom}]\!])}} \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\exists x.(\mathbf{bathroom}\ x \wedge \phi(x :: e))), \\
&\qquad\qquad \lambda e\phi.(\neg(\exists x.\mathbf{bathroom}\ x) \wedge \phi e)\rangle
\end{aligned}$$

$$\begin{aligned}
[\![(112)\text{-}2]\!]_{DN\text{-}TTDL} &= \overline{\overline{[\![\mathit{in_funny_place}]\!][\![\mathit{it}]\!]}} \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\mathbf{in_funny_place}\ (\mathsf{sel}\ e) \wedge \phi e), \\
&\qquad\qquad \lambda e\phi.(\neg(\mathbf{in_funny_place}\ (\mathsf{sel}\ e)) \wedge \phi e)\rangle
\end{aligned}$$

The discourse incrementation is similar as the standard bathroom example:

$$\begin{aligned}
[\![(112)]\!]_{DN\text{-}TTDL} &= [\![(112)\text{-}1]\!]_{DN\text{-}TTDL} \overline{\overline{\vee}} [\![(112)\text{-}2]\!]_{DN\text{-}TTDL} \\
&= \lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}A \mathbin{\overline{\overline{\wedge}}} \overline{\overline{\neg}}B))[\![(112)\text{-}1]\!]_{DN\text{-}TTDL}[\![(112)\text{-}2]\!]_{DN\text{-}TTDL} \\
&= \mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}[\![(112)\text{-}1]\!]_{DN\text{-}TTDL} \mathbin{\overline{\overline{\wedge}}} \overline{\overline{\neg}}[\![(112)\text{-}2]\!]_{DN\text{-}TTDL})) \\
&\twoheadrightarrow_\beta \langle \lambda e\phi.(\exists x.(\mathbf{bathroom}\ x) \vee \mathbf{in_funny_place}\ (\mathsf{sel}\ e) \wedge \phi e), \\
&\qquad \lambda e\phi.(\neg(\exists x.\mathbf{bathroom}\ x) \wedge \neg(\mathbf{in_funny_place}\ (\mathsf{sel}\ e)) \wedge \phi e)\rangle
\end{aligned}$$

With the empty initial context, the representation for the discourse is:

$$\begin{aligned}
[\![D_{(112)}]\!]_{DN\text{-}TTDL} &= \mathbf{update}_{DN\text{-}TTDL}\ \mathbf{start}\ [\![(112)]\!]_{DN\text{-}TTDL} \\
&\twoheadrightarrow_\beta \lambda e\phi.(\exists x.(\mathbf{bathroom}\ x \vee \mathbf{in_funny_place}\ (\mathsf{sel}\ \mathsf{nil})) \wedge \phi\ \mathsf{nil})
\end{aligned}$$

This time, the candidate list for the choice operator is empty. Hence the anaphoric relation in (112) is successfully blocked in DN-TTDL.

Implicit Double Negations

This is the last subsection for illustration, we will examine implicit double negations. As discussed in section 6.2.3, by defining the derived logical connectives (implication, disjunction, and universal quantification) in the conventional way, DN-TTDL fails to account for the infelicitous anaphoras under implicit double negation, see examples (107), (108), and (109). However, we may remedy this by redefining the dynamic connectives, as shown in formulas 6.90, 6.91, and 6.92.

Now let's first take a look at example (107). With the new dynamic disjunction, the logical representation of the first sentence is computed as follows:

$$
\begin{aligned}
[\![(107)\text{-}1]\!]_{DN\text{-}TTDL} &= \neg^{d}_{DN\text{-}TTDL}[\![(11)]\!]_{DN\text{-}TTDL} \\
&= \neg^{d}_{DN\text{-}TTDL}([\![(11)\text{-}1]\!]_{DN\text{-}TTDL}\overline{\overline{\vee}}[\![(11)\text{-}2]\!]_{DN\text{-}TTDL}) \\
&= \neg^{d}_{DN\text{-}TTDL}((\lambda AB.\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}A\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}B))) \\
&\qquad\qquad [\![(11)\text{-}1]\!]_{DN\text{-}TTDL}[\![(11)\text{-}2]\!]_{DN\text{-}TTDL}) \\
&\twoheadrightarrow_{\beta} \neg^{d}_{DN\text{-}TTDL}(\mathbf{closure}_{DN\text{-}TTDL}(\overline{\overline{\neg}}(\overline{\overline{\neg}}[\![(11)\text{-}1]\!]_{DN\text{-}TTDL}\ \overline{\overline{\wedge}}\ \overline{\overline{\neg}}[\![(11)\text{-}2]\!]_{DN\text{-}TTDL}))) \\
&\twoheadrightarrow_{\beta} \langle\lambda e\phi.(\neg(\forall x.(\mathbf{bathroom}\ x \to \mathbf{in\_funny\_place}\ (\mathsf{sel}(x :: e)))) \wedge \phi e), \\
&\qquad \lambda e\phi.(\forall x.(\mathbf{bathroom}\ x \to \mathbf{in\_funny\_place}\ (\mathsf{sel}(x :: e))) \wedge \phi e)\rangle
\end{aligned}
$$

This time, the left contexts of both projections are not updated. Hence although the pronoun in the second disjunct can be resolved as the referent x, no anaphor outside disjunction is possible. With formulas 6.91 and 6.92, examples (108) and (109) can be respectively accounted for in a similar manner. We will ignore the detailed computations for them here.

In what follows, we will look at a final example. Remind that at the end of section 5.3.3, we have shown that two negations does not always cancel each other out with example (99). In order to avoid the problem of plurality, we slightly modify the pair (99), giving rise to the following examples:

(113) a. A student$_i$ passed the examination. He$_i$ studied hard.
b. Not every student$_i$ failed the examination. *He$_i$ studied hard.

The anaphoric link, which crosses sentence boundary, is felicitous in (113-a) while problematic in (113-b), despite the fact that the latter does contain a double negation (*not* and *fail*). As discussed in section 6.2.2, with the old interpretation of the dynamic universal quantifier, we may draw the logical equivalence in formula 6.31, repeated as follows:

$$\overline{\overline{\exists}}(\lambda x.\overline{\overline{\neg}}\phi) = \overline{\overline{\neg}}\overline{\overline{\forall}}(\lambda x.\phi) \tag{6.31}$$

This implies that the first sentences in (113-a) and (113-b) are exactly the same under DN-TTDL. However, they are only truth-conditionally equivalent. From the dynamic point of view, (113-a)-1 and (113-b)-1 have different potential to take future anaphoric expressions. With what we have presented above, one may notify that the double negation in (113-b) is implicit, which can not be simply eliminated. Hence the logical equivalence 6.31 shall not hold unconditionally.

If we interpret *fail* as *not pass*, the lexical entry for *fail* can be derived from the one for *pass* as follows (a negation $\neg$ is inserted in front of the sub-formula "$\mathbf{pass}\ x$" in the representation of *pass*):

$$[\![pass]\!]_{DN\text{-}TTDL} = \lambda S.S(\lambda x.\langle\lambda e\phi.(\mathbf{pass}\ x \wedge \phi e), \lambda e\phi.(\neg(\mathbf{pass}\ x) \wedge \phi e)\rangle)$$

$$
\begin{aligned}
[\![fail]\!]_{DN\text{-}TTDL} &= \lambda S.S(\lambda x.\langle \lambda e\phi.(\neg(\mathbf{pass}\ x) \wedge \phi e), \lambda e\phi.(\neg(\neg(\mathbf{pass}\ x)) \wedge \phi e)\rangle) \\
&= \lambda S.S(\lambda x.\langle \lambda e\phi.(\neg(\mathbf{pass}\ x) \wedge \phi e), \lambda e\phi.(\mathbf{pass}\ x \wedge \phi e)\rangle)
\end{aligned}
$$

By employing the new definition of the universal quantifier (formula 6.92), we can achieve the semantics of the first sentence in (113-b) as follows:

$$
\begin{aligned}
[\![\text{(113-b)-1}]\!]_{DN\text{-}TTDL} = &\ \overline{\overline{[\![not]\!]([\![fail]\!]([\![every]\!][\![student]\!]))}} \\
&\twoheadrightarrow_{\beta} \langle \lambda e\phi.(\exists x.(\mathbf{student}\ x \wedge \mathbf{pass}\ x) \wedge \phi e), \\
&\qquad\qquad \lambda e\phi.(\neg(\exists x.(\mathbf{student}\ x \wedge \mathbf{pass}\ x)) \wedge \phi e)\rangle
\end{aligned}
$$

After processing sentence (113-b)-1, the left context (in both projections) will not be updated. There is no need to conduct the discourse incrementation any more. Hence the infelicitous anaphora in (113-b) can be successfully blocked in DN-TTDL.

Chapter 7

Modality

Contents

The goal of this chapter is to investigate modality in natural languages, especially epistemic modality, from the perspective of formal semantics. There are mainly two problems we shall tackle:

1. What is the accessibility of discourse referents within modal context?

2. How modal utterances are semantically interpreted with respect to preceding contexts?

These two problems are not unrelated with each other. The accessibility of anaphors, especially for inter-sentential ones, is largely dependent on the way that subsequent sentences are interpreted. So they will mainly be discussed at the same time.

In section 3.1.3, we have already introduced some fundamental notions of possible world semantics. In the current chapter, we will first discuss modality in more detail from the linguistic perspective. Then we will present the theory of modality developed by Angelika Kratzer Kratzer (1977, 1981, 1986, 1991). Meanwhile, we will review two previous works on the problem of modal subordination Asher and Pogodalla (2011a);

Roberts (1989), they have already been mentioned in section 5.3.5. After that, we shall propose an adaptation of TTDL, which treats the modal subordination in the traditional montagovian style. A formal link between the new adaptation and TTDL will be established. Finally, we close this chapter by merging the two adaptations proposed in this thesis.

7.1 Modality in Natural Language

As discussed in section 3.1.3, modality is a semantic notion which is concerned with possibility and necessity. In linguistics, modality enables people to talk about things beyond the actual here and now von Fintel (2006). It is reflected on the set of phenomena where notions such as belief, attitude and obligation are attached to natural language sentences. Modality, which has been pervasively attested across almost all languages, can be established by a wide range of grammatical categories and constructions. Take English for example, there are modal auxiliaries (e.g., *must*, *may*, *should*, *might*), modal adjective and adverbs (e.g., *it is possible ...*, *possibly*, *necessarily*, *probably*), conditionals (e.g., *if ... then ...*), propositional attitude verbs (e.g., *believe*, *know*, *hope*), etc. For the sake of simplicity, we only consider the first two categories, namely modal auxiliaries and modal adverbs, in this thesis.

One aspect of the semantics of modality is **modal force**, namely the strength of a modal, i.e., possibility and necessity. For the contrast of the two, see example (61). Remind that in section 3.1.3, we have symbolized possibility with $\Diamond$, necessity with $\Box$. Both operators are treated as quantifiers ranging over possible worlds: $\Diamond$ as existential, $\Box$ as universal. Because of that, possibility and necessity are also called **existential force** and **universal force** (respectively). The force of a modal expression is inherently contained in its lexical meaning. For instance, modals such as *may*, *might* and *could* always denote a possibility; while modals such as *must*, *should* and *would* always denote a necessity one.

Another aspect on the semantics of modality is **modal flavor**, it indicates the particular sort of premise information, e.g., epistemic, deontic, etc., with respect to which a modal is interpreted. This notion is motivated by the fact that it is insufficient to interpret modal expressions only relative their modal forces. According to this, modalities can be classified into different sub-types. Let's take the following sentences for example, where the modal is considered to be ambiguous:

(114) a. All Maori children <u>must</u> learn the names of their ancestors.
b. The ancestors of the Maoris <u>must</u> have arrived from Tahiti. Kratzer (1977)

Both (114-a) and (114-b) contain the same modal *must*, so each of them expresses a universal force. However, the meaning of *must* varies from one sentence to another. For instance, in (114-a), the modal *must* refers to an obligation or a duty that the Maori children should obey or fulfill, it is called a **deontic** modality; in (114-b), the same modal denotes some knowledge or belief, it is called an **epistemic** modality. This distinction can be revealed in an explicit way by paraphrasing (114) as follows, where an *in view of ...* adverbial phrase is added at the beginning of each sentence:

(115) a. In view of what their tribal duties are, the Maori children <u>must</u> learn the names of their ancestors.

b. In view of what is known, the ancestors of the Maoris must have arrived from Tahiti. Kratzer (1977)

The modal *must* in (114-a) means "necessary in view of what their tribal duties are"; while *must* in (114-b) means "necessary in view of what is known". A similar contrast can be found in the following examples:

(116) a. According to his dating coach, John must dance at parties.
b. Since John hangs out with Linda at parties, he must dance at parties. Starr (2012)

By barely looking at the modalized sentence *John/he must dance at parties*, which is shared by both discourses in (116), we are not able to tell whether it refers to an obligation (deontic), or a piece of knowledge (epistemic), or maybe something else. However, with the help of the prefixed adverbial phrases in (116), we can unambiguously determine that the shared modalized sentence expresses a deontic modality in (116-a), while it expresses an epistemic one in (116-a).

Actually, besides the deontic and epistemic modality as we have shown in the above examples, there are also other types of modality that a modal expression can express, such as bouletic (wishes or desires), teleological (goals), circumstantial (circumstances), etc., all of which are called the flavor of a modal[1]. For instance, all the following examples involve the same modal expression *have to*, which denotes different modalities:

(117) a. It has to be raining. [after observing people coming inside with wet umbrellas; epistemic modality]
b. Visitors have to leave by six pm. [hospital regulations; deontic]
c. You have to go to bed in ten minutes. [stern father; bouletic]
d. I have to sneeze. [given the current state of one's nose; circumstantial]
e. To get home in time, you have to take a taxi. [telelological] von Fintel (2006)

For more examples, please refer to Kratzer (1977); Portner (2009). Different from the modal force, which solely comes from the lexical meaning of a modal, the modal flavor depends on the specific situation where the modal is applied. Sometimes, it is given by linguistic means, where there are noticeable indicators such as the adverbial phrases *in view of ...* and *according to ...* in (115) and (116); most of the time however, no indicators are explicitly presented, then the readers have to resolve the most appropriate flavor based on clues from the context of use, for instance, as in (114) and (117).

In order to interpret modal expressions in formal systems such as MPL, we need to correctly handle both semantic aspects. First of all, the treatment of modal force is relatively straightforward. We employ the two modal operators for modeling the two forces: $\Diamond$ corresponds to possibility, $\Box$ corresponds to necessity. Besides, the modal flavor, which is a specification on the sort of modality, needs to be modeled as well. In standard modal logic, what we can do is to assign each different modal a different set of possible worlds which it quantifies over, namely, to associate each modal a corresponding accessibility relation. For instance, from examples (115) and (116), we know that the modal *must* can be interpreted with respect to either an epistemic or a deontic flavor. Then we propose two accessibility relations: $\mathbf{R_{epi}}$ and $\mathbf{R_{deo}}$. The former is assigned to

[1]The names of these different flavors may vary from author to author.

the epistemic *must*, the latter to the deontic *must*. Assume w_1 and w_2 are two possible worlds, now both $\mathbf{R_{epi}}(w_1, w_2)$ and $\mathbf{R_{deo}}(w_1, w_2)$ denote that w_2 is accessible from w_1. The former means that the relation is epistemic, namely w_2 is a world such that it is compatible with all knowledges or beliefs at w_1. The latter means the relation is deontic, namely w_2 is such a world that it satisfies all obligations or rules at w_1.

With the above configurations, the epistemic *must* and the deontic *must* are semantically represented as $\Box_{\mathbf{epi}}$ and $\Box_{\mathbf{deo}}$, respectively. Assume $M = \langle F, I\rangle$ is a Kripke model, where $F = \langle W, \mathbf{R_{epi}}, \mathbf{R_{deo}}\rangle$ is a frame, and I is the interpretation function. The notion of frame as previously defined in definition 3.1.20 is enriched by including various accessibility relations. Let $w \in W$ is a possible world, then the two necessity modal operators will obtain the following semantic interpretations:

- $[\![\Box_{\mathbf{epi}}\phi]\!]^{M,w}_{MPL} = 1$ iff $\forall w' \in W$: if $\mathbf{R_{epi}}(w, w')$ then $[\![\phi]\!]^{M,w'}_{MPL} = 1$;
- $[\![\Box_{\mathbf{deo}}\phi]\!]^{M,w}_{MPL} = 1$ iff $\forall w' \in W$: if $\mathbf{R_{deo}}(w, w')$ then $[\![\phi]\!]^{M,w'}_{MPL} = 1$.

In practice, if ϕ is a proposition, $\Box_{\mathbf{epi}}\phi$ usually stands for *it is known that* ϕ, while $\Box_{\mathbf{deo}}\phi$ often stands for *it is required/commanded that* ϕ. Likewise, the corresponding possibility operators, namely the epistemic possibility and the deontic possibility are defined in an analogous manner:

- $[\![\Diamond_{\mathbf{epi}}\phi]\!]^{M,w}_{MPL} = 1$ iff $\exists w' \in W$: $\mathbf{R_{epi}}(w, w')$ and $[\![\phi]\!]^{M,w'}_{MPL} = 1$;
- $[\![\Diamond_{\mathbf{deo}}\phi]\!]^{M,w}_{MPL} = 1$ iff $\exists w' \in W$: $\mathbf{R_{deo}}(w, w')$ and $[\![\phi]\!]^{M,w'}_{MPL} = 1$.

This strategy can also be readily applied to examples where multiple modalities are involved in a single discourse. By way of illustration, let's look at the following example:

(118) John <u>must</u> donate to charity, and he <u>might</u> do so. Starr (2012)

The modality in the first sentence of (118) is deontic, the one in the second is epistemic. Then with the above proposed frame $F = \langle W, \mathbf{R_{epi}}, \mathbf{R_{deo}}\rangle$, we can translate (118) into $\Box_{\mathbf{deo}}\phi \wedge \Diamond_{\mathbf{epi}}\phi$, where ϕ is the unmodalized proposition expressed by *John donates to charity*.

Basically, the introduction of different accessibility relations can successfully resolve the ambiguity among modal expressions. They keep the same surface form but differ in their flavors. However, from a generalization point of view, the above solution is not satisfactory enough. In the next section, we will present Kratzer's theory on modality, which aims to give a **unified** analysis on different types of modality (e.g., epistemic, deontic, bouletic, etc.).

Before closing this section, we would like to remark that for a modal expression, there are in fact only a limited number of flavors which it can be associated with. For instance, *might* is almost exclusively used to express epistemic modality; while *must* can express both epistemic and deontic modality, but not bouletic. In addition, among all the possibilities, a modal also has its own preference on the kind of modality it expresses. And the situation becomes even more complicated once we look into modality cross-linguistically Palmer (2001). As a consequence, one interesting research topic is to establish a taxonomy, containing the relationships between modals and their flavors. However, this is outside the domain of our work, see Hacquard (2006); Portner (2009) for more information.

7.2 Kratzer's Theory of Modality

In this section, we will first introduce one classical theory of modals, which has been developed by Angelika Kratzer Kratzer (1977, 1981, 1986, 1991). The reason for presenting this theory is twofold. On the one hand, it is the most studied work on this topic; on the other hand, it has been serving as the foundation for a large number of subsequent works on modality. At the end of the section, we will briefly examine two developments from Kratzer's theory, which are specifically proposed to deal with anaphoras under modality.

7.2.1 Conversational Background

In the previous section, we have presented a solution to model modal expressions, in which modals of different flavors are associated with different accessibility relations. That is to say, we assume that modals are polysemous. For instance, the epistemic *must* and the deontic *must* are assigned different semantic representations. Since a modal can possibly be used in many different ways, see example (117), it is impractical to assign a complete range of semantic entries for each expression. Moreover, occurrences of the same modal apparently share a huge shade of meaning, in particular, the modal force aspect. Hence it would be more elegant and reasonable if there exists a uniform treatment. This contributes one of the most essential motivations of Kratzer's theory: to tackle the problem of lexical ambiguity among modals.

In her theory, Kratzer proposes that modals are context-dependent, rather than ambiguous between various flavors. As we mentioned before, the *must* in examples (114-a) and (114-b) means "necessary in view of what their tribal duties are" and "necessary in view of what is known", respectively. However, if we understood modal in this way, the adverbial phrases as in examples (115) and (116) would be redundant, since modals carry all the necessary information, while this is not the case. So Kratzer's strategy is to make a clear-cut division on the two aspects of modal semantics that we presented above, that is to say, the force of a modal is all its meaning, as to the flavor, which is not part of the meaning of a modal any more, is fixed by the context. We will explain this in more detail below.

A modal sentence, as far as Kratzer concerns, is interpreted in a modular way such that it consists of three parts: a **neutral** modal operator, a background context, and a proposition under discussion. The last parameter is relatively easy to understand, it is the proposition governed by the corresponding modal operator. For instance, in example (61), the proposition we are discussing is the one expressed by *Sandy is at home*, in example (116), it is the one expressed by *John/he dances at parties*. Then, as to the modal operator, which is uniquely determined by the modal expression, is neutral in the sense that it only denotes the modal force, namely, whether it is existential or universal. For instance, Kratzer does not distinguish $\Diamond_{\mathbf{epi}}$ from $\Diamond_{\mathbf{deo}}$, there is only one possibility modal operator $\Diamond$; similarly, there is also only one necessity modal operator $\Box$. The background context is the foundation for the uniform interpretation of various types of modality. It indicates the particular flavor that a modal is applied to. In other words, it restricts the domain of worlds which modal operators quantify over. As we mentioned above, the background can be indicated by explicit markers such as adverbial phrases, but more typically, it is supplied by the context of use, hence it should be considered as a parameter of the context, rather than the meaning of the modal.

Before diving into the technical details of Kratzer's theory, let's look back into possible world semantics and revisit some basic notions such as interpretations and truth. As presented in section 3.1.3, with respect to a Kripke model, the interpretation of a modalized proposition at a possible world is a truth value, see definition 3.1.23. Another way to grasp the definition is to view intentional propositions as sets of possible worlds, or equivalently, sets of possibilities.

> *Informational content can be understood in terms of possibilities. The information admits some possibilities and excludes others. Its content is given by the division of possibilities into the admitted ones and the excluded ones. The information is that some one of these possibilities is realized, not any of those.* Lewis (1983)

If we take "possibilities" in the above citation as "possible worlds", then definition 3.1.23 can accordingly be paraphrased into the following form:

Definition 7.2.1. Let $M = \langle F, I \rangle$ be a Kripke model, where $F = \langle W, \mathbf{R} \rangle$ is a frame, I is an interpretation function, $w \in W$ a possible world, $\phi \in \mathbb{F}$ an MPL formula. The interpretation of ϕ under M, in notation $[\![\phi]\!]^M_{MPL}$, is defined inductively as follows:

1. $[\![p]\!]^M_{MPL} = \{w \mid I_w(p) = 1\}$, if $p \in \mathcal{A}$;
2. $[\![\neg\phi]\!]^M_{MPL} = W - [\![\phi]\!]^M_{MPL}$;
3. $[\![\phi \wedge \psi]\!]^M_{MPL} = [\![\phi]\!]^M_{MPL} \cap [\![\psi]\!]^M_{MPL}$;
4. $[\![\Diamond\phi]\!]^M_{MPL} = \{w \mid \exists w' \in W : \mathbf{R}(w, w') \text{ and } w' \in [\![\phi]\!]^M_{MPL}\}$;
5. $[\![\Box\phi]\!]^M_{MPL} = \{w \mid \forall w' \in W : \text{if } \mathbf{R}(w, w') \text{ then } w' \in [\![\phi]\!]^M_{MPL}\}$.

The notion of truth and satisfaction are modified in a corresponding way, see the contrast between definitions 3.1.24 and 7.2.2:

Definition 7.2.2. Let $M = \langle F, I \rangle$ be a Kripke model, where $F = \langle W, \mathbf{R} \rangle$ is a frame, I is an interpretation function, $w \in W$ a possible world, $\phi \in \mathbb{F}$ an MPL formula. We say that ϕ is **true** at w under M, or equivalently, M **satisfies** ϕ at w, in notation $M, w \models_{MPL} \phi$, iff $w \in [\![\phi]\!]^M_{MPL}$.

As shown in definitions 7.2.1 and 7.2.2, we are able to define the semantics of propositions in terms of possible worlds. For instance, assume ϕ is a MPL formula, its interpretation $[\![\phi]\!]^M_{MPL}$ is identified with the set of worlds such that ϕ is true in each of its members. Let W be the set of all possible worlds, the interpretation is a subset of W, namely $[\![\phi]\!]^M_{MPL} \subseteq W$. Likewise, assume Λ is a set of (modal) propositions, its interpretation $[\![\Lambda]\!]^M_{MPL}$ is thus the set containing interpretations of all its elements, which is a subset of the power set of W, namely $[\![\Lambda]\!]^M_{MPL} \subseteq \mathcal{P}(W)$, where $\mathcal{P}$ is the function returning the power set of an input set.

Following Kratzer (1981), we will define the following three logical concepts, which are concerned with relations between (modal) propositions and sets of propositions.

Definition 7.2.3. Let $\phi \in \mathbb{F}$ be a formula, $\Lambda \subseteq \mathbb{F}$ a set of formulas, W the set of all possible worlds.

- Consequence/Entailment

  ϕ is a **consequence** of Λ, or equivalently, ϕ **follows from** Λ, or equivalently, Λ **entails** ϕ, iff for every possible world $w \in W$, if all propositions of Λ are true at w, ϕ is also true at w.

- Consistency

  Λ is **consistent** iff there is a possible world $w \in W$ such that all propositions of Λ are true at w.

- Compatibility

  ϕ is **compatible** with Λ iff the union of $\{\phi\}$ and Λ, namely $\{\phi\} \cup \Lambda$, is consistent.

Now let's go back to Kratzer's theory. The background information provided by the context plays a key role in a unified analysis of various modalities. In order to model it, Kratzer proposes the notion of **conversational background**. Generally speaking, a conversational background stands for the entity denoted by adverbial phrases such as *in view of* and *according to*. It provides a particular premise, with respect to which a modal sentence will be evaluated. This premise can be formalized as a set of propositions (knowledges or obligations), and it is sensitive to the world. For instance, take the epistemic conversational background *in view of what is known* in (114-b), it gives a set of propositions known at the utterance world, which are different from world to world (people may know different things in different world). Analogously, take the deontic conversational background *according to his dating coach* in (116-a) for example, it supplies a set of commands from the coach that John should follow, which also differ from world to world. We formalize conversational background as follows:

Definition 7.2.4. A conversational background is a function from possible worlds to sets of (modal) propositions.

For instance, assume f is a conversational background, $w \in W$ a possible world, then $f(w) = \{\phi_1, \phi_2, ...\}$ is a set of propositions which contributes the background information at w. In other words, all propositions in $f(w)$, namely ϕ_1, ϕ_2, ..., are necessarily true[2] at w. Then the semantics of formulas in modal logic, namely definition 7.2.1, can be characterized by combining definition 7.2.3 and the above formation of conversational background, as follows.

Definition 7.2.5. Let W be a set of possible worlds, I an interpretation function, $\phi \in \mathbb{F}$ a MPL formula, f a conversational background. The interpretation of ϕ with respect to W, I, and f, in notation $[\![\phi]\!]^{W,I,f}_{MPL}$, is defined inductively as follows:

1. $[\![p]\!]^{W,I,f}_{MPL} = \{w \mid I_w(p) = 1\}$, if $p \in \mathcal{A}$;
2. $[\![\neg\phi]\!]^{W,I,f}_{MPL} = W - [\![\phi]\!]^{W,I,f}_{MPL}$;
3. $[\![\phi \wedge \psi]\!]^{W,I,f}_{MPL} = [\![\phi]\!]^{W,I,f}_{MPL} \cap [\![\psi]\!]^{W,I,f}_{MPL}$;
4. $[\![\Diamond\phi]\!]^{W,I,f}_{MPL} = \{w \mid w \text{ is compatible with } f(w)\}$;
5. $[\![\Box\phi]\!]^{W,I,f}_{MPL} = \{w \mid w \text{ is a consequence of } f(w)\}$.

[2]Whether $\phi_1, \phi_2, ...$ are knowledges, or obligations, or goals, depends on the particular type of the conversational background f.

As we can see, 7.2.5 is an alternative definition of the semantics of modality. Different from definition 7.2.1, it is achieved in terms of conversational background, rather than accessibility relation.

7.2.2 Accessibility Relation & Conversational Background

In the previous subsection, we provided a brief introduction to conversational background, including its motivation and definition. And we have shown that it can be used in place of the accessibility relation for defining the semantics of modals. In this subsection, we further investigate conversational background, in particular, its correspondence with the accessibility relation.

As introduced in the previous subsection, a conversational background f provides a set of propositions for a given world w. In fact, there is a further transformation we can make on $f(w)$, which will largely simplify the computation: to turn $f(w)$ into a single proposition. The idea is as follows. Assume $f(w) = \{\phi_1, \phi_2, ...\}$, where ϕ_1, ϕ_2, ..., are propositions. Then if there is a world u such that all propositions ϕ_1, ϕ_2, ..., are true at it, the conjunction $\phi_1 \wedge \phi_2 \wedge ...$ which is made up of all propositions in the set, ought to be true at u as well. So $f(w)$ and $\phi_1 \wedge \phi_2 \wedge ...$ can be used in the same way with respect to the logical relations we are interested in: in definition 7.2.3, only the worlds where **all propositions** of a set are true are concerned with. We use $\wedge f(w)$ as a notation for such a conjunction, namely:

$$\wedge f(w) = \phi_1 \wedge \phi_2 \wedge ... \tag{7.1}$$

Like usual propositions, the interpretation of the conjunction $\wedge f(w)$ is also a set of possible world, which is in fact the **intersection** of the interpretations from all the conjuncts. In a more succinct way:

$$[\![\wedge f(w)]\!]^M_{MPL} = [\![\phi_1]\!]^M_{MPL} \cap [\![\phi_2]\!]^M_{MPL} \cap ... \tag{7.2}$$

or

$$[\![\wedge f(w)]\!]^M_{MPL} = \{u \mid \forall \phi.(\phi \in f(w) \rightarrow u \in [\![\phi]\!]^M_{MPL})\} \tag{7.3}$$

Note that in the original reference Kratzer (1981), Kratzer uses the notation $\cap f(w)$ to refer to the intersection set $[\![\wedge f(w)]\!]^M_{MPL}$. We will stick to our notation in this thesis for reasons of uniformity.

In modal logic, the accessibility relation is given as part of the model. In the following, we shall see how it can be defined in terms of conversational background.

Definition 7.2.6. Let f be a conversational background, $w, u \in W$ possible worlds. We say u is **accessible** from w with respect to f, in notation $\mathbf{R}_f(w, u)$, iff $u \in [\![\wedge f(w)]\!]^M_{MPL}$, or equivalently, iff all propositions of $f(w)$ are true at u. Hence the set of worlds which are **accessible** from w with respect to f is $[\![\wedge f(w)]\!]^M_{MPL}$.

This means, we can specify the meaning of modals in terms of the accessibility relation determined by the conversational background under consideration. Then once again, we re-define the semantics of modal logic, in particular the ones for modal operators. Rules 4 and 5 in definition 7.2.5 can be paraphrased with the following rules:

4. $[\![\Diamond\phi]\!]^{W,I,f}_{MPL} = \{w \mid \exists w' \in W : \mathbf{R}_f(w, w') \text{ and } w' \in [\![\phi]\!]^{W,I,f}_{MPL}\}$;

5. $[\![\Box\phi]\!]^{W,I,f}_{MPL} = \{w \mid \forall w' \in W : \text{if } \mathbf{R}_f(w,w') \text{ then } w' \in [\![\phi]\!]^{W,I,f}_{MPL}\}$.

or equivalently:

4. $[\![\Diamond\phi]\!]^{W,I,f}_{MPL} = \{w \mid \exists w' \in W : w' \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL} \text{ and } w' \in [\![\phi]\!]^{W,I,f}_{MPL}\}$;

5. $[\![\Box\phi]\!]^{W,I,f}_{MPL} = \{w \mid \forall w' \in W : \text{if } w' \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL} \text{ then } w' \in [\![\phi]\!]^{W,I,f}_{MPL}\}$.

So far as we have shown, the notion of conversational background is a parameter representing various ways that a modal may depend on the context. In addition, we can use it to investigate properties of particular readings of modals, i.e., putting a host of different conditions on conversational backgrounds. That is to say, we can classify conversational backgrounds into different categories based on the properties they may have Kaufmann et al. (2006).

Definition 7.2.7. Let W be a set of possible worlds, I an interpretation function, f a conversational background.

- f is **consistent** iff $\forall w \in W : [\![\wedge f(w)]\!]^{W,I,f}_{MPL} \neq \emptyset$;
- f is **realistic** iff $\forall w \in W : w \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL}$;
- f is **totally realistic** iff $\forall w \in W : [\![\wedge f(w)]\!]^{W,I,f}_{MPL} = \{w\}$;
- f is **positively introspective** iff $\forall w, w' \in W$: if $w' \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ then $[\![\wedge f(w')]\!]^{W,I,f}_{MPL} \subseteq [\![\wedge f(w)]\!]^{W,I,f}_{MPL}$.

Some remarks on the above properties. First of all, let's look at consistency, which is often taken for granted for all conversational backgrounds in linguistic theories. If a conversational background is inconsistent, it means there exists some possible world which does not have any accessible world. Then a possibility modality will always be false while a necessity modality will always be true in it, because the modal quantifiers simply range over an empty set of worlds, which is undesirable.

As to realism, a conversational background is realistic if it assigns to every possible world a set of propositions that are not only necessarily true at it, but also are actually true at it. It is an essential condition which distinguishes epistemic modality from other types of modality, such as the deontic one: an epistemic modality is realistic because established knowledge must be true propositions, e.g., if something is known, then it must be true; while a deontic modality is not realistic because laws or obligations can be broken, e.g., if some regulations should be obeyed, it might still be the case that there are people who fail in obeying them. As a result, in epistemic modality, one will find that a modalized proposition of necessity, e.g., (119-a), is stronger than an unmodalized proposition, e.g., (119-b), which is stronger than a modalized proposition of possibility, e.g., (119-c).

(119) a. Jockl must have been the murderer (in view of what we know).
b. Jockl is the murderer.
c. Jockl might have been the murderer (in view of what we know). Kratzer (1991)

Total realism is a special case of realism, where the conversational background assigns to each world only one accessible world, which is itself.

Finally, we will look at positive introspection. It is also a property typically found in epistemic modality, with the assumption that if somebody knows something, he knows that he knows it. But there is still no consensus on whether positive introspection is a condition on epistemic modality or not. Some researchers propose that it is more accurate to say if somebody **knows** something, he **believes** that he knows it von Fintel and Heim (2011). However, that will lead us into the philosophical discussion on the difference between belief and knowledge, which is outside the domain of this thesis. For more information, one may refer to Williamson (2002).

In what follows, we try to draw a correspondence between the properties of accessibility relations and the conditions on conversational background. Hence definition 3.1.26 can be recast in terms of conversational background as follows:

Definition 7.2.8. Let W be a set of possible worlds, I an interpretation function, f a conversational background, $\mathbf{R}_f$ the accessibility relation with respect to f.

- $\mathbf{R}_f$ is **serial** iff f is consistent;
- $\mathbf{R}_f$ is **reflexive** iff f is realistic;
- $\mathbf{R}_f$ is **transitive** iff f is positively retrospective;
- $\mathbf{R}_f$ is **identical** iff f is totally realistic.

Below, we shall briefly explain each item in definition 7.2.8.

- **Seriality - Consistency**
  - We say $\mathbf{R}_f$ is **serial** iff for any possible world $w \in W$, there is a possible world $w' \in W$ such that w' is accessible from w, namely $\mathbf{R}_f(w, w')$;
  - We say f is **consistent** iff for any possible world $w \in W$, $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ is not an empty set.

  As discussed before, $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ is the set of possible worlds that are accessible from w. The fact that $[\![\wedge f(w')]\!]^{W,I,f}_{MPL}$ is a nonempty set indicates there is at least some possible world that is accessible from w (corresponding to the notion of seriality).

- **Reflexivity - Realism**
  - We say $\mathbf{R}_f$ is **reflexive** iff for any possible world $w \in W$, it is the case that w is accessible from itself, namely $\mathbf{R}_f(w, w)$ always holds;
  - We say f is **realistic** iff for any possible world $w \in W$, it is the case that $w \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL}$.

  As discussed before, $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ is the set of possible worlds that are accessible from w. The fact that w is an element of $[\![\wedge f(w)]\!]^{M}_{MPL}$ indicates w is always accessible from itself (corresponding to the notion of reflexivity).

- **Identity - Total Realism**

– We say $\mathbf{R}_f$ is **identical** iff for any possible worlds $w, w' \in W$, if w' is accessible from w, namely $\mathbf{R}_f(w, w')$ holds, then w' is identical with w. Thus every possible world is **always** and **only** accessible from itself;

– We say f is **totally realistic** iff for any possible world $w \in W$, it is the case that the set $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ equals the set $\{w\}$.

As discussed before, $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ is the set of possible worlds that are accessible from w. The fact that $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ is equivalent to $\{w\}$ indicates w is always and only accessible from itself (corresponding to the notion of identity). As one might have noticed, identity is a stricter case of reflexivity such that the only accessibility relation is self-accessibility.

- **Transitivity - Positive Introspection**

– We say $\mathbf{R}_f$ is **transitive** iff for any possible worlds $w, w', w'' \in W$, if w' is accessible from w, at the same time w'' is accessible from w', namely $\mathbf{R}_f(w, w')$ and $\mathbf{R}_f(w', w'')$ both hold, then w'' is accessible from w, namely $\mathbf{R}_f(w, w'')$ also holds;

– We say f is **positively introspective** iff

$$\forall w, w' \in W : (w' \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL}) \to ([\![\wedge f(w')]\!]^{W,I,f}_{MPL} \subseteq [\![\wedge f(w)]\!]^{W,I,f}_{MPL})$$

As discussed before, $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ and $[\![\wedge f(w')]\!]^{W,I,f}_{MPL}$ are respectively the set of possible worlds that are accessible from w and w'. The fact that $[\![\wedge f(w')]\!]^{W,I,f}_{MPL}$ is a subset of $[\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ when $w' \in [\![\wedge f(w)]\!]^{W,I,f}_{MPL}$ indicates the possible worlds that are accessible from w' are also accessible from w if w' is accessible from w (corresponding to the notion of transitivity).

As a summary, Kratzer's theory as we have presented so far, is a contextualized version of the standard modal logic such as MPL, it is called the **relative modality**. Different readings of a modal expression are reduced to the specification of a single modal force, together with various context-dependent conversational backgrounds. Hence we are able to interpret modals in a uniquely unambiguous way. We have also shown that correlated notions, such as the accessibility relation, together with its properties, can be recast in terms of conversational background correspondingly.

However, in natural language, modality is not a dichotomy of plain possibility and necessity, it is a graded concept and can be compared between one another, for instance, there are other modal forces such as *good possibility*, *slight possibility*, etc. Here are some specific linguistic examples:

(120) a. It is <u>barely possible</u> to climb Mount Everest without oxygen.
b. It is <u>easily possible</u> to climb Mount Toby.
c. They are <u>more likely</u> to climb the West Ridge than the Southeast Face.
d. It would be <u>more desirable</u> to climb the West Ridge by the Direct Route. Kratzer (1991)

In a relative modality, possibility is defined in terms of compatibility, see definition 7.2.5, which is an absolute concept. However, in order to account for example (120), we need to incorporate a new version of compatibility which can be tuned in a scalable fashion. Hence, Kratzer proposes that modal expressions should be interpreted with respect to two conversational backgrounds: one, as we introduced above, is called the **modal base**, it provides the background information, namely a set of accessible worlds; the other is called the **ordering source**, which imposes an ordering on the accessible worlds, i.e., some worlds are more accessible than others.

This machinery will not only resolve the problem of graded modality, but also cope with a series of other modality-related problems Kratzer (1991); Schoubye (2011), such as the inconsistencies, conditionals, etc. In this thesis, we will sidestep the ordering source, and only consider the modal base usage of conversational background. Interested readers may refer back to the original reference for more information Kratzer (1981).

7.2.3 Previous Works on Modal Subordination

In the field of discourse semantics, modal subordination has constantly attracted researchers' attention Sells (1985); Roberts (1987, 1989); Van Rooij (2005). As already discussed in section 5.3.5, modal subordination is the phenomenon whereby indefinite NPs introduced in a modal context can serve as antecedent for anaphoric expressions which occur in some subsequent modalized sentences. For instance, see example (12) and (13), which are repeated as follows:

(12) If John bought a book$_i$, he'll be home reading it$_i$ by now. It$_i$'ll be a murder mystery. Roberts (1989)

(13) A thief$_i$ might break into the house. He$_i$ would take the silver. Roberts (1989)

In either (12) or (13), the anaphor (i.e., *it* or *he*) does not occur within the scope of the modality in the first sentence, but is otherwise in a separate modal utterance. These particular anaphoric references, which are considered impossible in view of standard dynamic frameworks (e.g., DRT, DPL, and TTDL), turn out to be fairly acceptable. In order to account for examples as such, Craige Roberts develops an approach which combines Kratzer's theory of modality, as introduced in section 7.2, with dynamic semantics, among which she chooses DRT Roberts (1987, 1989). Technically, Roberts adds the two modal operators $\Box$ and $\Diamond$ to the syntax of the standard DRT, resulting in the following two new rules for DRS condition:

1. $K_1 \Diamond K_2$ is a DRS-condition, if K_1 and K_2 are DRSs;
2. $K_1 \Box K_2$ is a DRS-condition, if K_1 and K_2 are DRSs.

In both $K_1 \Diamond K_2$ and $K_1 \Box K_2$, K_2 is at a level subordinate to K_1. Thus discourse referents of K_1 are accessible from K_2. By convention, the symbol $\Diamond$ and $\Box$ denote the possibility and necessity modal operator, respectively. Typically, they are the semantic translation of the modals *might* and *would*. The DRS on their right hand side stands for the proposition that is within the scope of the modal operator. The DRS on the left hand side is the premise information for interpreting the modality, which restricts the domain of quantification. It functions exactly the same as the set of propositions provided by the conversational background in Kratzer's theory. Roughly speaking, from the semantic point of view, $K_1 \Diamond K_2$ is true iff there exists some possible world where K_1 is satisfied, and K_2 is satisfied in that world as well; $K_1 \Box K_2$ is true iff for every possible world, if K_1 is satisfied in it, then K_2 is also satisfied in it. For a more detailed description on the formal framework, please refer to the original reference Roberts (1989).

Based on the above syntactic rules, Roberts proposes the **accommodation** approach[3], where the contextually provided hypothetical common ground is accommodated as the antecedent of the modally subordinated clause. Consequently, (12) is paraphrased as "if John bought a book, he'll be home reading it by now; and if John bought a book, it'll be a murder mystery", namely the premise of the first sentence is copied into the second part of the discourse. Thus, we will obtain the DRS which corresponds to (12) as follows (where the linking conditions are added):

$$K_{(12)}: \quad \begin{array}{|c|}\hline \\ \hline \begin{array}{|c|}\hline x \\ \hline \textbf{book}\ x \\ \textbf{buy john}\ x \\ \hline \end{array} \ \Box \ \begin{array}{|c|}\hline y,\ z \\ \hline \textbf{read}\ y\ z \\ y \doteq \textbf{john} \\ z \doteq x \\ \hline \end{array} \\ \begin{array}{|c|}\hline x \\ \hline \textbf{book}\ x \\ \textbf{buy john}\ x \\ \hline \end{array} \ \Box \ \begin{array}{|c|}\hline u \\ \hline \textbf{murder\_mystery}\ u \\ u \doteq x \\ \hline \end{array} \\ \hline \end{array}$$

As shown above, the anaphoric pronoun *it* in the second modal sentence can be successfully linked to the antecedent indefinite NP *a book*, which is hidden in standard DRT. Similarly, this machinery also works for examples where different types of modals are involved. Take example (13) for instance, by accommodating the sub-DRS of the preceding sentence as the hypothetical common ground of the second sentence, we paraphrase (13) as "it is possible that a thief will break into the house; in all the worlds where a thief breaks into the house, he undoubtedly takes the silver", and we can formally end up with a representation as follows:

$$K_{(13)}: \quad \begin{array}{|c|}\hline \\ \hline \Diamond \ \begin{array}{|c|}\hline x \\ \hline \textbf{thief}\ x \\ \textbf{break\_into}\ x \\ \hline \end{array} \\ \begin{array}{|c|}\hline x \\ \hline \textbf{thief}\ x \\ \textbf{break\_into}\ x \\ \hline \end{array} \ \Box \ \begin{array}{|c|}\hline y \\ \hline \textbf{take\_silver}\ y \\ y \doteq x \\ \hline \end{array} \\ \hline \end{array}$$

Again, since the referents in the left hand side DRS are accessible from the right hand side DRS, the anaphoric pronoun *he* in the second sentence can be resolved with x, which is introduced by the indefinite NP *a thief* from the former context. Please note that we consider the modality in (13) as epistemic, that is to say, the common ground information for the first modalized sentence may be contextually implied by phrases such as *in view of what is known*, and *given that the house has poor security*. Since the conversational background of an epistemic modality is realistic, the premise propositions (i.e., what is

[3] Another approach, which is called **insertion**, has also been proposed Roberts (1989). However, since the insertion approach is less general, we will not discuss it here.

known) are also true at the evaluation world, namely they are implicitly contained in the main DRS. That is why sub-DRS on the left hand side of $\diamond$, which ought to be included in the upper portion of $K_{(13)}$, can be omitted.

In a nutshell, we summarize Roberts's accommodation strategy as follows, assume there are three sentences S_1, S_2 and S_3. Then a discourse of the form "*If* S_1*, will* S_2*. Will* S_3*.*"[4] will be translated into the DRS $(K_{S_1} \Box K_{S_2}) \wedge (K_{S_1} \Box K_{S_3})$, and a discourse of the form "*Might* S_1*. Would* S_2*.*" will be translated into the DRS $\diamond K_{S_1} \wedge (K_{S_1} \Box K_{S_2})$ Kibble (1994). These two rules exactly correspond to the structures of examples (12) and (13), respectively.

Although Roberts's proposition manages to explain the accessibility between anaphor and its antecedent across modal contexts, it suffers from some criticisms. First of all, the truth conditions of the representation in Roberts's solution are questionable. For instance, let's take a look at the example from Asher and Pogodalla (2011a), which is in a similar form to example (13):

(121) A wolf$_i$ might walk in. It$_i$ would growl. Asher and Pogodalla (2011a)

Assume ϕ denotes the proposition of the non-modalized version of the first sentence: *a wolf walks in*, ψ denotes the non-modalized version of the second sentence: *it growls*. Since (121) is in the form "*Might* S_1*. Would* S_2*.*", following the accommodation approach of Roberts, it is mapped into the following representation in MPL:

$$\diamond\phi \wedge \Box(\phi \rightarrow \psi) \tag{7.4}$$

This looks exactly the same as $K_{(13)}$. However, a careful consideration might reveal that this solution is not satisfactory: by accommodating ϕ as the antecedent of the necessity modality, we enforce ψ to hold in all possible worlds where ϕ holds. That is to say, for any wolf that walks in, it would growl. But the following can obviously serve as a counter example:

(122) A wolf$_i$ might walk in. It$_i$ would growl. A second wolf$_j$ might then walk in, but it$_j$ wouldn't growl. Asher and Pogodalla (2011a)

No awkwardness is brought about by the second wolf in (122), which does not growl as indicated in the context. However, if we adopt the result from Roberts's accommodation approach, we will end up with a contradiction, because all the wolves which enter must growl according to DRS 7.4. As a consequence, a more appropriate representation of Example (121), suggested by Asher and Pogodalla (2011a), ought to be

$$\diamond(\phi \wedge \Box(\phi \rightarrow \psi)) \tag{7.5}$$

As can be inferred from formula 7.5, on the one hand, the background proposition for the second modalized sentence, which is the one expressed by *a wolf walks in*, namely ϕ, is accommodated; on the other hand, the second modality, namely the *would* claim, is embedded under the first modality, namely the *might* modality. This will yield the correct semantics for discourses of the form "*Might* S_1*. Would* S_2*.*", such as example (13) and (122).

In addition, there is another defect of Roberts's proposal. According to Roberts, a discourse referent introduced under modality may be accessed from subsequent modal

[4] Note that this is only a sequence structure, and not a "*if...then...else...*" one.

context. However, if any following utterance switches from nonfactual mood to factual mood, the accessibility will cease. This conclusion is supported by previous examples such as (104) and (105). However, it does not always seem to be the case:

(123) John might buy $\underline{\text{a house}}_i$. He earns enough to get a mortgage. He could rent $\underline{\text{it}}_i$ out for the Festival. Kibble (1994)

(124) $\underline{\text{A wolf}}_i$ might walk in. We would be safe because $\underline{\text{John}}_j$ has $\underline{\text{a gun}}_k$. $\underline{\text{He}}_j$ would use $\underline{\text{it}}_k$ to shoot $\underline{\text{it}}_i$. Stone (1999)

In example (123), a factual sentence is inserted in the middle of the modalized context. This does not affect the discourse referents, which are introduced in the scope of modality, to be accessed from subsequent modally subordinated clauses: the pronoun *it* in the last sentence of (123) is resolved with the indefinite *a house* in the first sentence. Thus, we can re-enter the modal context even after jumping out of it. As to example (124), it describes two situations: a hypothetical situation where a wolf walks in, and an actual situation where John has a gun. As revealed by the last sentence of the discourse, anaphors in a subsequent hypothetical scenario may refer to referents in both situations. However, this is impossible according to Roberts's theory: the referent corresponding to the gun should be inaccessible. That is because referents introduced in the actual situation will not be updated into the modal context.

In order to resolve the problems in Roberts's theory, Asher and Pogodalla (2011a) proposes a solution based on TTDL. The basic idea is to enrich the notion of left context with Kratzer's theory on modality. Firstly, Asher and Pogodalla (2011a) considers the left context as a record, consisting of a list of discourse referents $\mathsf{m\_ref}$, and the current modal base base, which contains the propositions that are necessarily true. If we use γ to denote the type of left contexts, γ' to denote the type of lists of variables, ι and o have their conventional meanings, then the left context is represented as follows:

$$\gamma = \{\mathsf{m\_ref} : \gamma'; \mathsf{base} : o\} \tag{7.6}$$

For instance, assume e is a left context, then $e.\mathsf{m\_ref}$ returns a set of entities, $e.\mathsf{base}$ returns a modal base. Note that the modal base is of type o because a previous transformation, which turns a set of propositions into a single conjunction, as shown in formula 7.1, is also conducted here. The notion of right context is the same as in TTDL, it is a continuation of the left context, so its type is $\gamma \to o$. Sentences and discourses are still interpreted with respect to both the left and right contexts, hence they are of type $\Omega = \gamma \to (\gamma \to o) \to o$, see formulas 4.29 and 4.30. In addition, to save space in the lexical entries, the operators in the framework of Asher and Pogodalla (2011a) can be defined in terms of the constants in the previous signature Σ_{TTDL} (definition 4.4.1). We use the subscript AP to denote "Asher-Pogodalla" in the following formulas.

$$::_{AP} \triangleq \lambda xe.\{\mathsf{m\_ref} = x :: (e.\mathsf{m\_ref}); \mathsf{base} = e.\mathsf{base}\} \tag{7.7}$$

$$\mathsf{sel}_{AP} \triangleq \lambda e.\mathsf{sel}(e.\mathsf{m\_ref}) \tag{7.8}$$

$$\wedge_{AP} \triangleq \lambda Ae.\{\mathsf{m\_ref} = e.\mathsf{m\_ref}; \mathsf{base} = A \wedge e.\mathsf{base}\} \tag{7.9}$$

$$\mathsf{nil}_{AP} \triangleq \{\mathsf{m\_ref} = \mathsf{nil}; \mathsf{base} = \top\} \tag{7.10}$$

Here are some explanations on the above definitions. The novel constructor $::_{AP}$ is used to insert a variable into a left context, more specifically, into the referent list contained in a context. The novel choice operator sel_{AP} is used to pick up a variable from the left context, more specifically, from the referent list contained in a context. The novel conjunction $\wedge_{AP}$ is used to interpolate a proposition into a left context, more specifically, into the modal based contained in a context. Finally, the new empty left context nil_{AP} denotes a void record: its referent list is empty, its base is a tautology. Remind that in the original TTDL, a left context is a list of individuals, so the empty context is simply represented by the constant nil.

The following lexical entries[5] are provided for computing the semantics of example (121), where $[\![\ ]\!]_{AP}$ denotes the function which assigns representations to natural language expressions in the framework of Asher and Pogodalla (2011a):

$$
\begin{aligned}
[\![enter]\!]_{AP} &= \lambda S.S(\lambda xe\phi.(\mathbf{walk\_in}\ x \wedge \phi(\mathbf{walk\_in}\ x \wedge_{AP} e)))\\
[\![growl]\!]_{AP} &= \lambda S.S(\lambda xe\phi.(\mathbf{growl}\ x \wedge \phi(\mathbf{growl}\ x \wedge_{AP} e)))\\
[\![a]\!]_{AP} &= \lambda PQe\phi.\exists x.Px(x ::_{AP} e)(\lambda e'.Qxe'\phi)\\
[\![wolf]\!]_{AP} &= \lambda xe\phi.(\mathbf{wolf}\ x \wedge \phi(\mathbf{wolf}\ x \wedge_{AP} e))\\
[\![it]\!]_{AP} &= \lambda Pe\phi.P(\mathsf{sel}_{AP}\ e)e\phi
\end{aligned}
$$

$$[\![might]\!]_{AP} = \lambda Ae\phi.\Diamond(e.\mathsf{base} \wedge Ae\phi) \tag{7.11}$$

$$[\![would]\!]_{AP} = \lambda Ae\phi.\Box(e.\mathsf{base} \rightarrow Ae\phi) \tag{7.12}$$

Based on the above lexicon, the representations of the two sentences in (121) are compositionally obtained as follows:

$$
\begin{aligned}
[\![(121)\text{-}1]\!]_{AP} &= [\![might]\!]_{AP}([\![enter]\!]_{AP}([\![a]\!]_{AP}[\![wolf]\!]_{AP}))\\
&\twoheadrightarrow_\beta \lambda e\phi.\Diamond(e.\mathsf{base} \wedge \exists x.(\mathbf{wolf}\ x \wedge \mathbf{walk\_in}\ x \wedge\\
&\qquad \phi(\mathbf{wolf}\ x \wedge_{AP} \mathbf{walk\_in}\ x \wedge_{AP} (x ::_{AP} e))))
\end{aligned}
$$

$$
\begin{aligned}
[\![(121)\text{-}2]\!]_{AP} &= [\![would]\!]_{AP}([\![growl]\!]_{AP}[\![it]\!]_{AP})\\
&\twoheadrightarrow_\beta \lambda e\phi.\Box(e.\mathsf{base} \rightarrow (\mathbf{growl}(\mathsf{sel}_{AP}\ e) \wedge \phi(\mathbf{growl}(\mathsf{sel}_{AP}\ e) \wedge_{AP} e)))
\end{aligned}
$$

To obtain the semantic representation of the whole discourse, we use the update function (formula 4.33), or equivalently, the dynamic conjunction (formula 4.34) in TTDL. That is because conjunction is the default relation for sentence sequencing. As a result:

[5] In the original reference Asher and Pogodalla (2011a), the authors treat *might* and *would* as VP operators, namely they map a VP to another VP. In order to keep uniform with our previous illustrations, such as example (61) in section 3.1.3, we will treat them as sentential operators, which take a sentence and return a modalized sentence. This modification will not change the final representation.

$$\begin{aligned}
[\![(121)]\!]_{AP} &= [\![(121)\text{-}1]\!]_{AP} \wedge^{d}_{TTDL} [\![(121)\text{-}2]\!]_{AP}\\
&= \lambda ABe\phi.Ae(\lambda e'.Be'\phi)\\
&\quad\quad (\lambda e\phi.\Diamond(\exists x.(\mathbf{wolf}\ x \wedge \mathbf{walk_in}\ x \wedge \phi(\mathbf{wolf}\ x \wedge_{AP} \mathbf{walk_in}\ x \wedge_{AP} (x ::_{AP} e)))))\\
&\quad\quad (\lambda e\phi.\Box(e.\mathsf{base} \to (\mathbf{growl}(\mathsf{sel}_{AP}\ e) \wedge \phi(\mathbf{growl}(\mathsf{sel}_{AP}\ e) \wedge_{AP} e))))\\
&\twoheadrightarrow_{\beta} \lambda e\phi.\Diamond(\exists x.(\mathbf{wolf}\ x \wedge \mathbf{walk_in}\ x \wedge\\
&\quad\quad\quad\quad \Box((\mathbf{wolf}\ x \wedge \mathbf{walk_in}\ x) \to (\mathbf{growl}(\mathsf{sel}_{AP}(x ::_{AP} e)) \wedge\\
&\quad\quad\quad\quad\quad\quad \phi(\mathbf{growl}(\mathsf{sel}_{AP}(x ::_{AP} \mathsf{nil}_{AP})) \wedge_{AP}\\
&\quad\quad\quad\quad\quad\quad\quad\quad (\mathbf{wolf}\ x \wedge_{AP} \mathbf{walk_in}\ x \wedge_{AP} (x ::_{AP} e)))))))
\end{aligned}$$

There is only one candidate referent that the choice operator sel_{AP} may pick up, and we assume it does so. Then by passing the empty left context nil_{AP} (formula 7.10) and the empty right context **stop** (formula 4.36) to the above formula, we can end up with a more succinct representation.

$$\begin{aligned}
[\![(121)]\!]_{AP}\ \mathsf{nil}\ \mathbf{stop} \twoheadrightarrow_{\beta} \Diamond(\exists x.(&\mathbf{wolf}\ x \wedge \mathbf{walk_in}\ x \wedge\\
&\Box((\mathbf{wolf}\ x \wedge \mathbf{walk_in}\ x) \to (\mathbf{growl}\ x))))
\end{aligned}$$

The above representation is in the form of 7.5 rather than 7.4, hence it renders the desired semantic interpretation for discourse (121).

One may have noticed that in the lexical entries of modal verbs (i.e., formulas 7.11 and 7.12), the continuation is embedded in the scope of the modal operator. This setup fails to account for the second defect of Roberts's theory, namely the interactions between factual and modal contexts. In order to deal with that, Asher and Pogodalla (2011a) proposes to further enrich the left context. Besides the discourse referents in the modal world, the list of referents introduced in the factual world, in notation $\mathsf{f\_ref}$, is also inserted in the previous record structure (formula 7.6). This engenders a new interpretation of the left context:

$$\gamma = \{\mathsf{m\_ref} : \gamma'; \mathsf{base} : o; \mathsf{f\_ref} : \gamma'\} \tag{7.13}$$

It has been configured in such a way that anaphoric expressions in the factual world can retrieve variables in both $\mathsf{m\_ref}$ and $\mathsf{f\_ref}$, while for anaphoric expressions within a modality, only referents in $\mathsf{m\_ref}$ are accessible. In addition, the authors suggest to interpret sentences and discourses with the following updated semantic types (s and d denote the syntactic category of sentence and discourse, respectively):

$$[\![s]\!]_{AP} = [\![d]\!]_{AP} = \gamma \to (\gamma \to o) \to (\gamma \to o) \to (o \to o \to o) \to o$$

In the new way of interpretation, two continuations are used: one containing facts about the actual world, the other containing facts about live possibilities. Correspondingly, at the end of the interpretation, a pair of propositions are returned: one for the factual world, the other for the epistemic possible worlds. Here we shall omit the detailed illustrations, such as the lexical entries and representations of various discourses. For more information, please refer to the original article.

Generally speaking, as an extension of TTDL, Asher and Pogodalla (2011a) implements and improves the theory of modal subordination Roberts (1989) within continuation semantics. On the one hand, examples where different modals are involved, typically in the form "*Might* S_1. *Would* S_2.", such as (13) and (121), are provided a more precise semantic interpretation. On the other hand, referents in the factual world and modal world are distinguished such that a more fine-grained notion of referent accessibility is achieved.

In the next section, we will provide an adaptation of TTDL, which also focuses on the problem of modal subordination. We will explicitly spell out the semantics of modal operators on the syntactic level by integrating possible world variables, constants and quantifiers in the vocabulary. Further more, the logical representations in the new framework are rich enough to contain the detailed possible worlds hierarchy, including the set of all possible worlds, their accessibility relations, and the set of satisfied propositions in each world.

7.3 Adaptation of TTDL to Modality

In this section, we will integrate epistemic modality within the continuation-based dynamic framework TTDL as introduced earlier in section 4.4, we will call the new framework **Modal TTDL** (**M-TTDL**). As explained in section 7.2.1, a conversational background is a function from possible worlds to sets of propositions, which are the ones that are **necessarily** true at the given world. They serve as the common ground information, or premise assumption, for subsequent modally subordinated utterances. Hence our strategy for achieving M-TTDL is to enrich the context of TTDL with the notion of conversational background Kratzer (1981), in particular the modal base.

In the following, we will first present the formal framework, including the particular signature for M-TTDL, and the typing informations, such as the way in which (modal) proposition, left context, right context, etc., are respectively interpreted; then we will define some preliminary functions that will facilitate future presentation; after that, we propose the formal framework, including the syntax and semantics; finally, the lexical entries, together with the treatments of some puzzling examples will be provided.

7.3.1 Formal Framework

Same as its ancestor systems, i.e., TTDL and DN-TTDL, the adaptation M-TTDL is also a framework based on the simply typed λ-calculus. For all the formal details, please refer back to section 3.2. Below, we shall specify the signature of M-TTDL, which distinguishes it from other correlated frameworks, such as TTDL and DN-TTDL.

Since M-TTDL is concerned with the notion of possible world, which is missing in both TTDL and DN-TTDL, we need a different signature from the previous ones. Types and constants that are correlated with possible worlds ought to be incorporated in M-TTDL. For instance, we keep the two conventional ground types in M-TTDL: ι for individuals, and o for truth values. Besides, a third primitive type s is employed for possible worlds. As to γ, which is the type denoting lists of discourse referents, is abandoned because the context in M-TTDL will contain propositions (the modal base) rather than variables. In the following, we provide a formal characterization of the new signature. Please note that only the types of logical constants are specified. The particular type of a non-logical constant will be indicated when it is employed.

Definition 7.3.1. The signature $\Sigma_{M\text{-}TTDL}$ is defined as follows:

$$\begin{aligned}\Sigma_{M\text{-}TTDL} = \langle\{\iota, o, s\}, \{&\top, \wedge, \neg, {}^{\iota}\exists, {}^{s}\exists, \mathsf{sel}, \mathbf{H}\},\\ &\{\top : o, \wedge : o \to o \to o, \neg : o \to o, {}^{\iota}\exists : (\iota \to o) \to o,\\ &{}^{s}\exists : (s \to o) \to o, \mathsf{sel} : (\iota \to o) \to o \to \iota, \mathbf{H} : s\}\rangle\end{aligned}$$

Now let's take a close look at the logical constants. On the first place, we abandon in $\Sigma_{M\text{-}TTDL}$ the familiar list constructor _::_ and the empty list of referents nil, because the left context in M-TTDL is made up of propositions (the modal base), rather than variables. In addition, with respect to the modification on the left context, the choice operator sel is also changed accordingly. In previous systems, it is used to pick up a variable from a list of referents (of type $\gamma \to \iota$). But in M-TTDL, it will do the same job with respect to an input property (of type $\iota \to o$) and the current modal base (of type o). The former is the criteria based on which sel makes its decision. This explains the semantic type of sel as defined in $\Sigma_{M\text{-}TTDL}$. Further more, we distinguish between the quantifier over individuals ${}^{\iota}\exists$ and the one over possible worlds ${}^{s}\exists$. Their difference are revealed from their corresponding types. Some other conventional logical constants, such as $\to$ (implication), $\vee$ (disjunction), $\forall$ (universal quantifier), are defined same as before with the above primitives, see formula 3.1, 3.2, and 3.5. Please note that corresponding to the two existential quantifiers, there are also a pair of universal quantifiers: ${}^{\iota}\forall$ and ${}^{s}\forall$, the former ranges over individual variables, the latter over possible world variables. Finally, the possible world constant $\mathbf{H}$ denotes the current world. It will be used to provide the world of evaluation at the end of the semantic interpretation.

For the rest of this subsection, we will focus on the typing information in M-TTDL. The way to interpret left context, right context, and propositions will be elucidated sequentially. As we mentioned above, ι and o are still the types for individuals and truth values, respectively. However in modal systems, such as MPL in section 3.1.3, a (modal) proposition is interpreted as a set of possible worlds, rather than a truth value. Hence its type should be $s \to o$. Hereinafter, we abbreviate it as o_i, namely:

$$o_i \triangleq s \to o \tag{7.14}$$

Correspondingly, the semantic type of 1-place predicates, such as **man** and **walk_in**, is updated to $\iota \to o_i$; the type of 2-place predicates, such as **beat** and **eat**, is updated to $\iota \to \iota \to o_i$.

To explain the interpretation of the left context, we first propose the concept of **environment**. It is an ordered pair consisting of two modal propositions: the **background information** and the **base information**. The purpose of an environment is twofold: on the one hand, it encodes the propositions necessarily true at the given world, which is the background information; on the other hand, it enables to pass updated propositions from a possible world to accessible ones, which is the base information. Both the background and the base are propositions, they are hence of type o_i. As a result, the type of an environment is $(o_i \times o_i)$. If we use T_{env}, T_{bk}, and T_{ba} to denote the type of environment, background, and base, respectively, we can draw the following formulas:

$$T_{bk} = T_{ba} = o_i$$

$$T_{env} = T_{bk} \times T_{ba} = o_i \times o_i$$

Based upon the notion of environment, we thus define another concept: **general-**

ized environment, which is in parallel with the conversational background in Kratzer's theory. As we know, the conversational background is a function from possible worlds to sets of propositions (or equivalently, the conjunction consisting of all propositions). Analogously, the generalized environment is a mapping from possible worlds to environments. This means if we apply a generalized environment to a particular world, it will yield the environment at that world. Consequently, if we use T_{genv} to denote the type of generalized environments, it can be represented as follows:

$$T_{genv} = s \to T_{env}$$

In fact, the generalized environment can be regarded as an enhanced version of the conversational background. By applying it to a possible world argument, we obtain a pair of (modal) propositions. The first element, namely the background proposition, is exactly equivalent to the current modal base: it is the conjunction of all propositions that are necessarily satisfied/recognized at that possible world. And the background can be incrementally updated during the discourse processing, when new logical contents/propositions which are necessarily true in that world are provided. The second element of the pair, namely the base proposition, serves as a "buffer": appearing in the form of a conjunction as well, it consists of the propositions to be updated to accessible worlds. Its content will be reset after the updating in order to avoid information duplication. An illustration will be provided in section 7.3.6.

Besides environment and generalized environment, we need to introduce the concept of the **salient world**, or equivalently, the **world of interest**, for the process of discourse incrementation. Its purpose is to record the current position of the processing in the overall possible worlds hierarchy, this will determine in which world the propositions expressed by subsequent utterances are to be integrated. Note that this is different from the world of evaluation (the world where the sentence is uttered) we introduced in section 3.1.3.

With the above notions, we establish the left context in M-TTDL by encapsulating the salient world and the generalized environment in an ordered pair. By convention, we use γ_i to symbolize the type of left context, then:

$$\gamma_i \triangleq s \times T_{genv} \tag{7.15}$$

If we unfold γ_i with all primitive types, we will obtain the following typing information:

$$\begin{aligned} \gamma_i &= s \times (s \to T_{env}) \\ &= s \times (s \to (o_i \times o_i)) \\ &= s \times (s \to ((s \to o) \times (s \to o))) \end{aligned} \tag{7.16}$$

Same as in TTDL and DN-TTDL, the **right context** in M-TTDL is interpreted as a function from left contexts to (modal) propositions, hence its semantic type is $\gamma_i \to o_i$. Similarly, if we unfold it, we will obtain:

$$\begin{aligned}\gamma_i \to o_i &= (s \times T_{genv}) \to o_i \\ &= (s \times (s \to T_{env})) \to o_i \\ &= (s \times (s \to (o_i \times o_i))) \to o_i \\ &= (s \times (s \to ((s \to o) \times (s \to o)))) \to (s \to o)\end{aligned}$$

Accordingly, a dynamic proposition in M-TTDL is interpreted as a function which takes a left context and a right context, and returns a (modal) proposition. Both sentences and discourses will be treated in the same manner. Assume s and d are syntactic categories of sentences and discourses, respectively, then:

$$[\![s]\!] = \gamma_i \to (\gamma_i \to o_i) \to o_i \tag{7.17}$$

$$[\![d]\!] = \gamma_i \to (\gamma_i \to o_i) \to o_i \tag{7.18}$$

Again, we abbreviate the complex type with a compact term Ω_i, namely:

$$\Omega_i \triangleq \gamma_i \to (\gamma_i \to o_i) \to o_i \tag{7.19}$$

By unfolding formula 7.19, we can obtain the following result:

$$\begin{aligned}\Omega_i &= \gamma_i \to (\gamma_i \to o_i) \to o_i \\ &= (s \times T_{genv}) \to ((s \times T_{genv}) \to o_i) \to o_i \\ &= (s \times (s \to T_{env})) \to ((s \times (s \to T_{env})) \to o_i) \to o_i \\ &= (s \times (s \to ((s \to o) \times (s \to o)))) \to \\ &\quad ((s \times (s \to ((s \to o) \times (s \to o)))) \to (s \to o)) \to \\ &\quad (s \to o)\end{aligned} \tag{7.20}$$

As we may observe from formula 7.20, the type of dynamic propositions in M-TTDL is rather complicated, particularly it involves a number of occurrences of possible worlds (of type s) in different positions. However, by looking at the folded form, i.e., formula 7.19, it is clearly a member of the continuation semantic family.

Up until now, we have presented the typing information in M-TTDL. In what follows, we will first introduce some functions which are concerned with the modal base, possible worlds, and correlated concepts. They are cornerstones for our future presentation. Afterwards, we will provide the dynamic logic in M-TTDL, as well as the systematic dynamic translation.

7.3.2 Elementary Functions

In this subsection, we will introduce some fundamental functions which are concerned with the above introduced concepts such as environment, generalized environment, context, etc. These functions shall be presented in various groups, based on the particular semantic object they are working on. They will largely be used to construct lexical entries, which we will see in the succeeding subsection.

Modalized Logical Constants

First of all, let's first have a look at a set of modalized logical constants, which are defined in terms of the constants in the signature $\Sigma_{M\text{-}TTDL}$ (definition 7.3.1). These terms will save space and provide a better readability in subsequent function definitions.

- Modal conjunction[6]: $o_i \to o_i \to o_i$

$$\wedge_i \triangleq \lambda ABi.(Ai \wedge Bi) \tag{7.21}$$

  The operator $\wedge_i$ is the modal counterpart of $\wedge$. It takes two modal propositions as input, and returns another modal proposition, which is the conjunction consisting of the logical contents in the input.

- Modal negation: $o_i \to o_i$

$$\neg_i \triangleq \lambda Ai.\neg(Ai) \tag{7.22}$$

  The operator $\neg_i$ is the modal counterpart of $\neg$. It takes a modal proposition as input, and returns its modal negation.

- Modal existential quantifier for individuals: $(\iota \to o_i) \to o_i$

$${}^{\iota}\exists_i \triangleq \lambda Pi.{}^{\iota}\exists(\lambda x.Pxi) \tag{7.23}$$

  The operator ${}^{\iota}\exists_i$ is the modal counterpart of ${}^{\iota}\exists$. It takes a modal individual property (of type $\iota \to o_i$) as input, and returns an existentially quantified modal proposition.

- Modal tautology: o_i

$$\top_i \triangleq \lambda i.\top \tag{7.24}$$

  The tautology $\top$ is of type o, it always denotes the truth value 1. Its counterpart in modal systems: $\top_i$, which returns 1 at each possible world, is of type o_i.

Environment and Salient World Manipulation

After the functions on modal propositions, let's turn to the ones which deal with salient world and environment.

- Retrieve the salient world: $\gamma_i \to s$

$$\mathbf{woi} \triangleq \lambda e.\pi_1 e \tag{7.25}$$

  The function **woi** is relatively straightforward. It takes a left context e as input and returns its salient world, which is simply the first projection of e.

[6] This is the description of the function, which is followed by its corresponding semantic type. In subsequent function introductions, we will stick with the same notation.

- Retrieve the generalized environment: $\gamma_i \to T_{genv}$

$$\mathbf{genv} \triangleq \lambda e.\pi_2 e \tag{7.26}$$

Contrast to the previous function **woi**, the function **genv** takes a left context e and returns its generalized environment, which corresponds to the second projection of the input e.

- Retrieve the environment: $\gamma_i \to s \to T_{env}$

$$\mathbf{env} \triangleq \lambda ei.(\mathbf{genv}\ e\ i) \tag{7.27}$$

The function **env** is established upon **genv** (formula 7.26). It takes a left context and a possible world, and returns a specific environment at the input world.

- Modify the salient world: $\gamma_i \to s \to \gamma_i$

$$\mathbf{change\_woi} \triangleq \lambda ei.\langle i, (\mathbf{genv}\ e)\rangle \tag{7.28}$$

The function **change_woi** takes a left context e and a possible world i as input. It yields a new left context, where the salient world is modified to the input world i, the generalized environment is the one of the input left context.

- Retrieve the background: $\gamma_i \to s \to o_i$

$$\mathbf{bkgd} \triangleq \lambda ei.\pi_1(\mathbf{env}\ e\ i) \tag{7.29}$$

The function **bkgd** takes a left context e (a Cartesian product consisting of a salient world and a generalized environment) and a possible world i as input. It yields a modal proposition, which is the background (the first element of the environment) of the left context e at the given world i.

- Retrieve the base: $\gamma_i \to s \to o_i$

$$\mathbf{base} \triangleq \lambda ei.\pi_2(\mathbf{env}\ e\ i) \tag{7.30}$$

The function **base** takes a left context e (a Cartesian product consisting of a salient world and a generalized environment) and a possible world i as input. It yields a modal proposition, which is the base (the second element of the environment) of the left context e at the given world i.

Context Manipulation

In this subsection, we will see the functions which manipulate generalized environments and contexts. First, we define the following notation:

Definition 7.3.2. Let $w, w' \in W$ be possible worlds, G a generalized environment, E an environment. The notation $G[w := E]$ stands for a generalized environment such that:

$$G[w := E](w') = \begin{cases} E & \text{if } \mathbf{R}(w, w'), \\ G(w') & \text{otherwise.} \end{cases}$$

As indicated in definition 7.3.2, $G[w := E]$ is itself a generalized environment, whose interpretation relies on the input possible world argument. If the input world is accessible to w, then environment E will be returned, otherwise, the generalized environment G is applied to the input world. The presentation of the following functions will base on the above notation.

- Update the generalized environment: $T_{genv} \to s \to T_{env} \to T_{genv}$

$$\mathbf{up\_genv} \triangleq \lambda GiE.G[i := E] \tag{7.31}$$

  The function **up_genv** takes three arguments as input:

  1. A generalized environment G, which is of type T_{genv};
  2. A possible world i, which is of type s, it denotes the target world at which the generalized environment is to be updated;
  3. A to-be-updated environment E, which of type T_{env}.

  It thus yields another generalized environment, namely $G[i := E]$.

- Update the left context: $\gamma_i \to s \to o_i \to \gamma_i$

$$\begin{aligned} \mathbf{up\_context} \triangleq \lambda eiA.\langle(\mathbf{woi}\ e), & \\ & \mathbf{up\_genv} \\ & \quad (\mathbf{genv}\ e) \\ & \quad i \\ & \quad \langle A \wedge_i (\mathbf{bkgd}\ e\ i), A \wedge_i (\mathbf{base}\ e\ i)\rangle \\ & \rangle \end{aligned} \tag{7.32}$$

  The function **up_context** takes three arguments as input:

  1. A left context e, which is of type γ_i;
  2. A possible world i at which the update process takes place, it is of type s;
  3. A modal proposition A, which is the to-be-updated logical content, it is of type o_i.

  It yields an updated left context, with the logical content of the modal proposition A added in both the background and the base of e at world i.

- Copy the left context: $\gamma_i \to s \to s \to \gamma_i$

$$\begin{aligned}\textbf{copy\_context} \triangleq \lambda eij.\langle(\textbf{woi}\ e),&\\ \textbf{up\_genv}&\\ &(\textbf{genv}\ e)\\ &j\\ &(\textbf{env}\ e\ i)\\ \rangle&\end{aligned} \tag{7.33}$$

The function **copy_context** takes a left context e and two possible worlds i and j as input. It yields a left context, which has the same salient world as the input left context, but the original environment at world i will be copied to all worlds that are accessible from j.

- Reset the base in a left context: $\gamma_i \to s \to \gamma_i$

$$\begin{aligned}\textbf{reset\_base} \triangleq \lambda ei.\langle(\textbf{woi}\ e),&\\ \textbf{up\_genv}&\\ &(\textbf{genv}\ e)\\ &i\\ &\langle(\textbf{bkgd}\ e\ i), \top_i\rangle\\ \rangle&\end{aligned} \tag{7.34}$$

During the discourse processing, we will have to reset the base at various steps (particularly, when the proposition in the base has already been used) in order to avoid information duplication. The above function **reset_base** helps to achieve this goal. Basically, **reset_base** takes a left context e and a possible world i as input. It yields another left context, which contains the same salient world, and a modified generalized environment, where the base information is reset.

- The empty left context: γ_i

$$\mathsf{nil}_i \triangleq \langle \mathbf{H}, \lambda i.\langle \top_i, \top_i\rangle\rangle \tag{7.35}$$

The term nil_i represents the void left context in M-TTDL. It is a context at the current world $\mathbf{H}$, and both background and base propositions in the environment are the modal tautology $\top_i$. It is similar to the nil in TTDL, DN-TTDL, as well as the nil_{AP} (formula 7.10) in Asher and Pogodalla (2011a).

- The empty right context: $\gamma_i \to o_i$

$$\mathbf{stop}_i \triangleq \lambda e.\top_i \tag{7.36}$$

Analogous to the term **stop** (formula 4.36) in previous frameworks, the above term $\mathbf{stop}_i$ is an empty right context in M-TTDL. It takes a left context as input, no matter what its value is, it always returns the modal tautology $\top_i$. As discussed in section 7.3.1, the way that the context is unfolded is rather complex in M-TTDL. Thus at the end of the discourse processing, $\mathbf{stop}_i$ may be employed together with nil_i in order to obtain a more concise and compact logical representation. We will see its application in section 7.3.6.

7.3.3 Dynamic Translation

In this subsection, we will continue with the formal details of M-TTDL, focusing on the dynamic logic and the systematic dynamic translation.

First of all, as we explained before, M-TTDL parallels TTDL in the aspect of the way to interpret sentences and discourses: both of them are functions from left contexts to right contexts to propositions (contrast formulas 4.29, 4.30 with 7.17, 7.18). As a result, we can reuse the update function in TTDL (formula 4.33) for the incremental construction of the discourse in M-TTDL, namely:

$$\mathbf{update}_{M\text{-}TTDL} \triangleq \mathbf{update}_{TTDL} \tag{7.37}$$

Note that in both TTDL and M-TTDL, a dynamic proposition is interpreted as a function which takes a left and right context, and returns a truth value. For instance, in TTDL, its type is $\gamma \to (\gamma \to o) \to o$, while in M-TTDL, it is $\gamma_i \to (\gamma_i \to o_i) \to o_i$ (the types of the latter are indexed with i because the notion of possible world is incorporated). For more explanations, please refer back to section 4.4.2. Because of that, as the default connective between sentences in a discourse, the dynamic conjunction in M-TTDL is defined exactly the same as in TTDL:

$$\wedge^d_{M\text{-}TTDL} \triangleq \lambda ABe\phi.Ae(\lambda e'.Be'\phi) \tag{7.38}$$

In order to negate a dynamic proposition in M-TTDL, we propose the following negation operator:

$$\neg^d_{M\text{-}TTDL} \triangleq \lambda Ae\phi.\neg_i(A\; e\; \mathbf{stop}_i) \wedge_i \phi e \tag{7.39}$$

When a proposition is negated, its context change potential will be restrained. This explains the modalized empty continuation $\mathbf{stop}_i$ in the above definition. It prevents the information in the left context to be updated in future discourse. Contrasting $\neg^d_{M\text{-}TTDL}$ (formula 7.39) with $\neg^d_{TTDL}$ (formula 4.37), we see that the two operators are in a completely similar structure, except for that the logical constants in $\neg^d_{TTDL}$ (i.e., $\neg$, $\top$ and $\wedge$) are substituted by their modal counterparts in $\neg^d_{M\text{-}TTDL}$ (i.e., $\neg_i$, $\top_i$ and $\wedge_i$).

For the dynamic existential quantifier (the one which ranges over individual variables) in M-TTDL, we propose the following definition:

$${}^{\iota}\exists^d_{M\text{-}TTDL} \triangleq \lambda Pe\phi.({}^{\iota}\exists_i(\lambda x.Pxe\phi)) \tag{7.40}$$

Compared with its predecessors, namely $\exists^d_{TTDL}$ (formula 4.38) and $\exists^d_{DN\text{-}TTDL}$ (formula 6.28), the job of ${}^{\iota}\exists^d_{M\text{-}TTDL}$ is less crucial. The quantifier ${}^{\iota}\exists^d_{M\text{-}TTDL}$ does not update variables to the left context, because the structure of the left context is totally changed. In M-TTDL, the left context consists of propositions rather than invididuals. For more discussion, please refer back to section 7.3.1.

Based on the above analysis, we will now present the systematic dynamic translation in M-TTDL. To distinguish the translations in M-TTDL from the previous ones in TTDL and DN-TTDL, we introduce the m-bar notation.

Notation 7.3.1. We use the m-bar notation, for instance, $\overline{\tau}^{\mathsf{m}}$ or $\overline{M}^{\mathsf{m}}$, to denote the **dynamic translation** of a type τ or a λ-term M in M-TTDL.

The dynamic translation of types in M-TTDL is in a parallel structure with the ones in TTDL and DN-TTDL. One may compare the following definition with definitions 4.4.2 and 6.2.1.

Definition 7.3.3. The **dynamic translation of a type** $\tau \in T$: $\overline{\tau}^{\mathsf{m}}$, is defined inductively as follows:

1. $\overline{\iota}^{\mathsf{m}} = \iota$;
2. $\overline{o_i}^{\mathsf{m}} = \Omega_i$;
3. $\overline{\sigma \to \tau}^{\mathsf{m}} = \overline{\sigma}^{\mathsf{m}} \to \overline{\tau}^{\mathsf{m}}$, where $\tau, \sigma \in T$.

The detailed unfolding of Ω_i can be found in formula 7.20. Again, for the dynamic translation of λ-terms, we need to define the two functions: $\mathbb{D}^m$ and $\mathbb{S}^m$, which will be used to translate non-logical constants. These two function in M-TTDL are slightly different from their previous versions in TTDL (definition 4.4.3) and DN-TTDL (definition 6.2.2).

Definition 7.3.4. The **dynamization function** $\mathbb{D}^m_\tau$, which takes an input λ-term A of type $(\gamma_i \to \tau)$, returns an output λ-term A' of type $\overline{\tau}^{\mathsf{m}}$; the **staticization function** $\mathbb{S}^m_\tau$, which takes an input λ-term A' of type $\overline{\tau}^{\mathsf{m}}$, returns an output λ-term A of type $\gamma_i \to \tau$. In the following formulas, e denotes a variable of type γ_i.

- $\mathbb{D}^m_\tau$ is defined inductively on type τ as follows:
  1. $\mathbb{D}^m_\iota A = A\ \mathsf{nil}_i$;
  2. $\mathbb{D}^m_{o_i} A = \lambda e\phi i.(Aei \wedge \phi(\mathbf{up\_context}\ e\ i\ (Ae))i)$;
  3. $\mathbb{D}^m_{\alpha\to\beta} A = \lambda x.\mathbb{D}^m_\beta(\lambda e.Ae(\mathbb{S}^m_\alpha xe))$.
- $\mathbb{S}^m_\tau$ is defined inductively on type τ as follows:
  1. $\mathbb{S}^m_\iota A' = \lambda e.A'$;
  2. $\mathbb{S}^m_{o_i} A' = \lambda e.A'\ e\ \mathbf{stop}_i$;
  3. $\mathbb{S}^m_{\alpha\to\beta} A' = \lambda e.(\lambda x.\mathbb{S}^m_\beta(A'(\mathbb{D}^m_\alpha(\lambda e'.x)))e)$.

In previous frameworks, i.e., TTDL and DN-TTDL, the change of context is achieved through the dynamic existential quantifier (formulas 4.38 and 6.28). However, since the left context is interpreted differently in M-TTDL, the function $D^m_{o_i}$ is designed in a way such that it changes the current left context by inserting the dynamized modal proposition into the environment. For more discussions on the general cases of $\mathbb{D}^m$ and $\mathbb{S}^m$, please refer back to section 4.4.2. Below, we present the dynamic translation of λ-terms in M-TTDL, which is similar to that in TTDL and DN-TTDL, compare the following definition with definitions 4.4.4 and 6.2.3:

Definition 7.3.5. The **dynamic translation of a λ-term** M (of type τ): $\overline{\overline{M}}$, which is another λ-term of type $\overline{\overline{\tau}}$, is defined as follows:

1. $\overline{x}^{\mathsf{m}} = x$, if $x \in \mathcal{X}$;
2. $\overline{\mathbf{a}}^{\mathsf{m}} = \mathbb{D}^{m}_{\tau}(\lambda e.\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}_{NL}$;
3. $\overline{\wedge}^{\mathsf{m}} = \wedge^{d}_{M\text{-}TTDL}$, see formula 7.38;
4. $\overline{\neg}^{\mathsf{m}} = \neg^{d}_{M\text{-}TTDL}$, see formula 7.39;
5. $\overline{{}^{\iota}\exists}^{\mathsf{m}} = {}^{\iota}\exists^{d}_{M\text{-}TTDL}$, see formula 7.40;
6. $\overline{(MN)}^{\mathsf{m}} = (\overline{M}^{\mathsf{m}}\,\overline{N}^{\mathsf{m}})$;
7. $\overline{(\lambda x.M)}^{\mathsf{m}} = (\lambda x.\overline{M}^{\mathsf{m}})$.

For the dynamic translation of other logical constants such as $\vee$ (disjunction), $\rightarrow$ (implication), and $\forall$ (universal quantifier), we can apply the corresponding rules in definition 7.3.5 to their derived terms. Take implication for instance:

$$\begin{aligned}
\overline{A \rightarrow B}^{\mathsf{m}} &= \overline{\neg(A \wedge \neg B)}^{\mathsf{m}} \\
&= \overline{\neg}^{\mathsf{m}}(\overline{A}^{\mathsf{m}}\overline{\wedge}^{\mathsf{m}}(\overline{\neg}^{\mathsf{m}}\overline{B}^{\mathsf{m}})) \\
&= \neg^{d}_{M\text{-}TTDL}(\overline{A}^{\mathsf{m}} \wedge^{d}_{M\text{-}TTDL} (\neg^{d}_{M\text{-}TTDL}\overline{B}^{\mathsf{m}})) \\
&\twoheadrightarrow_{\beta} \lambda e\phi.\neg_i(\overline{A}^{\mathsf{m}} e(\lambda e'.\neg_i(\overline{B}^{\mathsf{m}}\ e'\ \mathbf{stop}_i))) \wedge_i \phi e
\end{aligned} \tag{7.41}$$

As to the semantics of M-TTDL, it follows from TTDL, which is also the same as in FOL, see section 4.4.2 for the detailed interpretations.

The rest of this chapter is organized as follows. In the next subsection, we will provide the specific lexical entries around modality, which are mainly established based on the functions introduced in section 7.3.2. Then in section 7.3.5, we will focus on the relation between M-TTDL and TTDL: they are proved to have the same empirical predictions when no modality is concerned. Finally, applications of M-TTDL will be illustrated with specific linguistic examples in section 7.3.6.

7.3.4 Lexical Entries for Modals

Based on the above analysis, in particular the fundamental functions in section 7.3.2, we will propose the core of M-TTDL in this subsection, namely the specific lexical entries for modal expressions. We will first present the logical representations of the two modal operators: $\Diamond$ and $\Box$, which express possible modality and necessary modality, respectively; then we will introduce the **at** function, which explicitly indicates the world at which a dynamic proposition is to be evaluated; finally, two semantic entries corresponding to the epistemic modals in natural language: *might* and *would*, will be established based on the preceding knowledge.

Possibility Modal Operator

The modal operator $\Diamond$ takes a dynamic proposition A (of type Ω_i) as input, and returns another dynamic proposition $\Diamond A$, which contains an existential modality. Hence the operator $\Diamond$ should be of type $\Omega_i \to \Omega_i$. Its entry is presented as follows:

$$\begin{aligned}\Diamond \triangleq \lambda Ae\phi i.{}^s\exists j.(&\mathbf{R}\ i\ j \wedge \\ &\mathbf{base}\ e\ i\ j \wedge \\ &A\ (\mathbf{copy\_context}\ e\ i\ j) \\ &\quad(\lambda e'j'.\phi\ (\mathbf{reset\_base}\ (\mathbf{change\_woi}\ e'\ j')i)\ i\ ) \\ &\quad j)\end{aligned} \tag{7.42}$$

The above entry can be understood as follows. The quantifier ${}^s\exists$ ranges over possible world variables, $\mathbf{R}$ denotes the accessible relation, so ${}^s\exists j.\mathbf{R}\ i\ j$ means there exists a possible world j which is accessible from world i. The modal base at world i, namely, the proposition that is necessarily true at the utterance world, should be satisfied in all of its accessible worlds, including j. This explains the sub-part **base** $e\ i\ j$, which serves as the common background in world j. As to the input proposition A, it is first applied to a left context, where the environment at the utterance world i is copied to the newly established possible world j, this corresponds to **copy_context** $e\ i\ j$. Then, a right context, where the base at world i is reset to the modal tautology $\top_i$, and the world of interest is switched to world j, is passed to A. Finally, the input proposition is evaluated at world j.

In conclusion, a dynamic proposition A is **possibly true**, namely $\Diamond A$ is satisfied, iff there is a possible world j, which is accessible from the utterance world i, such that:

- The propositions which are necessarily true at world i, namely the modal base at i, should be satisfied at world j;
- The possible world j inherits the generalized environment from the utterance world i, and the base information at the utterance world i is reset to $\top_i$;
- The salient world is updated to world j;
- The modalized proposition is evaluated at world j.

Now let's have a look at some example. Assume A is a dynamic proposition (of type Ω_i), where no modality is involved, a is the logical content of A (of type o_i). Namely A is constructed by translating a with respect to the dynamization rules, see definition 7.3.5. In order to illustrate how the above entry of $\Diamond$ works, we shall contrast the environment of proposition A and $\Diamond A$.

Assume **mb** is the modal base function, which returns the background information at a given world. For more detail, please refer back to Kratzer's theory in section 7.2. The possible worlds hierarchy for interpreting the dynamic modal proposition A is presented in figure 7.1, where circles are used to denote possible worlds, a solid line with an arrow indicates the accessibility relation, a dotted line means the accessibility relation is not specified. Besides, we use the red color to signify the salient world, assume the current left context is e, we will term the salient world w_s, namely $w_s = \mathbf{woi}\ e$. For the world of utterance, we uniquely term it i. Finally, we place the propositions that are true at each world besides it, e.g., A is besides world i in figure 7.1. In subsequent diagrams, we will stick to the same notation style.

By default, proposition A is uttered at world i. Because A is not concerned with any modality, it is true iff A is satisfied at i. Remark that $(\mathbf{mb}\ i) \wedge A$ and A have the

Fig. 7.1 Possible Worlds Hierarchy of A

same truth conditions, because $(\mathbf{mb}\ i)$ is already satisfied at i. As shown in figure 7.1, the interpretation of A will not change the salient world. In addition, since we do not have further information on the relation between the utterance world i and the salient world w_s, their accessibility is unspecified. Table 7.1 lists the detailed content of the environment at each possible world in question[7]:

Existing World	Environment
i	$\langle a \wedge_i (\mathbf{bkgd}\ e\ i), a \wedge_i (\mathbf{base}\ e\ i)\rangle$
w_s	$\langle(\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)), (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$

Table 7.1 Environment at Each World of A

As a summary, after the interpretation of A, both elements in the environment at world i, namely the background and base are updated with a; while the environment at the salient world w_s is not modified.

Let's turn to $\Diamond A$, its possible worlds hierarchy is depicted in figure 7.2:

Fig. 7.2 Possible Worlds Hierarchy of $\Diamond A$

Again, the proposition $\Diamond A$ is uttered at world i. A is possibly true at the utterance world i, or equivalently, $\Diamond A$ is true at i, iff $(\mathbf{mb}\ i) \wedge A$ is true at an accessible world from i, e.g., j, where $(\mathbf{mb}\ i)$ is the background information at i. As presented above, the lexical entry of $\Diamond$ modifies the salient world to the newly established world j. The detailed content of environment at each world is listed respectively in table 7.2:

The interpretation of $\Diamond A$ requires an accessible possible world from the utterance world, in which A is satisfied. Further more, its logical content a is updated to the environment of the salient world. At the same time, the base information at the evaluation world is reset to a modal tautology to avoid information duplication.

Necessity Modal Operator

The modal operator $\Box$ is the one which creates a modality of universal force. It takes a dynamic proposition, A (of type Ω_i) for instance, as input, and returns a modalized

[7] Like the notation in figures, the red color is used to denote salient world in tables, which will be employed in the following tables as well.

Existing World	Environment
i	$\langle(\textbf{bkgd}\ e\ i), \top_i\rangle$
j	$\langle a \wedge_i (\textbf{bkgd}\ e\ i), a \wedge_i (\textbf{base}\ e\ i)\rangle$

Table 7.2 Environment at Each World of $\Diamond A$

proposition $\Box A$. Hence same as $\Diamond$, the operator $\Box$ should also be of type $\Omega_i \to \Omega_i$. The entry is presented as follows:

$$\begin{aligned}\Box \triangleq \lambda Ae\phi i.(^{s}\forall j.(\mathbf{R}\ i\ j \to \\ &(\textbf{base}\ e\ i\ j \to \\ &\quad (A\ (\textbf{copy\_context}\ e\ i\ j)\ \textbf{stop}_i\ j)))) \\ \wedge\ \phi e i\end{aligned} \tag{7.43}$$

The above entry can be understood as follows. The quantifier $^{s}\forall$ ranges over possible world variables, so $^{s}\forall j.\mathbf{R}\ i\ j$ means for every possible world j that is accessible from the utterance world i. The modal base at world i, namely, the proposition that is necessarily true at the utterance world, should be satisfied in all of its accessible worlds. This explains the sub-part **base** $e\ i\ j$. Different from in $\Diamond$, where the modal base is conjuncted, it plays the role of antecedent of an implication in the necessity modality. As to the input proposition A, similar to $\Diamond$, it is first applied to a left context, where the environment at the utterance world i is copied to the newly established possible world j, this corresponds to **copy_context** $e\ i\ j$. However, due to the semantics of the necessity modality, referents introduced in the scope of $\Box$ shall not be accessed from subsequent context. As a result, the empty continuation $\textbf{stop}_i$ is passed to A, and the current left context will not be modified after processing the modalized proposition.

In conclusion, a dynamic proposition A is **necessarily true**, namely $\Box A$ is satisfied, iff for every possible world j, if j is accessible from the utterance world i, then:

- The propositions which are necessarily true at world i, namely the modal base at i, should be satisfied and serve as premise assumption at world j;
- The possible world j inherits the generalized environment from the utterance world i;
- Information at every possible world j is not able to be passed to subsequent sentences;
- The left context is not changed, the utterance world of the modalized proposition is still i, and proposition A is evaluated at world j.

Now we turn to some example. Like $\Diamond A$, $\Box A$ is also established upon A. We may compare the following information with the one presented in figure 7.1 and table 7.1, which will not be repeated below any more. The possible worlds hierarchy of interpreting $\Box A$ is illustrated in figure 7.3:

Same as above, the utterance world is i. A is necessarily true at the utterance world i, or equivalently, $\Box A$ is true at i, iff $(\textbf{mb}\ i) \to A$ is true at every accessible world from

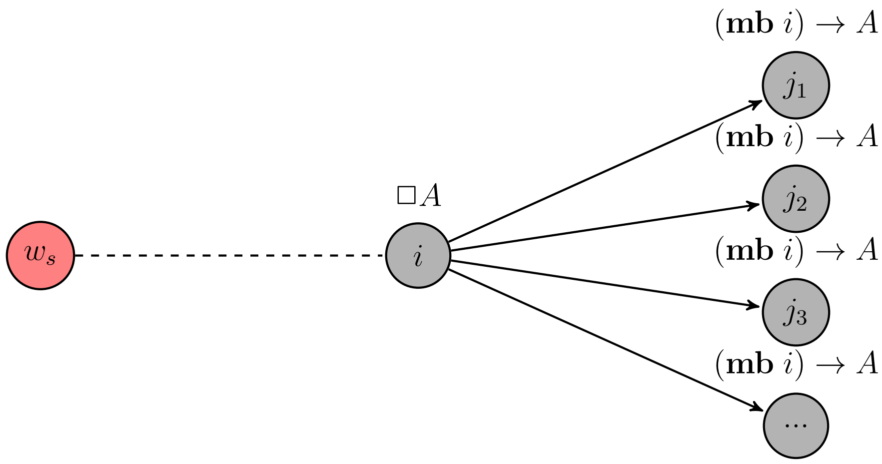


Fig. 7.3 Possible Worlds Hierarchy of $\Box A$

i, where $(\mathbf{mb}\ i)$ is the background information at i. As discussed above, the lexical entry of $\Box$ does not modify the salient world, which is still the original w_s. Also, the relation between the salient world and the utterance world is not specified. As regard to the detailed content of environment at each possible world, we can refer to table 7.3:

Existing World	**Environment**
i	$\langle(\mathbf{bkgd}\ e\ i), (\mathbf{base}\ e\ i)\rangle$
j	$\langle(\mathbf{bkgd}\ e\ j), (\mathbf{base}\ e\ j)\rangle$
w_s	$\langle(\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)), (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$

Table 7.3 Environment at Each World of $\Box A$

After interpreting a necessity modality, the environment at every existing world, such as the salient world w_s, the utterance world i, and all its accessible worlds j, keep unchanged. Hence the operator $\Box$ does not modify the context change potential of the preceding discourse. On the one hand, it does not change the salient world; on the other hand, it does not modify the context at any possible world.

Evaluation "at" Some Possible World

As discussed earlier in section 3.1.3, one huge difference between the interpretation of propositions in classical logic and modal logic is that, a proposition is evaluated at a specific possible world in the latter system. Below, we will introduce the function **at**, which aims to associate a (modal) proposition with a possible world.

Intuitively, the **at** function picks up a particular world for a proposition, where it is to be evaluated. Hence its semantic type ought to be: $s \to \Omega_i \to \Omega_i$, where s denotes the target world, the first Ω_i denotes the input proposition, and the second Ω_i is the output proposition with the world information interpolated. We propose its detailed semantic entry as follows:

$$
\begin{aligned}
\mathbf{at} \triangleq\ & \lambda j A e \phi i. \\
& \quad \mathbf{if}\ (j = i) \\
& \quad\quad (A\ e\ \phi\ i) \\
& \quad\quad (\mathbf{base}\ e\ i\ j \wedge \\
& \quad\quad\quad A\ (\mathbf{up\_context}\ e\ j\ (\mathbf{base}\ e\ i)) \\
& \quad\quad\quad\quad (\lambda e' j'.\phi\ (\mathbf{reset\_base}\ e'\ i) i) \\
& \quad\quad\quad\quad j)
\end{aligned} \tag{7.44}
$$

In the above formula, the two input arguments, j and A, stand for the target world of evaluation and the dynamic proposition to be evaluated, respectively. The rest of the entry can be understood as follows. Firstly, the operator **if** is a logical constant, it is used to determine whether the target world j is identical to the current utterance world i or not. If the two worlds happen to coincide, the second argument will be returned. No modification is needed in this case: the proposition is by default evaluated in the utterance world. Otherwise, if the proposition is to be evaluated in another world than the utterance world i, the third argument will be returned. In this case, the base at i is updated to the context of the target world j by the context update function (we do not use the function **copy_context** because it will overwrite the environment at world j). Further more, same as for $\diamond$, after employing the modal base at world i, we reset it as the modal tautology, this explains the sub-part **reset_base** e' i.

In conclusion, a dynamic proposition A, which is uttered at world i, is interpreted true at another possible world j, iff

- The base proposition at the utterance world i is passed to the target world j;
- The context at world j is updated with the base proposition from world i;
- The base of the utterance world i is reset after being employed;
- The logical content of proposition A is evaluated at the target world j.

Finally, same as above, we provide an illustration, which elucidates the environment of **at** **H** A, where **H** is a possible world constant, A is the dynamic proposition to be evaluated. Again, we assume a is the logical content of A. Since **at** **H** A is built upon A, we may contrast the following analysis with the information in figure 7.1 and table 7.1, which will not be repeated here any more.

First of all, the possible worlds hierarchy of **at** **H** A is illustrated in figure 7.4:

We have to distinguish two cases: if the utterance world i is equal to the target world **H**, **at** **H** A and A are identical formulas, that is to say, **at** **H** A being true at **H** is equivalent to A being true at i, as shown in the upper part of figure 7.4; otherwise, if i and **H** are different worlds, **at** **H** A is true at i means $(\mathbf{mb}\ i) \wedge A$ is true at **H**, where $(\mathbf{mb}\ i)$ is the background information at the utterance world i, as shown in the lower portion of figure 7.4. The **at** function merely evaluates a dynamic proposition at another world, it does not change the default salient world w_s. Also no explicit accessibility relation among possible worlds, such as between i and w_s, between i and **H**, can be induced from **at**.

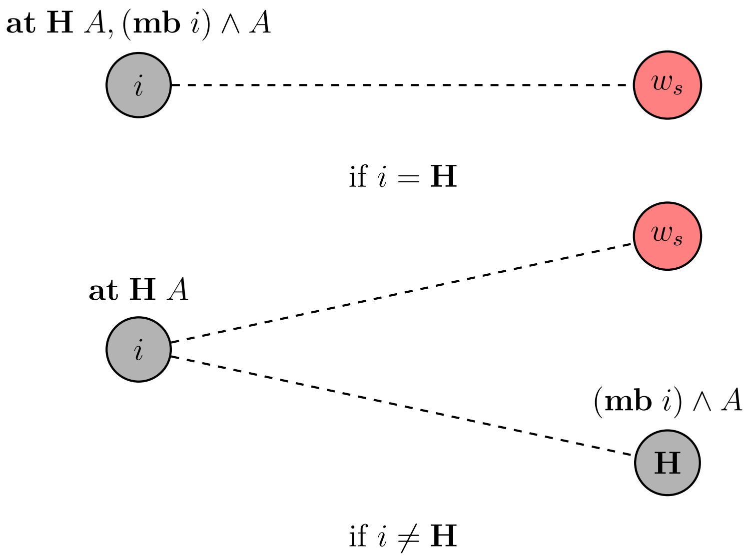


Fig. 7.4 Possible Worlds Hierarchy of **at H** A

As to the detailed content of the environment at each world, we will also have to distinguish the above mentioned two cases. These two situations are listed separately in table 7.4:

Existing World		Environment
$i = \mathbf{H}$	$\mathbf{H}$	$\langle a \wedge_i (\mathbf{bkgd}\ e\ i), a \wedge_i (\mathbf{base}\ e\ i)\rangle$
	w_s	$\langle(\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)), (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$
$i \neq \mathbf{H}$	i	$\langle(\mathbf{bkgd}\ e\ i), \top_i\rangle$
	w_s	$\langle(\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)), (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$
	$\mathbf{H}$	$\langle a \wedge_i (\mathbf{base}\ e\ i) \wedge_i \mathbf{bkgd}(e, \mathbf{H}),$ $a \wedge_i (\mathbf{base}\ e\ i) \wedge_i \mathbf{base}(e, \mathbf{H})\rangle$

Table 7.4 Environment at Each World of **at H** A

On the one hand, when the target world **H** is identical to the utterance world i, the result of **at H** A is the same as the one for dynamic proposition A, see table 7.1. This is exactly what we expect. On the other hand, when **H** is different from i, the base proposition of i, together with the logical content of A, will be updated to the environment of **H**.

Modal Expressions

In this subsection, we will propose the semantic entries for the linguistic expressions which trigger epistemic modality, namely the modals: *might* and *would*. The technical details will largely depend on what we have introduced above. Basically, we will show how to build up complex entries with $\Diamond$, $\Box$, and **at**. We shall start with *might*, then address *would*.

The modal verb *might*, which introduces the epistemic possibility modality, is of type

$\Omega_i \to \Omega_i$. We propose the lexical entry of *might* as follows:

$$[\![might]\!]_{M\text{-}TTDL} = \lambda Ae\phi i.(\mathbf{at}\ (\mathbf{woi}\ e)\ (\Diamond A))e\phi i \tag{7.45}$$

The intuition behind the above formula is as follows: when we say something might happen, it means that at a particular world (the salient world), the proposition is possibly true, which logically denotes that the proposition is satisfied at some accessible world. This is exactly the meaning born within the entry of *might* as in formula 7.45: a dynamic proposition A is possibly true with regard to the salient world.

The unfolding of 7.45 is rather tedious, we will not do it here. Instead, similar as previously, we will provide the possible worlds hierarchy and the environment status of $[\![might]\!]_{M\text{-}TTDL}A$ as an illustration. Since *might* involves function **at**, we will have to determine the identity between the salient world w_s and the utterance world i. The following analysis, which will be divided into two separate cases, is analogous to the one for **at**. In addition, since $[\![might]\!]_{M\text{-}TTDL}A$ is built upon A, we can compare the following analysis with the information in figure 7.1 and table 7.1, which will not be repeated here any more.

The possible worlds hierarchy of interpreting $[\![might]\!]_{M\text{-}TTDL}A$ is illustrated in figure 7.5, in which we abbreviate $[\![might]\!]_{M\text{-}TTDL}A$ as $\mathbf{M}\ A$. Its environment status is listed in table 7.5.

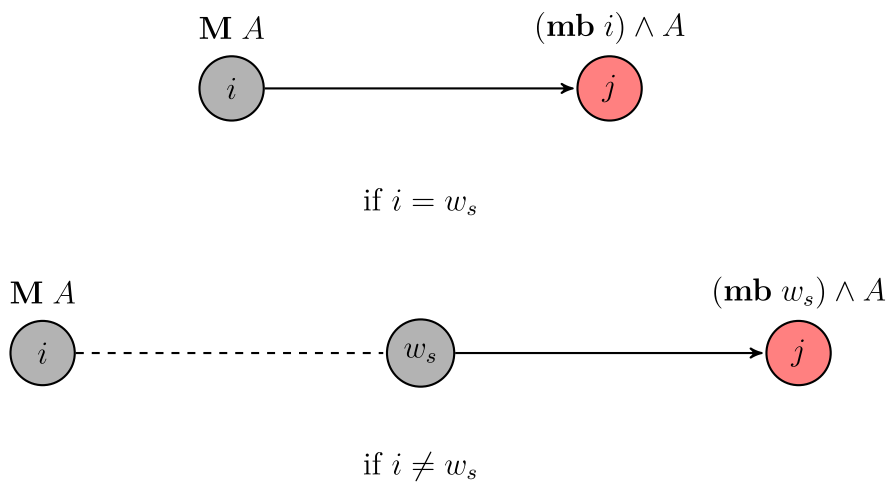


Fig. 7.5 Possible Worlds Hierarchy of $[\![might]\!]_{M\text{-}TTDL}A$

Existing World		Environment
$i = w_s$	i	$\langle(\mathbf{bkgd}\ e\ i), \top_i\rangle$
	j	$\langle a \wedge_i (\mathbf{bkgd}\ e\ i), a \wedge_i (\mathbf{base}\ e\ i)\rangle$
$i \neq w_s$	i	$\langle(\mathbf{bkgd}\ e\ i), \top_i\rangle$
	w_s	$\langle(\mathbf{base}\ e\ i) \wedge_i (\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)), \top_i\rangle$
	j	$\langle a \wedge_i (\mathbf{base}\ e\ i) \wedge_i (\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)),$ $a \wedge_i (\mathbf{base}\ e\ i) \wedge_i (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$

Table 7.5 Environment at Each World of $[\![might]\!]_{M\text{-}TTDL}A$

If we compare the effect of $[\![might]\!]_{M\text{-}TTDL}$ and $\Diamond$, namely figure 7.5 and figure 7.2, table 7.5 and table 7.2, we can find out that $[\![might]\!]_{M\text{-}TTDL}A$ and $\Diamond A$ generate the same result when the default salient world w_s is identical to the utterance world i; however, if the two worlds are different, $[\![might]\!]_{M\text{-}TTDL}A$ and $\Diamond A$ will generate different results. This is because when interpreting $[\![might]\!]_{M\text{-}TTDL}A$, a new possible world j will be established over the salient world w_s, rather than over the utterance world i, as shown in the lower part of figure 7.5.

Same as *might*, the entry for the modal verb *would* is also of type $\Omega_i \to \Omega_i$. Its representation is also made up of previous introduced entries:

$$[\![would]\!]_{M\text{-}TTDL} = \lambda Ae\phi i.(\mathbf{at}\ (\mathbf{woi}\ e)\ (\Box A))e\phi i \tag{7.46}$$

The intuition behind the above formula is as follows: when we say something would happen, it means at the salient world, the proposition is necessarily true, which logically denotes that the proposition is satisfied at every accessible world. This is exactly the meaning born within the entry of *would* as in formula 7.46: a dynamic proposition A is necessarily true with regard to the salient world.

Likewise, we are not going to unfold 7.46, but an analogous illustration containing the possible worlds structure and the environment status will be provided. Same as $[\![might]\!]_{M\text{-}TTDL}A$, $[\![would]\!]_{M\text{-}TTDL}A$ is also built upon A, we can compare the following analysis with the information in figure 7.1 and table 7.1.

The possible worlds hierarchy of interpreting $[\![would]\!]_{M\text{-}TTDL}A$ is illustrated in figure 7.6, in which we abbreviate $[\![would]\!]_{M\text{-}TTDL}A$ as $\mathbf{W}\ A$:

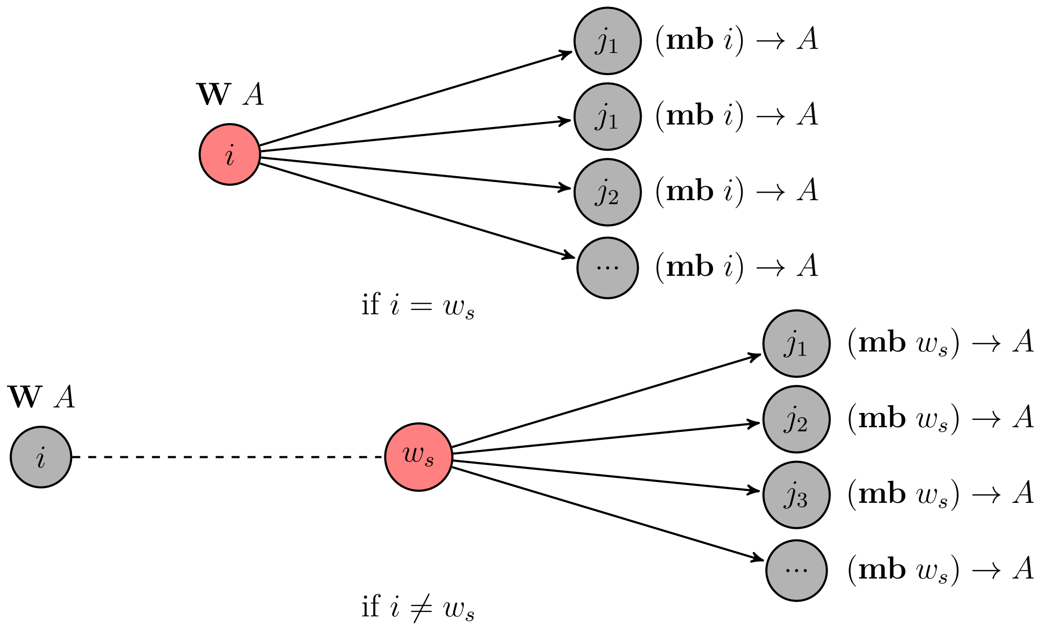


Fig. 7.6 Possible Worlds Hierarchy of $[\![would]\!]_{M\text{-}TTDL}A$

The environment status for $[\![would]\!]_{M\text{-}TTDL}A$ is listed in table 7.6. The two situations, one where i equals w_s, and the other where they are different, are separately presented.

As motivated by the comparison between *might* and $\Diamond$, a corresponding contrast can be drawn between $[\![would]\!]_{M\text{-}TTDL}$ and $\Box$: $[\![would]\!]_{M\text{-}TTDL}A$ and $\Box A$ yield the same result when the default salient world w_s is identical to the utterance world i, see figure 7.6 and

Existing World		**Environment**
$i = w_s$	i	$\langle(\mathbf{bkgd}\ e\ i), (\mathbf{base}\ e\ i)\rangle$
	j	$\langle(\mathbf{bkgd}\ e\ j), (\mathbf{base}\ e\ j)\rangle$
$i \neq w_s$	i	$\langle(\mathbf{bkgd}\ e\ i), \top_i\rangle$
	w_s	$\langle(\mathbf{base}\ e\ i) \wedge_i (\mathbf{bkgd}\ e\ (\mathbf{woi}\ e)),$ $(\mathbf{base}\ e\ i) \wedge_i (\mathbf{base}\ e\ (\mathbf{woi}\ e))\rangle$
	j	$\langle(\mathbf{bkgd}\ e\ j), (\mathbf{base}\ e\ j)\rangle$

Table 7.6 Environment at Each World of $[\![would]\!]_{M\text{-}TTDL}A$

figure 7.3, table 7.6 and table 7.3; while when the two worlds are different, the dynamic proposition $\Box A$ is still uttered at world i, but since the salient world w_s is not i, possibly due to some modality in the preceding discourse, the necessity from $\Box A$ will be established over w_s, rather than over i.

The entries for *might* and *would* share many properties, especially the way they affect the possible worlds hierarchy in different situations. However, as we may notice at the same time, $[\![might]\!]_{M\text{-}TTDL}$ has the potential to update the salient world, while $[\![would]\!]_{M\text{-}TTDL}$ does not. In addition, $[\![might]\!]_{M\text{-}TTDL}$ always resets the base of the salient world, the one based on which new worlds are built upon, while $[\![would]\!]_{M\text{-}TTDL}$ does not.

7.3.5 From TTDL to M-TTDL

In the previous chapter when we were introducing DN-TTDL, we have shown that if a discourse does not contain any multi-negation, its logical representation in DN-TTDL and TTDL are identical (see section 6.2.2). This implies that DN-TTDL can be successfully applied to account for the anaphoric links in inter-sentential anaphora and donkey sentence.

The goal of this subsection is to examine the relation between TTDL and M-TTDL. We would like to know whether M-TTDL is akin to DN-TTDL, such that it can also cover the paradigm phenomena that dynamic semantics systems are designed to solve. Because formulas in TTDL and M-TTDL have very different forms, it is difficult to compare the two systems in a straightforward way. Hence we will first introduce a bridging framework, which is syntactically similar to TTDL. It is called **Propositional TTDL** (**P-TTDL**), because the left context in the new framework is updated with propositions, rather than discourse referents[8]. This is exactly the case in M-TTDL, see formula 7.16. More specifically, P-TTDL is defined upon M-TTDL by discarding the notion of modality. In other words, P-TTDL is a simplified variant of M-TTDL such that it is not concerned with possible worlds.

First of all, before presenting the formal details of P-TTDL, we define a mapping function, which is used to abstract variables from a proposition.

Definition 7.3.6. Given the signature Σ_0, let $M \in \mathbb{F}_{\Sigma_0}$ be a simply typed λ-calculus formula (definition 7.3.9). The mapping function ${}^o\mathsf{M}^\gamma : o \to (\gamma \to \gamma)$ takes M as input, and returns a function (of type $\gamma \to \gamma$) , which appends variables occurring in M to a list of referents, namely:

[8]Please note that P-TTDL is very similar to the framework GL presented in Lebedeva (2012).

$$^{o}\mathsf{M}^{\gamma}(M) = \lambda e.(x_1 :: (... :: (x_n :: e)))$$

where for all x, if x occurs in M, then $x \in \{x_1, ..., x_n\}$.

The above function ensures that the left context in P-TTDL is in the form of a list of discourse referents (of type γ), although it is updated with propositions. This permits a more direct comparison between P-TTDL and TTDL on the syntactic level. The function $^{o}\mathsf{M}^{\gamma}$ will be used to define the dynamization function in P-TTDL, which we will see shortly below.

Generally speaking, P-TTDL shares with TTDL most of the technical details, such as the signature (definition 4.4.1), the way in which contexts and propositions are interpreted (formula 4.31). The dynamic logics of the two frameworks are also alike. For instance, P-TTDL reuses the dynamic conjunction (formula 4.34) and the dynamic negation (formula 4.37) in TTDL. However, the dynamic existential quantifier in P-TTDL is slightly different from the one in TTDL:

$$\exists^{d}_{P\text{-}TTDL} \triangleq \lambda Pe\phi.\exists(\lambda x.Pxe\phi) \tag{7.47}$$

As we can see, $\exists^{d}_{P\text{-}TTDL}$ is structurally the same as the dynamic quantifier $^{\iota}\exists^{d}_{M\text{-}TTDL}$ (formula 7.40) in M-TTDL: the former is the extensional version of the latter. However, unlike the dynamic quantifier $\exists^{d}_{TTDL}$ (formula 4.38) in TTDL, both $\exists^{d}_{P\text{-}TTDL}$ and $^{\iota}\exists^{d}_{M\text{-}TTDL}$ do not update the bound variable into the current left context. The reason has already been mentioned above: M-TTDL and P-TTDL updates propositions instead of referents into the context, so the dynamic existential quantifiers in M-TTDL and P-TTDL do not work in the same way as the one in TTDL.

Now let's turn to the dynamic translation in P-TTDL. To distinguish the translation in P-TTDL from the one in TTDL, we introduce the following new notation.

Notation 7.3.2. We use the dash-bar notation, for instance $\bar{\bar{M}}$, to denote the **dynamic translation** of a λ-term M in P-TTDL.

Same as in previous continuation-based frameworks (e.g., TTDL, DN-TTDL, and M-TTDL), the translation of non-logical constants is achieved through the dynamization function $\mathbb{D}$ and the staticization function $\mathbb{S}$. In P-TTDL, they are simultaneously defined on each other as follows.

Definition 7.3.7. The **dynamization function** $\mathbb{D}^{p}_{\tau}$, which takes an input λ-term A of type $(\gamma \to \tau)$, returns an output λ-term A' of type $\overline{\tau}$; the **staticization function** $\mathbb{S}^{p}_{\tau}$, which takes an input λ-term A' of type $\overline{\tau}$, returns an output λ-term A of type $(\gamma \to \tau)$.

- $\mathbb{D}^{p}_{\tau}$ is defined inductively on type τ as follows:
  1. $\mathbb{D}^{p}_{\iota}A = A\ \mathsf{nil}$;
  2. $\mathbb{D}^{p}_{o}A = \lambda e\phi.(Ae \wedge \phi(^{o}\mathsf{M}^{\gamma}(Ae)e))$;
  3. $\mathbb{D}^{p}_{\alpha\to\beta}A = \lambda x.\mathbb{D}^{p}_{\beta}(\lambda e.Ae(\mathbb{S}^{p}_{\alpha}xe))$.
- $\mathbb{S}^{p}_{\tau}$ is defined inductively on type τ as follows:
  1. $\mathbb{S}^{p}_{\iota}A' = \lambda e.A'$;
  2. $\mathbb{S}^{p}_{o}A' = \lambda e.A'\ e\ \mathbf{stop}$;

3. $\mathbb{S}^p_{\alpha\rightarrow\beta}A' = \lambda e.(\lambda x.\mathbb{S}^p_\beta(A'(\mathbb{D}^p_\alpha(\lambda e'.x)))e)$.

Note that the above definition is analogous to the one in M-TTDL (definition 7.3.4), the only difference is between $\mathbb{D}^p_o$ and $\mathbb{D}^m_{o_i}$. As we presented in section 7.3.1, the left context in M-TTDL is rather complex, see formula 7.16. However, since possible worlds are not involved in P-TTDL, it does not need such a left context of the same complexity. Hence in P-TTDL, we can directly insert the proposition in the left context without appealing to the function **up_context**. Moreover, the function ${}^o\mathrm{M}^\gamma$ is employed to transform the updated proposition into a list of updated referents.

Based on the above characterizations of $\mathbb{D}^p$ and $\mathbb{S}^p$, we can thus define the dynamic translation of λ-terms in P-TTDL. One can compare it with the rules of translation in TTDL (definition 4.4.4).

Definition 7.3.8. The **propositional dynamic translation of a λ-term** M (of type τ): $\overline{\overline{M}}$, which is another λ-term of type $\overline{\tau}$, is defined as follows:

1. $\overline{\overline{x}} = \overline{x}$, if $x \in \mathcal{X}$;

2. $\overline{\overline{\mathbf{a}}} = \mathbb{D}^p_\tau(\lambda e.\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}_{NL}$ and $\mathbf{a} : \tau$;

3. $\overline{\overline{\wedge}} = \overline{\wedge}$, see formula 4.34;

4. $\overline{\overline{\neg}} = \overline{\neg}$, see formula 4.37;

5. $\overline{\overline{\exists}} = \exists^d_{P\text{-}TTDL}$, see formula 7.47;

6. $\overline{\overline{(MN)}} = \overline{\overline{M}}\ \overline{\overline{N}}$;

7. $\overline{\overline{(\lambda x.M)}} = \lambda x.\overline{\overline{M}}$.

Up until now, we have finished presenting the framework P-TTDL. As one can see, according to its formal details (definition 7.3.7 and 7.3.8), P-TTDL is indeed the un-modalized version of M-TTDL: contexts in the two systems are updated in the same way (with propositions), also, their dynamic logics are in parallel. As a result, by comparing P-TTDL with TTDL, we can indirectly investigate the relation between M-TTDL and TTDL. This is what we are going to do immediately below.

Firstly, we draw the following lemma for the dynamization function $\mathbb{D}^p$, which will be used in future proofs.

Lemma 7.3.1. Given the signature Σ_0 (definition 3.2.13), let M_n be a λ-term of type $\underbrace{\iota \rightarrow \ldots \rightarrow \iota}_{n} \rightarrow o$, then:

$$\mathbb{D}^p_{\underbrace{\iota \rightarrow \ldots \rightarrow \iota}_{n}\rightarrow o}(\lambda e.M_n) = \lambda x_1 \ldots x_n.\mathbb{D}^p_o(\lambda e.M_n x_1 \ldots x_n) \tag{7.48}$$

As one might have noticed, the above lemma is in a completely similar structure as lemma 6.2.3, which was proposed for prior versions of the dynamization function: $\mathbb{D}$ and $\mathbb{D}^{dn}$. These two lemmas can be proved in exactly the same way (by induction). Hence we will omit the detailed proof for lemma 7.3.1, interested readers may refer back to section 6.2.2.

Definition 7.3.9. Given signature Σ_0 (definition 3.2.13), the set of simply typed λ-calculus formulas $\mathbb{F}_{\Sigma_0}$ is inductively defined as follows:

1. $\mathbf{P}t_1...t_n \in \mathbb{F}_{\Sigma_0}$, whenever $\mathbf{P} \in \mathcal{C}_{NL}$, $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and $\mathbf{P} : \underbrace{\iota \to ... \to \iota}_{n} \to o$, $t_1, ..., t_n : \iota$;
2. $\neg M_1 \in \mathbb{F}_{\Sigma_0}$, whenever $M_1 \in \mathbb{F}_{\Sigma_0}$;
3. $M_1 \wedge M_2 \in \mathbb{F}_{\Sigma_0}$, whenever $M_1, M_2 \in \mathbb{F}_{\Sigma_0}$;
4. $\exists(\lambda x.M_1) \in \mathbb{F}_{\Sigma_0}$, whenever $x \in \mathcal{X}$, $M_1 \in \mathbb{F}_{\Sigma_0}$.

In order to characterize the relation between P-TTDL and TTDL, we also need a novel notion of free variables, called alternative free variables.

Definition 7.3.10. Given the signature Σ_0, let $M \in \mathbb{F}_{\Sigma_0}$ be a simply typed λ-calculus formula (definition 7.3.9). The set of **alternative free variables** of M, in notation $AFV(M)$, is inductively defined in terms of the free variable function FV (definition 3.1.11 and 3.1.12) as follows:

1. $AFV(\mathbf{P}t_1...t_n) = FV(\mathbf{P}t_1...t_n)$;
2. $AFV(\neg M_1) = \emptyset$;
3. $AFV(M_1 \wedge M_2) = AFV(M_1) \cup AFV(M_2)$;
4. $AFV(\exists(\lambda x.M_1)) = AFV(M_1) - \{x\}$.

For a simply typed λ-calculus M, if $AFV(M)$ is an empty set, namely $AFV(\phi) = \emptyset$, we say M is an **alternative closed formula**. For the relation between the two functions FV and AFV, we can consult the following lemma.

Lemma 7.3.2. Given the signature Σ_0. For any simply typed λ-calculus formula $M \in \mathbb{F}_{\Sigma_0}$, the set of its alternative free variables is included in the set of its free variables, namely:

$$\forall M \in \mathbb{F}_{\Sigma_0} : AFV(M) \subseteq FV(M) \tag{7.49}$$

Proof. We prove the lemma by induction on the form of M.

1. Let $M = \mathbf{P}t_1...t_n$, where $\mathbf{P} \in \mathcal{C}_{NL}$, $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$. According to definition 7.3.10:

   $$AFV(M) = FV(M)$$

   Hence $AFV(M) \subseteq FV(M)$.

2. Let $M = \neg M_1$, where $M_1 \in \mathbb{F}_{\Sigma_0}$ is a simply typed λ-calculus formula. According to definition 7.3.10:

   $$AFV(M) = \emptyset$$

   Since the empty set $\emptyset$ is a subset of any other set, so $AFV(M) \subseteq FV(M)$.

3. Let $M = M_1 \wedge M_2$, where $M_1, M_2 \in \mathbb{F}_{\Sigma_0}$ are simply typed λ-calculus formulas. According to definition 3.1.12 and 7.3.10:

$$FV(M) = FV(M_1) \cup FV(M_2)$$
$$AFV(M) = AFV(M_1) \cup AFV(M_2)$$

By induction hypothesis, we know that:

$$AFV(M_i) \subseteq FV(M_i), \text{ where } i \in \{1, 2\}$$

The union of subsets is a subset of the union of their supersets. As a result, $AFV(M) \subseteq FV(M)$.

4. Let $M = \exists(\lambda x.\mathbf{P}x \wedge M_1)$, where $M_1 \in \mathbb{F}_{\Sigma_0}$ is a simply typed λ-calculus formula. According to definition 3.1.12 and 7.3.10:

$$\begin{aligned} FV(M) &= FV(\mathbf{P}x \wedge M_1) - \{x\} \\ &= FV(\mathbf{P}x) \cup FV(M_1) - \{x\} \\ &= \{x\} \cup FV(M_1) - \{x\} \\ &= FV(M_1) - \{x\} \end{aligned}$$

$$AFV(M) = AFV(M_1) - \{x\}$$

By induction hypothesis, we know that:

$$AFV(M_1) \subseteq FV(M_1)$$

By subtracting the same element in a subset and its superset, the inclusion still holds between the two new sets. As a result, $AFV(M) \subseteq FV(M)$.

As a result, given Σ_0, $AFV(M)$ is a subset of $FV(M)$ for every simply typed λ-calculus formula $M \in \mathbb{F}_{\Sigma_0}$. □

Finally, we propose the following lemma, which links the two frameworks: P-TTDL and TTDL, together. The lemma is immediately followed by its proof.

Lemma 7.3.3. Given the signature Σ_0, let $M \in \mathbb{F}_{\Sigma_0}$ be a simply typed λ-calculus formula (definition 7.3.9). Its dynamic translation under TTDL $\overline{M}$ and its dynamic translation under P-TTDL $\overset{\text{- - -}}{M}$ bear the following relation:

$$\overset{\text{- - -}}{M} = \lambda e\phi.\overline{M}(x_1 :: (... :: (x_m :: e)))\phi \tag{7.50}$$

where $\{x_1, ..., x_m\} = AFV(M)$. That is to say, the two dynamic terms $\overset{\text{- - -}}{M}$ and $\overline{M}$ are identical, except that the alternative free variables of M are updated into the left context of the former but not of the latter.

Proof. We prove the lemma by induction on the form of M.

1. Let $M = \mathbf{P}t_1...t_n$, where $\mathbf{P} \in \mathcal{C}_{NL}$, $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$. Since M is an atomic formula, according to definitions 7.3.10, 3.1.12 and 3.1.11, all variables occurring in M are free variables, and are alternative free variables as well, namely:

$$AFV(M) = FV(M) = \{x_1, ..., x_m\} = \{t_1, ..., t_n\} \cap \mathcal{X}, \text{ where } m \leq n \tag{7.51}$$

   (a) First we examine $\overline{M}$. Since $\mathbf{P}$ is of type $\underbrace{\iota \to ... \to \iota}_{n} \to o$, according to definition 4.4.4 and lemma 6.2.3:

$$\overline{\mathbf{P}} = \mathbb{D}_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}_o(\lambda e.\mathbf{P}x_1...x_n) \tag{7.52}$$

   Since $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and all of them are of type ι, according to definitions 4.4.4 and 4.4.3:

$$\overline{t_i} = t_i, \text{ for all } i \in \{1, ..., n\} \tag{7.53}$$

   Finally, according to definitions 4.4.4 and 4.4.3, formulas 7.52 and 7.53:

$$\begin{aligned}\overline{M} &= \overline{\mathbf{P}t_1...t_n} = \overline{\mathbf{P}}\ \overline{t_1}...\overline{t_n} = \overline{\mathbf{P}}t_1...t_n \\ &= \lambda x_1...x_n.\mathbb{D}_o(\lambda e.\mathbf{P}x_1...x_n)t_1...t_n \\ &\twoheadrightarrow_\beta \mathbb{D}_o(\lambda e.\mathbf{P}t_1...t_n) \\ &= \lambda e\phi.\mathbf{P}t_1...t_n \wedge \phi e\end{aligned} \tag{7.54}$$

   By updating the alternative free variables of M (formula 7.51) into the current left context of $\overline{M}$, we can obtain:

$$\begin{aligned}&\lambda e\phi.\overline{M}(x_1 :: (... :: (x_m :: e)))\phi \\ =&\lambda e\phi.(\lambda e'\phi'.\mathbf{P}t_1...t_n \wedge \phi' e')(x_1 :: (... :: (x_m :: e)))\phi \\ \twoheadrightarrow_\beta&\lambda e\phi.\mathbf{P}t_1...t_n \wedge \phi(x_1 :: (... :: (x_m :: e)))\end{aligned} \tag{7.55}$$

   (b) Then let's turn to $\overset{-\,-\,-}{M}$. Since $\mathbf{P}$ is of type $\underbrace{\iota \to ... \to \iota}_{n} \to o$, according to definition 7.3.8 and lemma 7.3.1:

$$\overset{-\,-\,-}{\mathbf{P}} = \mathbb{D}^p_{\underbrace{\iota \to ... \to \iota}_{n} \to o}(\lambda e.\mathbf{P}) = \lambda x_1...x_n.\mathbb{D}^p_o(\lambda e.\mathbf{P}x_1...x_n) \tag{7.56}$$

   Since $t_1, ..., t_n \in \mathcal{X} \cup \mathcal{C}_{NL}$, and all of them are of type ι, according to definitions 7.3.8 and 7.3.7:

$$\overset{-\,-\,-}{t_i} = t_i, \text{ for all } i \in \{1, ..., n\} \tag{7.57}$$

Finally, according to definitions 7.3.8 and 7.3.7, formulas 7.56 and 7.57:

$$
\begin{aligned}
\overline{\overline{M}} = \overline{\overline{\mathbf{P}t_1...t_n}} &= \overline{\overline{P}}\ \overline{\overline{t_1}}\,...\,\overline{\overline{t_n}} = \overline{\overline{P}}\,t_1...t_n \\
&= \lambda x_1...x_n.\mathbb{D}^p_o(\lambda e.Px_1...x_n)t_1...t_n \\
&\twoheadrightarrow_\beta \mathbb{D}^p_o(\lambda e.\mathbf{P}t_1...t_n) \\
&= \lambda e\phi.\mathbf{P}t_1...t_n \wedge \phi((^o\mathrm{M}^\gamma\ \mathbf{P}t_1...t_n)e) \\
&= \lambda e\phi.\mathbf{P}t_1...t_n \wedge \phi(x_1 :: (... :: (x_m :: e)))
\end{aligned}
\tag{7.58}
$$

Hence, by comparing formulas 7.55 and 7.58, we can draw that when $M = \mathbf{P}t_1...t_n$, $\overline{\overline{M}}$ and $\overline{M}$ only differ on the context information regarding the alternative free variables in M.

2. Let $M = \neg M_1$, where $M_1 \in \mathbb{F}_{\Sigma_0}$ is a simply typed λ-calculus formula. By induction hypothesis:

$$
\overline{\overline{M_1}} = \lambda e\phi.\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e)))\phi \tag{7.59}
$$

where $\{x_{11}, ..., x_{1m}\} = AFV(M_1)$. In addition, because M is a negation, according to definition 7.3.10, there is no alternative free variable in M, namely:

$$
AFV(M) = \emptyset \tag{7.60}
$$

(a) First we examine $\overline{M}$. According to definition 4.4.4, formula 4.37:

$$
\begin{aligned}
\overline{M} = \overline{\neg M_1} &= \overline{\neg}\,\overline{M_1} = \neg^d_{TTDL}\overline{M_1} \\
&= (\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)\overline{M_1} \\
&\rightarrow_\beta \lambda e\phi.\neg(\overline{M_1}\ e\ \mathbf{stop}) \wedge \phi e
\end{aligned}
\tag{7.61}
$$

Updating the alternative free variables of M into the current left context of $\overline{M}$ will not change $\overline{M}$ because $AFV(M)$ is simply an empty set (see formula 7.60).

(b) Then let's turn to $\overline{\overline{M}}$. According to definition 7.3.8, formulas 4.37 and 7.59:

$$
\begin{aligned}
\overline{\overline{M}} &= \overline{\overline{\neg M_1}} = \overline{\overline{\neg}}\ \overline{\overline{M_1}} = \neg^d_{TTDL}\overline{\overline{M_1}} \\
&= (\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)\overline{\overline{M_1}} \\
&\rightarrow_\beta \lambda e\phi.\neg(\overline{\overline{M_1}}\ e\ \mathbf{stop}) \wedge \phi e \\
&= \lambda e\phi.\neg((\lambda e'\phi'.\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e')))\phi')\ e\ \mathbf{stop}) \wedge \phi e \\
&\twoheadrightarrow_\beta \lambda e\phi.\neg(\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e)))\mathbf{stop}) \wedge \phi e
\end{aligned}
\tag{7.62}
$$

Since the empty continuation **stop** will block all the variables in the left context, the resulting terms in formulas 7.61 and 7.62 are the same, namely:

$$\begin{aligned} \overset{\text{- - -}}{M} &= \lambda e\phi.\neg(\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e)))\textbf{stop}) \wedge \phi e \\ &= \lambda e\phi.\neg(\overline{M_1}\ e\ \textbf{stop}) \wedge \phi e = \overline{M} \end{aligned} \tag{7.63}$$

As a result, when $M = \neg M_1$, $\overset{\text{- - -}}{M}$ and $\overline{M}$ only differ on the context information regarding the alternative free variables in M. Particularly, they are identical in this case (see formula 7.63) because $AFV(M) = \emptyset$.

3. Let $M = M_1 \wedge M_2$, where $M_1, M_2 \in \mathbb{F}_{\Sigma_0}$ are simply typed λ-calculus formulas. By induction hypothesis, we can draw formula 7.59 for M_1 and the following similar formula for M_2:

$$\overset{\text{- - - -}}{M_2} = \lambda e\phi.\overline{M}_2(x_{21} :: (... :: (x_{2m'} :: e)))\phi \tag{7.64}$$

where $\{x_{21}, ..., x_{2m'}\} = AFV(M_2)$. In addition, according to definition 7.3.10, the alternative free variables in M are the union of the alternative free variables in M_1 and M_2, namely:

$$AFV(M) = AFV(M_1) \cup AFV(M_2) = \{x_{11}, ..., x_{1m}, x_{21}, ..., x_{2m'}\} \tag{7.65}$$

(a) First we examine $\overline{M}$. According to definition 4.4.4 and formula 4.34:

$$\begin{aligned} \overline{M} = \overline{M_1 \wedge M_2} = \overline{M_1}\overline{\wedge}\overline{M_2} &= \overline{M_1} \wedge^d_{TTDL} \overline{M_2} \\ &= (\lambda ABe\phi.Ae(\lambda e'.Be'\phi))\ \overline{M_1}\ \overline{M_2} \\ &\twoheadrightarrow_\beta \lambda e\phi.\overline{M_1}e(\lambda e'.\overline{M_2}e'\phi) \end{aligned} \tag{7.66}$$

By updating the alternative free variables of M (formula 7.65) into the current left context of $\overline{M}$, we can obtain:

$$\begin{aligned} &\lambda e\phi.\overline{M}(x_{11} :: (... :: (x_{1m} :: (x_{21} :: (... :: (x_{2m'} :: e))))))\phi \\ =&\lambda e\phi.(\lambda e\phi.\overline{M_1}e(\lambda e'.\overline{M_2}e'\phi))(x_{11} :: (... :: (x_{1m} :: (x_{21} :: (... :: (x_{2m'} :: e))))))\phi \\ \twoheadrightarrow_\beta&\lambda e\phi.\overline{M_1}(x_{11} :: (... :: (x_{1m} :: (x_{21} :: (... :: (x_{2m'} :: e))))))(\lambda e'.\overline{M_2}e'\phi) \end{aligned} \tag{7.67}$$

(b) Then let's turn to $\overset{\text{- - -}}{M}$. According to definition 7.3.8, formulas 4.34, 7.59 and

7.64:

$$
\begin{aligned}
\overset{\text{- - -}}{M} &= \overset{\text{- - - - - - - - -}}{M_1 \wedge M_2} = \overset{\text{- - -}}{M_1} \,\overset{\text{- -}}{\wedge}\, \overset{\text{- - -}}{M_2} = \overset{\text{- - -}}{M_1} \wedge^{d}_{TTDL} \overset{\text{- - -}}{M_2} \\
&= (\lambda ABe\phi.Ae(\lambda e'.Be'\phi))\ \overset{\text{- - -}}{M_1}\ \overset{\text{- - -}}{M_2} \\
&\twoheadrightarrow_\beta \lambda e\phi.\overset{\text{- - -}}{M_1}\, e(\lambda e'.\overset{\text{- - -}}{M_2}\, e'\phi) \\
&= \lambda e\phi.(\lambda e\phi.\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e)))\phi)e \\
&\qquad\qquad (\lambda e'.(\lambda e\phi.\overline{M}_2(x_{21} :: (... :: (x_{2m'} :: e)))\phi)e'\phi) \\
&\twoheadrightarrow_\beta \lambda e\phi.(\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e))) \\
&\qquad\qquad (\lambda e'.(\overline{M}_2(x_{21} :: (... :: (x_{2m'} :: e')))\phi)))
\end{aligned}
\tag{7.68}
$$

As we can see from formula 7.68, elements of $AFV(M_1)$ and $AFV(M_2)$ are updated into the left context of $\overset{\text{- - -}}{M}$ sequentially. This is the same as updating all the variables all at once. So the resulting terms in formulas 7.67 and 7.68 are the same, namely:

$$
\begin{aligned}
\overset{\text{- - -}}{M} &= \lambda e\phi.(\overline{M}_1(x_{11} :: (... :: (x_{1m} :: e))) \\
&\qquad\qquad (\lambda e'.(\overline{M}_2(x_{21} :: (... :: (x_{2m'} :: e')))\phi))) \\
&= \lambda e\phi.\overline{M_1}(x_{11} :: (... :: (x_{1m} :: (x_{21} :: (... :: (x_{2m'} :: e))))))(\lambda e'.\overline{M_2}e'\phi) \\
&= \lambda e\phi.\overline{M}(x_{11} :: (... :: (x_{1m} :: (x_{21} :: (... :: (x_{2m'} :: e))))))\phi
\end{aligned}
\tag{7.69}
$$

Consequently, when $M = M_1 \wedge M_2$, $\overset{\text{- - -}}{M}$ and $\overline{M}$ only differ on the context information regarding the alternative free variables in M.

4. Let $M = \exists(\lambda x.M_1)$, where $x \in \mathcal{X}$, $M_1 \in \mathbb{F}_{\Sigma_0}$. By induction hypothesis:

$$
\overset{\text{- - -}}{M_1} = \lambda e\phi.\overline{M}_1(x :: (x_1 :: (... :: (x_m :: e))))\phi \tag{7.70}
$$

where $\{x, x_1, ..., x_m\} = AFV(M_1)$. In addition, according to definition 7.3.10, the alternative free variables in M are the alternative free variables in M_1 excluding x, namely:

$$
AFV(M) = AFV(M_1) - \{x\} = \{x_1, ..., x_m\} \tag{7.71}
$$

(a) First we examine $\overline{M}$. According to definition 4.4.4, formula 4.38:

$$
\begin{aligned}
\overline{M} &= \overline{\exists(\lambda x.M_1)} = \overline{\exists}(\lambda x.\overline{M_1}) = \exists^{d}_{TTDL}(\lambda x.\overline{M_1}) \\
&= (\lambda Pe\phi.\exists(\lambda x.Px(x :: e)\phi))(\lambda x.\overline{M_1}) \\
&\twoheadrightarrow_\beta \lambda e\phi.\exists(\lambda x.\overline{M}_1(x :: e)\phi)
\end{aligned}
\tag{7.72}
$$

By updating the alternative free variables of M (formula 7.71) into the current left context of $\overline{M}$, we can obtain:

$$
\begin{aligned}
&\lambda e\phi.\overline{M}(x_1 :: (\ldots :: (x_m :: e)))\phi \\
=&\lambda e\phi.(\lambda e'\phi'.\exists(\lambda x.\overline{M_1}(x :: e')\phi'))(x_1 :: (\ldots :: (x_m :: e)))\phi \\
\twoheadrightarrow_\beta&\lambda e\phi.\exists(\lambda x.\overline{M_1}(x :: (x_1 :: (\ldots :: (x_m :: e))))\phi)
\end{aligned}
\tag{7.73}
$$

(b) Then let's turn to $\overline{\overline{M}}$. According to definition 7.3.8, formulas 7.47, 7.70:

$$
\begin{aligned}
\overline{\overline{M}} &= \overline{\overline{\exists(\lambda x.M_1)}} = \overline{\overline{\exists}}(\lambda x.\overline{\overline{M_1}}) = \exists^d_{P\text{-}TTDL}(\lambda x.\overline{\overline{M_1}}) \\
&= (\lambda Pe\phi.\exists(\lambda x.Pxe\phi))(\lambda x.\overline{\overline{M_1}}) \\
&\twoheadrightarrow_\beta \lambda e\phi.\exists(\lambda x.\overline{\overline{M_1}}\,e\phi) \\
&= \lambda e\phi.\exists(\lambda x.(\lambda e'\phi'.\overline{M}_1(x :: (x_1 :: (\ldots :: (x_m :: e'))))\phi')e\phi) \\
&\twoheadrightarrow_\beta \lambda e\phi.\exists(\lambda x.\overline{M}_1(x :: (x_1 :: (\ldots :: (x_m :: e))))\phi)
\end{aligned}
\tag{7.74}
$$

Finally, by comparing formulas 7.73 and 7.74, we can draw that when $M = \exists(\lambda x.M_1)$, $\overline{\overline{M}}$ and $\overline{M}$ only differ on the context information regarding the alternative free variables in M.

As a result, given a simply typed λ-calculus formula M, $\overline{\overline{M}}$ differs from $\overline{M}$ only on the aspect that after interpreting $\overline{\overline{M}}$, the alternative free variables of M are updated into the current left context, while it is not the case for $\overline{M}$. □

In our semantic analysis, we are only interested in closed formulas, which do not contain any free variable. As indicated by lemma 7.3.2, for any simply typed λ-calculus formula, the set of its alternative free variables are included in its set of free variables. Hence closed formulas do not contain alternative free variable, as well (the only subset of an empty set is itself). Then according to lemma 7.3.3, the two frameworks: P-TTDL and TTDL, will always obtain the same result, because they only differ in the left context concerning alternative free variables. Since P-TTDL is defined as the unmodalized version of M-TTDL, we thus conclude that M-TTDL and TTDL are compatible in all cases where no modality is involved.

To conclude, TTDL is linked with P-TTDL by lemma 7.3.3, P-TTDL is linked with M-TTDL by its definition (complicating P-TTDL with the notion of possible world will result in M-TTDL). Relations between the three frameworks are depicted in figure 7.7.

Fig. 7.7 Relations between TTDL, P-TTDL, and M-TTDL

7.3.6 Illustration

Other Lexical Entries

With the systematic way of dynamization in section 7.3.3, we can obtain the semantic representation for other linguistic elements in a purely compositional way, exactly the same as what we did previously in sections 4.4.3 and 6.2.4.

We will take transitive verb (e.g., *beat*) as an example and conduct its translation step by step.

1. The standard entry for *beat*:

$$[\![beat]\!] = \lambda OS.S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))$$

   It takes two NPs and yields a proposition, its type is $((\iota \to o) \to o) \to ((\iota \to o) \to o) \to o$.

2. As shown in section 7.3.3:

$$\begin{aligned}\overline{[\![beat]\!]}^{\mathsf{m}} &= \overline{\lambda OS.S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))}^{\mathsf{m}} \\ &= \lambda OS.\overline{S(\lambda x.O(\lambda y.\mathbf{beat}\ x\ y))}^{\mathsf{m}} \\ &= \lambda OS.S(\lambda x.O(\lambda y.\overline{\mathbf{beat}}^{\mathsf{m}}\ x\ y))\end{aligned}$$

3. The predicate symbol **beat** is of type $\iota \to \iota \to o_i$, according to definition 7.3.4:

$$\begin{aligned}\overline{\mathbf{beat}}^{\mathsf{m}} &= \mathbb{D}^m_{\iota\to\iota\to o_i}(\lambda e.\mathbf{beat}) \\ &= \lambda x.\mathbb{D}^m_{\iota\to o_i}(\lambda e.(\lambda e'.\mathbf{beat})e(\mathbb{S}^m_\iota xe)) \\ &\to_\beta \lambda x.\mathbb{D}^m_{\iota\to o_i}(\lambda e.\mathbf{beat}\ x) \\ &= \lambda xy.\mathbb{D}^m_{o_i}(\lambda e.(\lambda e'.\mathbf{beat}\ x)(\mathbb{S}^m_\iota ye)) \\ &\to_\beta \lambda xy.\mathbb{D}^m_{o_i}(\lambda e.\mathbf{beat}\ x\ y) \\ &\to_\beta \lambda xye\phi i.((\lambda e'.\mathbf{beat}\ x\ y)ei \wedge \phi(\mathbf{up\_context}\ e\ i\ ((\lambda e'.\mathbf{beat}\ x\ y)e))i) \\ &\twoheadrightarrow_\beta \lambda xye\phi i.(\mathbf{beat}\ x\ y\ i \wedge \phi(\mathbf{up\_context}\ e\ i\ (\mathbf{beat}\ x\ y))i)\end{aligned}$$

4. As a result

$$\begin{aligned}\overline{[\![beat]\!]}^{\mathsf{m}} &= \lambda OS.S(\lambda x.O(\lambda y.\overline{\mathbf{beat}}^{\mathsf{m}}\ x\ y)) \\ &\twoheadrightarrow_\beta \lambda OS.S(\lambda x.O(\lambda ye\phi i.(\mathbf{beat}\ x\ y\ i \wedge \phi(\mathbf{up\_context}\ e\ i\ (\mathbf{beat}\ x\ y))i)))\end{aligned}$$

   Same as for previous entries involving elementary functions, we will not unfold the complete entry. In the above formula, O and S are both of the type of a dynamic NP, namely $(\iota \to \Omega_i) \to \Omega_i$, x and y are both of type ι, hence $\overline{[\![beat]\!]}^{\mathsf{m}}$ is of type $((\iota \to \Omega_i) \to \Omega_i) \to ((\iota \to \Omega_i) \to \Omega_i) \to \Omega_i$.

For the detailed translation of other syntactic categories, please refer to Appendix A.3.

Discourses

The purpose of this subsection is to show how the framework M-TTDL can be applied to handle linguistic examples, which are concerned with modal subordination. In what follows, we will compositionally compute the logical representation of the discourse based on the above proposed lexical entries.

Firstly, let's start with a simple example, where the anaphoric link is felicitous because modality is only involved in the second part of the discourse.

(125) A wolf$_i$ walks in. It$_i$ might growl.

The first sentence in example (125) does not contain modality. Its semantic representation in M-TTDL can be computed as follows:

$$[\![(125)\text{-}1]\!]_{M\text{-}TTDL} = \overline{[\![walk\_in]\!]([\![a]\!][\![wolf]\!])}^{\mathsf{m}}$$

The second part, where the modal *might* appears, is translated in the following way:

$$[\![(125)\text{-}2]\!]_{M\text{-}TTDL} = [\![might]\!]_{M\text{-}TTDL}\overline{([\![growl]\!][\![it]\!])}^{\mathsf{m}}$$

The detailed lexical entry for pronoun *it* can be found in section A.3 of appendix A. Remark that the choice operator sel in M-TTDL has a different type as the one in TTDL (see definition 7.3.1). We do not give the complete unfolding of the logical formulas because their sizes are rather huge. Instead, we will directly present the result of discourse incrementation, which will be applied to the empty left context nil_i, the empty continuation $\mathbf{stop}_i$, and the world constant $\mathbf{H}$. Same as before, assume the conjunction is the connective for sentence sequencing, then:

$$\begin{aligned}
&[\![(125)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&= ([\![(125)\text{-}1]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![(125)\text{-}2]\!]_{M\text{-}TTDL})\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&\twoheadrightarrow_\beta {}^{\iota}\exists x.(\mathbf{wolf}\ x\ \mathbf{H} \wedge \mathbf{walk_in}\ x\ \mathbf{H}\wedge\\
&\qquad\qquad {}^{s}\exists j.(\mathbf{R}\ \mathbf{H}\ j \wedge ((\mathbf{walk_in}\ x\ j \wedge \mathbf{wolf}\ x\ j)\wedge\\
&\qquad\qquad\qquad \mathbf{growl}\ (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ j))\ (\mathbf{walk_in}\ x\ j \wedge \mathbf{wolf}\ x\ j))\ j)))
\end{aligned}$$

In the above formula, the choice operator sel should select a variable from its second argument, namely the proposition $(\mathbf{walk\_in}\ x\ j \wedge \mathbf{wolf}\ x\ j)$, based on the criteria from its first argument, namely $\lambda x.\neg(\mathbf{human}\ x\ j)$. Since variable x is the only candidate, the above representation can be further reduced into:

$$\begin{aligned}
&[\![(125)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}\ \mathbf{H}\\
&\twoheadrightarrow_\beta {}^{\iota}\exists x.(\mathbf{wolf}\ x\ \mathbf{H} \wedge \mathbf{walk_in}\ x\ \mathbf{H}\wedge\\
&\qquad\qquad {}^{s}\exists j.(\mathbf{R}\ \mathbf{H}\ j \wedge ((\mathbf{walk_in}\ x\ j \wedge \mathbf{wolf}\ x\ j) \wedge \mathbf{growl}\ x\ j)))
\end{aligned}$$

Assume $\mathbf{H}$ is the world of utterance, the semantics of the above formula is: there is a wolf which walks in at the actual world $\mathbf{H}$, and at an accessible possible world j, there is also a wolf which walks in, and it also growls at j. This is exactly what (125) means.

In addition, the framework M-TTDL can successfully block the infelicitous anaphoras as in the following examples, where the referents are introduced within the scope of modal operators:

(126) a. $\underline{\text{A wolf}_i}$ might walk in. *$\underline{\text{It}_i}$ growls.
b. $\underline{\text{A wolf}_i}$ would walk in. *$\underline{\text{It}_i}$ growls.

The interpretations of the first two sentences are calculated as follows:

$$[\![\text{(126-a)-1}]\!]_{M\text{-}TTDL} = [\![might]\!]_{M\text{-}TTDL}\overline{([\![walk\_in]\!]([\![a]\!][\![wolf]\!]))}^{\mathsf{m}}$$
$$[\![\text{(126-b)-1}]\!]_{M\text{-}TTDL} = [\![would]\!]_{M\text{-}TTDL}\overline{([\![walk\_in]\!]([\![a]\!][\![wolf]\!]))}^{\mathsf{m}}$$

They share the same second part:

$$[\![\text{(126-a)-2}]\!]_{M\text{-}TTDL} = [\![\text{(126-b)-2}]\!]_{M\text{-}TTDL} = \overline{[\![growl]\!][\![it]\!]}^{\mathsf{m}}$$

The following steps are the same as for the previous example, we will give the final result directly. For (126-a):

$$\begin{aligned}
&[\![\text{(126-a)}]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&= ([\![\text{(126-a)-1}]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![\text{(126-a)-2}]\!]_{M\text{-}TTDL})\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&\twoheadrightarrow_\beta {}^s\exists j.(\mathbf{R}\ \mathbf{H}\ j \wedge {}^\iota\exists x.(\mathbf{wolf}\ x\ j \wedge (\mathbf{walk_in}\ x\ j \wedge\\
&\qquad\qquad \mathbf{growl}\ (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ \mathbf{H}))\ \top)\ \mathbf{H})))
\end{aligned}$$

The above formula means that there is an accessible world j from the actual world $\mathbf{H}$, in which a wolf walks in. And at the actual world, there is some individual that growls. But since the choice operator sel does not have a proper proposition from which it can pick up a referent, the anaphora in (126-a) can not be resolved. Assume A is the proposition expressed by *a wolf walks in*, B is the one expressed by *it growls*, $\mathbf{M}$ denotes the entry of *might*, then the possible worlds hierarchy of example (126-a) is depicted in figure 7.8:

Fig. 7.8 Possible Worlds Hierarchy of Example (126-a)

The anaphor *it* occurs at world $\mathbf{H}$, while the referent corresponding to *a wolf* is introduced in j. As a result, the anaphoric link can not be resolved. It is the similar case for (126-b), although the detailed logical representation is different:

$$\begin{aligned}
&[\![\text{(126-b)}]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&= [\![\text{(126-b)-1}]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![\text{(126-b)-2}]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H}\\
&\twoheadrightarrow_\beta {}^s\forall j.(\mathbf{R}\ \mathbf{H}\ j \rightarrow {}^\iota\exists x.(\mathbf{wolf}\ x\ j \wedge (\mathbf{walk_in}\ x\ j \wedge\\
&\qquad\qquad \mathbf{growl}\ (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ \mathbf{H}))\ \top)\ \mathbf{H})))
\end{aligned}$$

The above formula means that for all accessible worlds from the actual world $\mathbf{H}$, there is a wolf walking in at it. At the same time, there is some individual who growls at the actual world. But this growing individual can not be properly resolved as any referent. Its possible worlds hierarchy is provided in figure 7.9:

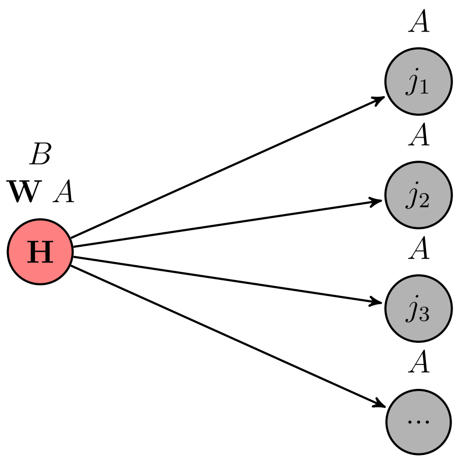


Fig. 7.9 Possible Worlds Hierarchy of Example (126-b)

No discourse referent is introduced at world **H**, where the anaphor *it* occurs. Hence the anaphora is problematic.

Now let's consider a more complex discourse, where modalities are involved in both component sentences. We repeat example (121) as follows:

(121) $\underline{\text{A wolf}_i}$ might walk in. $\underline{\text{It}_i}$ would growl. Asher and Pogodalla (2011a)

The first sentence of (121) is identical to (126-a)-1, hence:

$$[\![(121)\text{-}1]\!]_{M\text{-}TTDL} = [\![(126\text{-a})\text{-}1]\!]_{M\text{-}TTDL} = [\![might]\!]_{M\text{-}TTDL}\overline{([\![walk\_in]\!]([\![a]\!][\![wolf]\!]))}^{\mathsf{m}}$$

The representation for the second sentence can be achieved in a similar way:

$$[\![(121)\text{-}2]\!]_{M\text{-}TTDL} = [\![would]\!]_{M\text{-}TTDL}\overline{([\![growl]\!][\![it]\!])}^{\mathsf{m}}$$

Following the previous examples, the discourse incrementation for (121) is also straightforward. We will directly give the final result:

$$\begin{aligned}
&[\![(121)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
&= [\![(121)\text{-}1]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![(121)\text{-}2]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
&\twoheadrightarrow_\beta\ {}^s\exists j.(\mathbf{R}\ \mathbf{H}\ j \wedge {}^t\exists x.(\mathbf{wolf}\ x\ j \wedge \mathbf{walk_in}\ x\ j \wedge \\
&\qquad\qquad {}^s\forall k.(\mathbf{R}\ j\ k \to ((\mathbf{wolf}\ x\ k \wedge \mathbf{walk_in}\ x\ k) \to \\
&\qquad\qquad\qquad (\mathbf{growl}\ (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ k))\ (\mathbf{wolf}\ x\ k \wedge \mathbf{walk_in}\ x\ k))\ k)))))
\end{aligned}$$

Now the choice operator sel will select a non-human variable at world k from the proposition ($\mathbf{wolf}\ x\ k \wedge \mathbf{walk\_in}\ x\ k$), where x is the only possibility. Hence we can further reduce the above formula as follows:

$$\begin{aligned}
&[\![(121)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
&\twoheadrightarrow_\beta {}^s\exists j.(\mathbf{R}\ \mathbf{H}\ j \land {}^\iota\exists x.(\mathbf{wolf}\ x\ j \land \mathbf{walk_in}\ x\ j \land \\
&\qquad\qquad {}^s\forall k.(\mathbf{R}\ j\ k \to ((\mathbf{wolf}\ x\ k \land \mathbf{walk_in}\ x\ k) \to (\mathbf{growl}\ x\ k)))))
\end{aligned}$$

This means there exists a possible world j which is accessible from the actual world $\mathbf{H}$, a wolf walks in at j; and at every accessible world k from j, if the wolf walks in, then it growls. This corresponds to the semantics of the original discourse (121). Again, we provide its possible worlds hierarchy as follows.

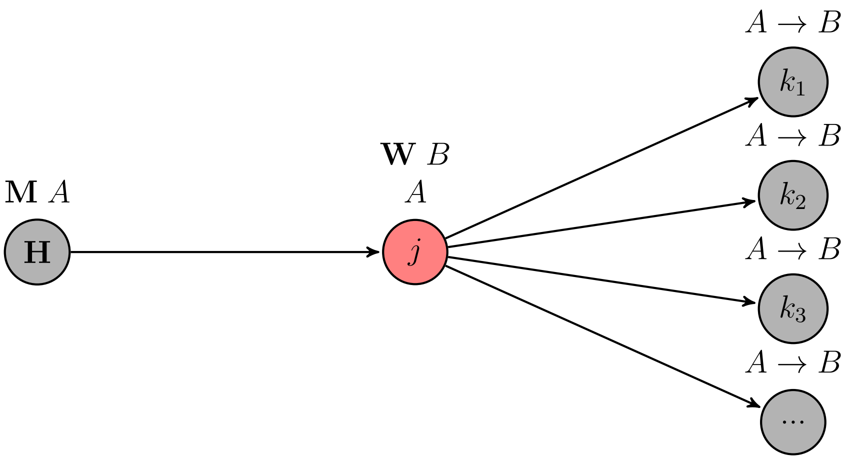


Fig. 7.10 Possible Worlds Hierarchy of Example (121)

Finally, we will examine a last example, which switches back and forth between the modal mode and the factual mode. For the sake of convenience, we simplify example (124) as follows:

(127) A wolf$_i$ might walk in. John has a gun$_j$. John would use it$_j$ to shoot it$_i$.

We first have to compute the semantic representation for each component sentence. As we can see, (127)-1 is exactly the same as (126-a)-1 and (121)-1, we will not repeat it here any more. For the remaining two sentences, we have:

$$[\![(127)\text{-}2]\!]_{M\text{-}TTDL} = \overline{[\![have]\!]([\![a]\!][\![gun]\!])[\![John]\!]}^{\mathsf{m}}$$

$$[\![(127)\text{-}3]\!]_{M\text{-}TTDL} = [\![would]\!]_{M\text{-}TTDL}\overline{([\![use]\!][\![it]\!]([\![shoot]\!][\![it]\!])[\![John]\!])}^{\mathsf{m}}$$

The semantic representation of the whole discourse is obtained by straightforwardly sequencing the three component sentences with dynamic conjunction. Same as before, we apply it to the empty left context nil_i, the empty continuation $\mathbf{stop}_i$, and the world constant $\mathbf{H}$. The reduced formula is as follows:

$$
\begin{array}{l}
[\![(127)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
= ([\![(127)\text{-}1]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![(127)\text{-}2]\!]_{M\text{-}TTDL}\overline{\wedge}^{\mathsf{m}}[\![(127)\text{-}3]\!]_{M\text{-}TTDL})\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
\twoheadrightarrow_\beta\ {}^s\exists j.(\mathbf{R}\ \mathbf{H}\ j) \wedge ({}^\iota\exists x.(\mathbf{wolf}\ x\ j) \wedge ((\mathbf{walk\_in}\ x\ j)\wedge \\
\qquad ({}^\iota\exists y.(\mathbf{gun}\ y\ \mathbf{H}) \wedge ((\mathbf{have\ john}\ y\ \mathbf{H}) \wedge (((\mathbf{have\ john}\ y\ j) \wedge (\mathbf{gun}\ y\ j))\wedge \\
\qquad (\forall k.(\mathbf{R}\ j\ k) \to ((((\mathbf{have\ john}\ y\ k) \wedge (\mathbf{gun}\ y\ k)) \wedge ((\mathbf{walk\_in}\ x\ k) \wedge (\mathbf{wolf}\ x\ k))) \to \\
\qquad\quad (\mathbf{use} \\
\qquad\qquad \mathbf{john} \\
\qquad\qquad (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ k)) \\
\qquad\qquad\qquad (((\mathbf{have\ john}\ y\ k) \wedge (\mathbf{gun}\ y\ k)) \wedge ((\mathbf{walk\_in}\ x\ k) \wedge (\mathbf{wolf}\ x\ k)))) \\
\qquad\qquad (\mathbf{shoot} \\
\qquad\qquad\quad \mathbf{john} \\
\qquad\qquad\quad (\mathsf{sel}\ (\lambda x.\neg(\mathbf{human}\ x\ k)) \\
\qquad\qquad\qquad\quad (((\mathbf{have\ john}\ y\ k) \wedge (\mathbf{gun}\ y\ k)) \wedge ((\mathbf{walk\_in}\ x\ k) \wedge (\mathbf{wolf}\ x\ k)))) \\
\qquad\qquad\quad k) \\
\qquad\qquad k))))))))
\end{array}
$$

As we can see, both choice operators can select a non-human variable at world k from the sub-formula $(((\mathbf{have\ john}\ y\ k) \wedge (\mathbf{gun}\ y\ k)) \wedge ((\mathbf{walk\_in}\ x\ k) \wedge (\mathbf{wolf}\ x\ k))$. Let's assume the first sel picks up y, the second picks up x, then the above formula can be further reduced to:

$$
\begin{array}{l}
[\![(127)]\!]_{M\text{-}TTDL}\ \mathsf{nil}_i\ \mathbf{stop}_i\ \mathbf{H} \\
\twoheadrightarrow_\beta\ {}^s\exists j.(\mathbf{R}\ \mathbf{H}\ j) \wedge ({}^\iota\exists x.(\mathbf{wolf}\ x\ j) \wedge ((\mathbf{walk\_in}\ x\ j)\wedge \\
\qquad ({}^\iota\exists y.(\mathbf{gun}\ y\ \mathbf{H}) \wedge ((\mathbf{have\ john}\ y\ \mathbf{H}) \wedge (((\mathbf{have\ john}\ y\ j) \wedge (\mathbf{gun}\ y\ j))\wedge \\
\qquad\quad ({}^s\forall k.(\mathbf{R}\ j\ k) \to ((((\mathbf{have\ john}\ y\ k) \wedge (\mathbf{gun}\ y\ k))\wedge \\
\qquad\qquad ((\mathbf{walk\_in}\ x\ k) \wedge (\mathbf{wolf}\ x\ k))) \to (\mathbf{use\ john}\ y\ (\mathbf{shoot\ john}\ x\ k)\ k))))))))
\end{array}
$$

The semantics of the above complex formula is: there is a possible world j accessible from the actual world $\mathbf{H}$, a wolf walks in at j; further more, John owns a gun at the actual world $\mathbf{H}$; in addition, in every possible world k which is accessible from j, if the wolf walks in, then John uses the gun to shoot the wolf. As a result, all the anaphoric links in discourse (127), which are across the modal mode and the factual mode, can be correctly accounted for.

7.4 Putting Everything Together

Up until now, we have proposed two adaptations of TTDL, namely DN-TTDL and M-TTDL. Both of them can be formally related with TTDL, i.e., theorem 6.2.4 for DN-TTDL and the intermediate systems P-TTDL for M-TTDL. For a complete description of the relations between all continuation-based dynamic frameworks so far in this thesis, one may refer to figure 7.11. The particular notations for the dynamic translations have been presented besides the corresponding systems.

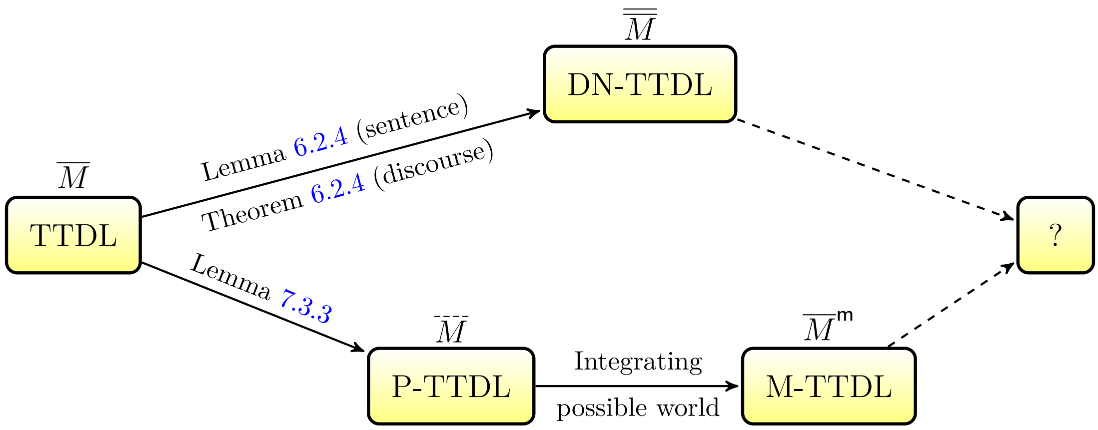


Fig. 7.11 Relations between All Frameworks

As indicated in figure 7.11, DN-TTDL and M-TTDL are developed upon TTDL from two directions. More specifically, DN-TTDL extends TTDL by pairing the standard dynamic translation with its negation; while M-TTDL integrates the notion of possible world and modifies the way in which contexts are interpreted. A natural question is whether these two adaptations can finally be converged into a more powerful system. In this section, we will propose such a continuation-based dynamic framework, which is called **Double Negation Modal TTDL** (**DNM-TTDL**). This synthesized system will capture the properties from both DN-TTDL and M-TTDL. Hence from a theoretical point of view, it provides the feasibility to treat both problematic phenomena (i.e., double-negation and modality) within a single framework. In what follows, we will present the formal details of DNM-TTDL.

First of all, DNM-TTDL shares the signature with M-TTDL, see definition 7.3.1. Moreover, contexts and propositions in DNM-TTDL are interpreted exactly the same as in M-TTDL, see formulas 7.16 and 7.20. Then inheriting from DN-TTDL, DNM-TTDL encapsulates two dynamic propositions in its semantic representation: one is the standard M-TTDL translation, the other is its dynamic negation. Hence, assume s and d are the syntactic categories of sentence and discourse, respectively, their semantic interpretation under DNM-TTDL are as follows:

$$[\![s]\!]_{DNM\text{-}TTDL} = \Omega_i \times \Omega_i \tag{7.75}$$

$$[\![d]\!]_{DNM\text{-}TTDL} = \Omega_i \tag{7.76}$$

For more discussion on Ω_i, please refer back to formula 7.20. We will use Ω_i^{dn} as an abbreviation for $\Omega_i \times \Omega_i$ in the following context, namely:

$$\Omega_i^{dn} \triangleq \Omega_i \times \Omega_i \tag{7.77}$$

As to the dynamic logic, DNM-TTDL is in a completely parallel form with DN-TTDL, except for that DN-TTDL is established on TTDL, while DNM-TTDL is established on M-TTDL. Since the dynamic negation $\neg^d_{DN\text{-}TTDL}$ (formula 6.23) does not involve any logical constant in TTDL, it can be reused in DNM-TTDL without modification. For the other operators, we can straightforwardly deduce their formulas as follows:

$$\begin{aligned}\mathbf{update}_{DNM\text{-}TTDL} &\triangleq \lambda DS.(\mathbf{update}_{M\text{-}TTDL}\ D\ (\pi_1 S))\\ &\twoheadrightarrow_\beta \lambda DSe\phi.De(\lambda e'.(\pi_1 S)e'\phi)\end{aligned} \tag{7.78}$$

$$\begin{aligned}\wedge^d_{DNM\text{-}TTDL} &\triangleq \lambda AB.\langle(\pi_1 A)\wedge^d_{M\text{-}TTDL}(\pi_1 B),\\ &\qquad \neg^d_{M\text{-}TTDL}((\pi_1 A)\wedge^d_{M\text{-}TTDL}(\pi_1 B))\rangle\\ &\twoheadrightarrow_\beta \lambda AB.\langle\lambda e\phi.(\pi_1 A)e(\lambda e'.(\pi_1 B)e'\phi),\\ &\qquad \lambda e\phi.\neg_i((\pi_1 A)e(\lambda e'.(\pi_1 B)\ e'\ \mathbf{stop}_i))\wedge_i \phi e\rangle\end{aligned} \tag{7.79}$$

$$\begin{aligned}\exists^d_{DNM\text{-}TTDL} &\triangleq \lambda P.\langle{}^\iota\exists^d_{M\text{-}TTDL}(\lambda x.\pi_1(Px)),\\ &\qquad \neg^d_{M\text{-}TTDL}({}^\iota\exists^d_{M\text{-}TTDL}(\lambda x.\pi_1(Px)))\rangle\\ &\twoheadrightarrow_\beta \lambda P.\langle\lambda e\phi.{}^\iota\exists_i(\lambda x.(\pi_1(Px))e\phi),\\ &\qquad \lambda e\phi.\neg_i({}^\iota\exists_i(\lambda x.(\pi_1(Px))\ e\ \mathbf{stop}_i))\wedge_i \phi e\rangle\end{aligned} \tag{7.80}$$

One may compare the above terms with their counterparts in DN-TTDL, i.e., formulas 6.26, 6.27, and 6.28, respectively. For more details on the constants in M-TTDL, such as $\mathbf{update}_{M\text{-}TTDL}$, $\wedge^d_{M\text{-}TTDL}$, $\neg^d_{M\text{-}TTDL}$, etc., please refer back to section 7.3.3.

Finally, let's proceed the systematic dynamic translation in DNM-TTDL. Again, we introduce a new notation in order to distinguish the translation from prior versions.

Notation 7.4.1. We use the double-m-bar notation, for instance, $\overline{\overline{\tau}}^{\mathsf{m}}$ or $\overline{\overline{M}}^{\mathsf{m}}$, to denote the **dynamic translation** of a type τ or a λ-term M in DNM-TTDL.

To translate types in DNM-TTDL, we have the following rules, which are similar to the ones for DN-TTDL (see definition 6.2.1):

Definition 7.4.1. The **dynamic translation of a type** $\tau \in T$: $\overline{\overline{\tau}}^{\mathsf{m}}$, is defined inductively as follows:

1. $\overline{\overline{\iota}}^{\mathsf{m}} = \iota$;
2. $\overline{\overline{o}}^{\mathsf{m}} = \Omega^{dn}_i$;
3. $\overline{\overline{\sigma \to \tau}}^{\mathsf{m}} = \overline{\overline{\sigma}}^{\mathsf{m}} \to \overline{\overline{\tau}}^{\mathsf{m}}$, where $\tau, \sigma \in T$.

As can be inferred from definition 7.4.1, the dynamic proposition in DN-TTDL is of type Ω^{dn}_i. Same as in previous continuation-based frameworks, before presenting the detailed dynamic translation of λ-terms, we will introduce the dynamization and staticization function: $\mathbb{D}^s$ and $\mathbb{S}^s$. These two functions are defined in terms of $\neg^d_{M\text{-}TTDL}$, $\mathbb{D}^m$, and $\mathbb{S}^m$, all of which are from M-TTDL. The detailed definition is analogous to the one in DN-TTDL (see definition 6.2.2). Like before, they will be used for the dynamization of non-logical constants.

Definition 7.4.2. The **dynamization function** $\mathbb{D}^s_\tau$, which takes an input λ-term A of type $(\gamma \to \tau)$, returns an output λ-term A' of type $\overline{\overline{\tau}}^{\mathsf{m}}$; the **staticization function** $\mathbb{S}^s_\tau$, which takes an input λ-term A' of type $\overline{\overline{\tau}}^{\mathsf{m}}$, returns an output λ-term A of type $(\gamma \to \tau)$.

- $\mathbb{D}^s_\tau$ is defined inductively on type τ as follows:
    1. $\mathbb{D}^s_\iota A = \mathbb{D}^m_\iota A$;
    2. $\mathbb{D}^s_o A = \langle \mathbb{D}^m_o A, \neg^d_{M\text{-}TTDL}(\mathbb{D}^m_o A)\rangle$;
    3. $\mathbb{D}^s_{\alpha\to\beta} A = \lambda x.\mathbb{D}^s_\beta(\lambda e.Ae(\mathbb{S}^s_\alpha xe))$.
- $\mathbb{S}^s_\tau$ is defined inductively on type τ as follows:
    1. $\mathbb{S}^s_\iota A' = \mathbb{S}^m_\iota A'$;
    2. $\mathbb{S}^s_o A' = \lambda e.\mathbb{S}^m_o(\pi_1 A')e$;
    3. $\mathbb{S}^s_{\alpha\to\beta} A' = \lambda e.(\lambda x.\mathbb{S}^s_\beta(A'(\mathbb{D}^s_\alpha(\lambda e'.x)))e)$.

The above definition allows us to come up with the dynamic translation for λ-terms in DNM-TTDL, which is a trivial adaptation of the ones in prior systems.

Definition 7.4.3. The **double negation dynamic translation of a λ-term** M (of type τ): $\overline{\overline{M}}^{\mathsf{m}}$, which is another λ-term of type $\overline{\overline{\tau}}^{\mathsf{m}}$, is defined as follows:

1. $\overline{\overline{x}}^{\mathsf{m}} = x$, if $x \in \mathcal{X}$;
2. $\overline{\overline{\mathbf{a}}}^{\mathsf{m}} = \mathbb{D}^s_\tau(\lambda e.\mathbf{a})$, if $\mathbf{a} \in \mathcal{C}_{NL}$ and $\mathbf{a} : \tau$;
3. $\overline{\overline{\wedge}}^{\mathsf{m}} = \wedge^d_{DNM\text{-}TTDL}$, see formula 7.79;
4. $\overline{\overline{\neg}}^{\mathsf{m}} = \neg^d_{DN\text{-}TTDL}$, see formula 6.23;
5. $\overline{\overline{{}^{\iota}\exists}}^{\mathsf{m}} = {}^{\iota}\exists^d_{M\text{-}TTDL}$, see formula 7.80;
6. $\overline{\overline{(MN)}}^{\mathsf{m}} = (\overline{\overline{M}}^{\mathsf{m}}\,\overline{\overline{N}}^{\mathsf{m}})$;
7. $\overline{\overline{(\lambda x.M)}}^{\mathsf{m}} = (\lambda x.\overline{\overline{M}}^{\mathsf{m}})$.

In conclusion, we can complete figure 7.11 by resolving the question mark with DNM-TTDL. This gives an overview of the work that has been realized in this thesis.

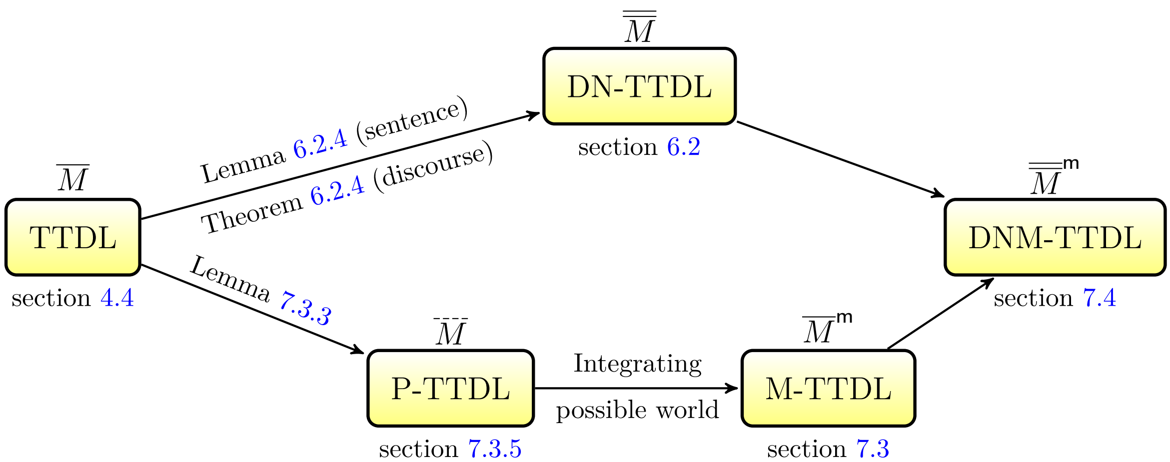


Fig. 7.12 Relations between All Frameworks (Completed)

Chapter 8

Conclusion

Contents

In this last chapter, we will first restate the major points of this thesis, and summarize the goals that have been achieved. After that, a final discussion, consisting of future directions of the present work, will be presented.

8.1 Summary

Anaphora is a critical machinery of natural language. It denotes the linguistic phenomenon whereby the interpretation of one expression, called the anaphor, depends on that of another, called the antecedent. To figure out the meaning of an utterance containing an anaphoric expression, one has to resolve the anaphora, namely, to correctly link the anaphor with its antecedent.

Generally speaking, this thesis has studied the semantics of one specific type of anaphora: inter-sentential pronominal anaphora, where both antecedent and anaphor are singular NPs, and they span across various sentences. More specifically, we have concentrated on one particular aspect of anaphora: the accessibility of the antecedent, namely the eligibility of an (indefinite) NP to serve as antecedent. Following the tradition of dynamic semantics, this thesis proposed to interpret linguistic expressions in a compositional and dynamic fashion. The interpretation generates an updated context, where felicitous anaphoric expressions can be licensed, while infelicitous ones can not.

In summary, the present work did not intend to establish a new semantic theory. Rather, it has aimed to adapt one specific framework: TTDL, to those cases which lack a proper treatment under the standard setup of contemporary dynamic theories. The motivation has originated from two particular problems: double negation and modal subordination. A formal account has been provided for each problem from the semantic perspective. In particular, DN-TTDL was proposed for double negation, M-TTDL was proposed for modal subordination. Moreover, the above two extensions were successfully integrated to form a more powerful system, called DM-TTDL. This integration has shown that the two independent problems in question could be treated in a unified system. All frameworks proposed in this thesis were established upon TTDL. They provide considerable evidences of TTDL's flexibility in handling complicated discourse phenomena.

8.2 Future Work

This thesis has given an account for the anaphoric links across double negation and modalized context. However, it is only a first step towards a comprehensive theory on the semantics of anaphora. In this section, we will discuss a number of possible directions with which it can be continued.

8.2.1 Discourse Structure

Besides negation, disjunction, modality, etc., various other factors have been observed to affect the accessibility of the antecedent. One of the most prominent is the discourse structure, or equivalently, the rhetorical structure.

Much of the work in discourse semantics has assumed that discourse is more than a conjunction of the constituent sentences, rather, it has structure Asher (1993); Grosz and Sidner (1986); Hobbs (1985); Mann and Thompson (1986). That is to say, constituent sentences of a discourse bear certain rhetorical relations among each other. For instance, common relations[1] include elaboration, topic, explanation, narration, background, contrast, etc. The two representative frameworks in this field are Rhetorical Structure Theory (RST) Mann and Thompson (1986) and Segmented Discourse Representation Theory (SDRT) Asher (1993). Although the set of discourse relations in the two theories are not exactly the same, they agree to classify relations into the following two classes: **subordinating** and **coordinating**. For example, elaboration, topic, explanation are subordinating; narration, background, and contrast are coordinating.

The intuition behind this classification is as follows. The function of a sentence over its context could be to introduce a new topic or to support and explain a topic. In the former case, the leading sentence subordinates the supporting sentences; in the latter case, the supporting sentences coordinate one another. If we picture the discourse structure, the difference between the two classes is the hierarchical connection they establish: a subordinating relation results in a vertical edge while a coordinating relation results in a horizontal one. Namely, if a sentence subordinates another, the former is on a higher level in the discourse structure than the latter; while if a sentence coordinates another, the two are on the same level.

It has been noticed that discourse structure has certain impact on the resolution of discourse anaphora. The theoretical foundation dates back to Polanyi (1988), which suggests that the accessible nodes in a discourse hierarchy are those which locate on the right frontier, and only accessible nodes are applicable in the updating of discourse structure. This is also known as the **Right Frontier Constraint**. We will illustrate it with the following example:

(128) a. John had <u>a great evening</u> last night.

[1] The taxonomy of discourse relations is still an open question.

b. He had a great meal.
c. He ate salmon$_i$.
d. He devoured lots of cheese.
e. He then won a dancing competition.
f. *It$_i$ was a beautiful pink. Asher and Vieu (2005)

First of all, let's look at the sub-discourse consisting of the first four sentences. (128-a) provides the main topic of the discourse that John had a great evening. (128-b) then elaborates (128-a) with the information that he had a great meal. After that, (128-c) and (128-d) further elaborate the meal that John had, i.e., salmon and cheese, respectively. And the relation between the two sentences is narration. As a result, we can build up the discourse hierarchy for the sub-clauses in figure 8.1.

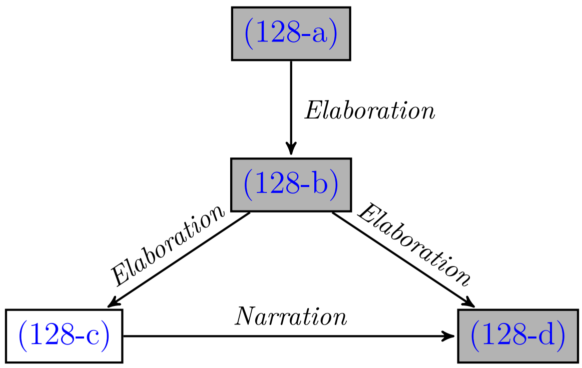


Fig. 8.1 Hierarchical Structure: (128-a)-(128-d)

The nodes that are on the right frontier are (128-a), (128-b), and (128-d), which are highlighted in figure 8.1. Then let's continue the discourse with the fifth sentence. (128-e) describes the dancing competition that John won. It is clearly not about the dinner any more. So (128-e) forms a narration relation to (128-b), both of which serve to elaborate the topic introduced by the first sentence. The updated discourse hierarchy is shown in figure 8.2.

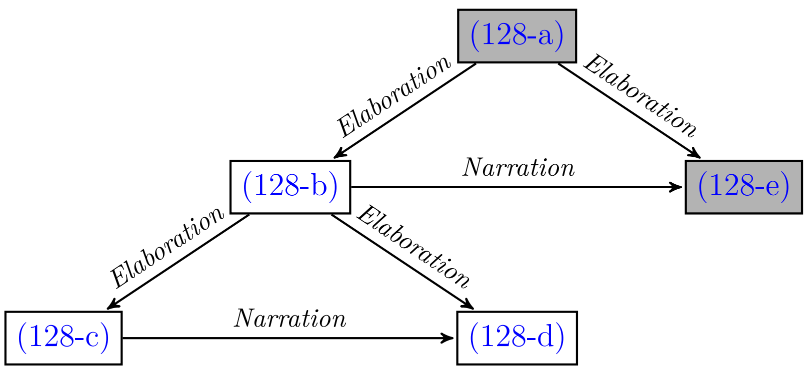


Fig. 8.2 Hierarchical Structure: (128-a)-(128-e)

In the new hierarchy, the nodes that are located on the right frontier are (128-a) and (128-e), which have been highlighted. According to the Right Frontier Constraint, successive utterances can only be attached to these two nodes. Namely, only referents in (128-a) and (128-e) can serve as antecedent for subsequent anaphors. This explains why the anaphoric pronoun *it* in (128-f) fails to be resolved. The most likely antecedent *salmon* was introduced in (128-c), which, however, is not on the right frontier of the discourse structure. Hence the anaphoric link can not be felicitously established.

In order to account for examples such as (128), the current TTDL-based dynamic frameworks should be incorporated with the notion of discourse structure. For each discourse referent in the left context, its location in the discourse hierarchy should also be stored. Then as the discourse proceeds, the referents which are not on the right frontier will be eliminated. One existing attempt following this line is Asher and Pogodalla (2011b), we thus suggest to enrich their work with the frameworks developed in this thesis.

8.2.2 Plurality

Another possible direction for further research is to extend our frameworks to plurality. It has been noticed that singular and plural anaphora differ in many aspects, they are not parallel phenomena. Since we confine ourselves to singular anaphora in the present work, the current TTDL-based theories are not adequate enough to be a general account of pronominal anaphora. The discourse referents in all TTDL-based frameworks are singular, so they fail to address the anaphoric links in the following discourses:

(129) a. John took Mary to Acapulco. They had a lousy time.
b. Last month John took Mary to Acapulco. Fred and Suzie were already there. The next morning they set off on their sailing trip. Kamp and Reyle (1993)

Firstly, the pronoun *they* does not fit with a singular NP antecedent, e.g., *John*, *Mary*. It expects a plural referent, in other words, a collection of (individual) discourse referents. Secondly, assume plural referents do exist in the context, there are no transparent construction of plural NPs that correspond to them. For instance, in (129-a) the two proper names are not in conjunction as *John and Mary*; similarly, we do not see the specific NP *John, Mary, Fred and Suzie* in (129-b). It seems that singular pronouns are resolved with particular earlier NPs, while antecedents of plural pronouns should be inferred from the context.

In addition, there is another interesting observation on plurality: universal quantification, which blocks the accessibility of referents within its scope, seems to allow plural anaphors in subsequent utterances. For instance[2]:

(130) a. Every student$_i$ wrote a paper. They$_i$ also read a book.
b. Every student wrote a paper$_i$. They$_i$ weren't very good. Nouwen (2003b)

In both discourses of (130), the first sentence only involves singular NPs, i.e., *every student* and *a paper*. Then, similar as in example (129), the antecedent for the plural pronoun *they* lacks a transparent construction. Moreover, in TTDL-based frameworks, universal quantification comes with the empty continuation, this means no variable can

[2] As pointed out by Elworthy (1992), the acceptability of anaphoric links in example (130) is controversial among native speakers. People who insist on syntactic agreement will regard them as infelicitous, people who emphasize more on the semantic aspect would judge them acceptable. Here we adopt the latter viewpoint.

out-live its scope. Because of these two reasons, the anaphoric links in (130) can not be addressed with the current setup of our dynamic frameworks.

In order to account for plural anaphora within TTDL-based theories, we will have to introduce the notion of plural discourse referent. Further more, because most plural referents are established through inference, as in example (129) and (130), we will also have to formally specify the particular inferential processes. A solution within DRT has been provided in Kamp and Reyle (1993). According to this proposal, the inferential process in (129) is called **summation**, the one in (130) is called **abstraction**. Based on that, we can further explore the possibility to integrate plural referent and the inferential processes in TTDL-based theories. Once an elementary mechanism is achieved, one may additionally study how TTDL can be adapted to account for more complex phenomena concerned with plurality, such as distributive and collective reading Gillon (1996), complement anaphora (also known as the compset reference) Geurts (1997); Nouwen (2003a), etc. One tentative proposal has been provided by Qian and Amblard (2013), although more refinements have to be made.

8.2.3 Accommodating Counterfactual

In chapter 7, we have proposed the adaptation M-TTDL for modal subordination, where the modal base is updated into the left context. While the present framework fails to cover the following example, whose first part is in factual mood, while the second part is modalized:

(131) John won't buy a car_i because he wouldn't have room for it_i in his garage. Partee (1973)

In TTDL-based frameworks, discourse referents introduced in the scope of (single) negation is not accessible from outside. So the indefinite *a car* in (131) should not serve as antecedent for any subsequent anaphor. However, the anaphoric link in (131) is fairly acceptable. As suggested by Partee (1973), the auxiliary *would* requires the presence of a subordinate clause with *if* or *unless*, except when it is used to express volition or habit. Hence (131) can be regarded as an abbreviation for the following sentence:

(132) John won't buy a car because if he did buy a car_i, he wouldn't have room for it_i in his garage. Partee (1973)

In the paraphrase (132), there are two occurrences of *a car*. The pronoun *it* is anaphorically related to the second occurrence rather than the first one. Then with the paraphrase, the anaphora can be successfully accounted for in M-TTDL. Similar examples include:

(133) a. I didn't submit a paper_i. They wouldn't have published it_i. Kibble (1994)
b. John didn't buy a mystery novel_i. He would be reading it_i by now. Krifka (2001)
c. Mary didn't buy a microwave_i. She would never use it_i. Frank (1997)
d. Fred didn't draw a picture_i. He would have made a mess of it_i. Frank (1997)

In like manner as example (132), discourses in (133) can be paraphrased as follows:

(134) a. I didn't submit a paper. If I had submitted a paper$_i$, they wouldn't have published it$_i$. Kibble (1994)
b. John didn't buy a mystery novel. If he had bought a mystery novel$_i$, he would be reading it$_i$ by now. Krifka (2001)
c. Mary didn't buy a microwave because if she had bought a microwave$_i$ she would never have used it$_i$. Frank (1997)
d. Fred didn't draw a picture because if he had drawn a picture$_i$ he would have made a mess of it$_i$. Frank (1997)

The interesting thing we can draw from the above examples is that, it is the counterfactual that has been accommodated in the modal base. In order to broaden the empirical coverage of M-TTDL on examples such as (131) and (133), we should at least tackle two fundamental questions. Firstly, the condition that triggers the accommodation ought to be precisely specified, e.g., the presence of the modal auxiliary *would*. More importantly, we will need to determine which factual proposition(s) should be negated. It seems that we always accommodate the counterfactual of the nearest preceding sentence, but this generalization has to be verified by more examples.

8.2.4 Others

Besides the three above aspects, many other directions can be considered as future research, as well. For instance, in the following set of examples, the lifespan of a discourse referent is longer than we would expect:

(135) a. You must write a letter$_i$ to your parents. It$_i$ has to be sent by airmail. The letter must get there by tomorrow. Karttunen (1969)
b. Mary wants to marry a rich man$_i$. He$_i$ must be a banker. Karttunen (1969)
c. Harvey courts a girl$_i$ at every convention. She$_i$ always comes to the banquet with him. The girl$_i$ is usually very pretty. Karttunen (1969)
d. A train$_i$ leaves every hour for Boston. It$_i$ always stops in New Haven. Sells (1985)
e. Every chess set comes with a spare pawn$_i$. It$_i$ is taped to the top of the box. Sells (1985)

At a first glance, the above examples, in particular, (135-c), (135-d), and (135-e), where no modality is involved, can be generalized as modal subordination. The quantifications in these examples can be treated in an analogous way as modality[3]: the quantification is over objects such as situation or time, while modality ranges over possible worlds. According to M-TTDL, the indefinite introduced under the scope of some modal operator is accessible to subsequent modal context. However, this rule is only admitted when the first modality is existential, which is not the case in any of (135). To account for examples such as (135), one will have to investigate the environments under which the scope of universal modality can be extended.

Further more, discourse referents in the present work are only concerned with individuals. However, abstract entities such as propositions, events, actions, etc., may also play the role of discourse referent. These abstract entities are usually not introduced by

[3]The second sentence in example (135-e) is assumed to contain a covert universal quantifier: it can be paraphrased as *it is always taped to the top of the box.*

NPs, but by clauses or sentences. And they are typically referred back by the neutral pronoun *it*, as shown in an earlier example in chapter 2:

(30) John insulted the ambassador$_i$. It$_i$ happened at noon. Gundel et al. (2005)

Naturally, the anaphor *it* is understood as referring to the event that John insulted the ambassador. To account for such kind of anaphora, the left context should be enriched with abstract discourse referents. As for the establishment of these referents, one will have to propose appropriate lexical entries for the action predicates Qian and Amblard (2011).

Finally, we have been treating accessibility as a dichotomic concept, namely, a discourse referent is either accessible or inaccessible. However, accessibility is intuitively a graded notion: among all accessible referents, some are more likely than others to serve as antecedent Ariel (1985, 1990, 2001). The likelihood is affected by a number of factors, such as the distance between the antecedent and the anaphor, the state of topicality, etc. Thus, in a more comprehensive theory of anaphora, accessible discourse referents should be accompanied by their corresponding degrees of accessibility.

Appendix A

Dynamic Translation of The Lexicon

In this appendix, we provide the systematic translations of static lexical entries to their dynamic counterparts in three frameworks: TTDL, DN-TTDL and M-TTDL. The presentation will base on the syntactic category of the lexicon for each framework.

A.1 The Dynamic Translation of TTDL

Proper Name

We take *John* as an example for proper name.

1. The standard entry for *John*:

$$[\![John]\!] = \lambda P.P\ \mathbf{john}$$

   It takes a property as input, and yields a proposition, its type is $(\iota \to o) \to o$.

2. According to definition 4.4.4:

$$\overline{[\![John]\!]} = \overline{\lambda P.P\ \mathbf{john}} = \lambda P.\overline{P\ \mathbf{john}} = \lambda P.\overline{P}\ \overline{\mathbf{john}} = \lambda P.P\ \overline{\mathbf{john}}$$

3. The individual constant **john** is of type ι, hence according to definition 4.4.3:

$$\overline{\mathbf{john}} = \mathbb{D}_\iota(\lambda e.\mathbf{john}) = \mathbf{john}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\overline{[\![John]\!]} = \lambda P.P\ \overline{\mathbf{john}} = \lambda P.P\ \mathbf{john}$$

   P is of type $\iota \to \Omega$, **john** is of type ι, hence $\overline{[\![John]\!]}$ is of type $(\iota \to \Omega) \to \Omega$.

Common Noun

We take *man* as an example for common noun.

1. The standard entry for *man*:

$$[\![man]\!] = \lambda x.\mathbf{man}\ x$$

   It takes an individual as input, and yields a proposition, its type is $\iota \to o$.

2. According to definition 4.4.4:

$$\overline{[\![man]\!]} = \overline{\lambda x.\mathbf{man}\ x} = \lambda x.\overline{\mathbf{man}\ x} = \lambda x.\overline{\mathbf{man}}\ x$$

3. The predicate constant **man** is of type $\iota \to o$, hence according to definition 4.4.3:

$$\begin{aligned}
\overline{\mathbf{man}} &= \mathbb{D}_{\iota\to o}(\lambda e.\mathbf{man}) \\
&= \lambda x.\mathbb{D}_o(\lambda e.(\lambda e'.\mathbf{man})e(\mathbb{S}_\iota xe)) \\
&\to_\beta \lambda x.\mathbb{D}_o(\lambda e.\mathbf{man}(\mathbb{S}_\iota xe)) \\
&= \lambda x.\mathbb{D}_o(\lambda e.\mathbf{man}\ x) \\
&= \lambda xe\phi.((\lambda e'.\mathbf{man}\ x)e \wedge \phi e) \\
&\to_\beta \lambda xe\phi.(\mathbf{man}\ x \wedge \phi e)
\end{aligned}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned}
\overline{[\![man]\!]} &= \lambda x.\overline{\mathbf{man}}\ x \\
&= \lambda x.(\lambda x'e\phi.(\mathbf{man}\ x' \wedge \phi e))x \\
&\to_\beta \lambda xe\phi.(\mathbf{man}\ x \wedge \phi e)
\end{aligned}$$

   x is of type ι, hence $\overline{[\![man]\!]}$ is of type $\iota \to \Omega$.

Intransitive Verb

We take *walk* as an example for intransitive verb.

1. The standard entry for *walk*:

$$[\![walk]\!] = \lambda S.S(\lambda x.\mathbf{walk}\ x)$$

   It takes a NP as input, and yields a proposition, its type is $((\iota \to o) \to o) \to o$.

2. According to definition 4.4.4:

$$\begin{aligned}
\overline{[\![walk]\!]} &= \overline{\lambda S.S(\lambda x.\mathbf{walk}\ x)} \\
&= \lambda S.S\overline{(\lambda x.\mathbf{walk}\ x)}
\end{aligned}$$

3. Similar to $\overline{\lambda x.\mathbf{man}\ x}$, we can infer

$$\overline{\lambda x.\mathbf{walk}\ x} = \lambda x e \phi.(\mathbf{walk}\ x \wedge \phi e)$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned} \overline{[\![walk]\!]} &= \lambda S.S\overline{(\lambda x.\mathbf{walk}\ x)} \\ &= \lambda S.S(\lambda x e \phi.(\mathbf{walk}\ x \wedge \phi e)) \end{aligned}$$

S is of the type of a dynamized NP, namely $(\iota \to \Omega) \to \Omega$, x is of type ι, hence $\overline{[\![walk]\!]}$ is of type $((\iota \to \Omega) \to \Omega) \to \Omega$.

Determiners

Existential Quantifier

We first look at the existential quantifier *a*.

1. The standard entry for *a*:

$$[\![a]\!] = \lambda PQ.\exists(\lambda x.Px \wedge Qx)$$

It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 4.4.4:

$$\begin{aligned} \overline{[\![a]\!]} &= \overline{\lambda PQ.\exists(\lambda x.Px \wedge Qx)} \\ &= \lambda PQ.\overline{\exists(\lambda x.Px \wedge Qx)} \\ &= \lambda PQ.\overline{\exists}(\lambda x.Px \mathbin{\overline{\wedge}} Qx) \\ &= \lambda PQ.\exists^d_{TTDL}(\lambda x.Px \wedge^d_{TTDL} Qx) \end{aligned}$$

3. According to formula 4.34 and 4.38:

$$\begin{aligned} \overline{[\![a]\!]} &= \lambda PQ.\exists^d_{TTDL}(\lambda x.Px \wedge^d_{TTDL} Qx) \\ &= \lambda PQ.(\lambda P' e\phi.\exists(\lambda x.P'x(x :: e)\phi))(\lambda x.(\lambda ABe\phi.Ae(\lambda e'.Be'\phi))(Px)(Qx)) \\ &\twoheadrightarrow_\beta \lambda PQ.(\lambda P' e\phi.\exists(\lambda x.P'x(x :: e)\phi))(\lambda x.(\lambda e\phi.(Px)e(\lambda e'.(Qx)e'\phi))) \\ &\twoheadrightarrow_\beta \lambda PQe\phi.\exists(\lambda x.Px(x :: e)(\lambda e'.Qxe'\phi)) \end{aligned}$$

P and Q are of type $\iota \to \Omega$, hence $\overline{[\![a]\!]}$ is of type $(\iota \to \Omega) \to (\iota \to \Omega) \to \Omega$.

Universal Quantifier

We turn to the universal quantifier *every*.

1. The standard entry for *every*:

$$\begin{aligned}[\![every]\!] &= \lambda PQ.\forall(\lambda x.(Px \to Qx))\\ &= \lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))\end{aligned}$$

   It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 4.4.4:

$$\begin{aligned}\overline{[\![every]\!]} &= \overline{\lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}\\ &= \lambda PQ.\overline{\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}\\ &= \lambda PQ.\overline{\neg}(\overline{\exists}(\lambda x.Px\overline{\wedge}\overline{\neg}(Qx)))\\ &= \lambda PQ.\neg^d_{TTDL}(\exists^d_{TTDL}(\lambda x.Px \wedge^d_{TTDL} \neg^d_{TTDL}(Qx)))\end{aligned}$$

3. According to formula 4.34, 4.37 and 4.38:

$$\begin{aligned}\overline{[\![every]\!]} &= \lambda PQ.\neg^d_{TTDL}(\exists^d_{TTDL}(\lambda x.Px \wedge^d_{TTDL} \neg^d_{TTDL}(Qx)))\\ &= \lambda PQ.(\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)((\lambda Pe\phi.\exists(\lambda x.Px(x :: e)\phi))\\ &\qquad (\lambda x.(\lambda ABe\phi.Ae(\lambda e'.Be'\phi))(Px)((\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)(Qx))))\\ &\twoheadrightarrow_\beta \lambda PQ.(\lambda Ae\phi.\neg(A\ e\ \mathbf{stop}) \wedge \phi e)((\lambda Pe\phi.\exists(\lambda x.Px(x :: e)\phi))\\ &\qquad (\lambda xe\phi.Pxe(\lambda e'.(\neg(Q\ x\ e'\ \mathbf{stop}) \wedge \phi e'))))\\ &\twoheadrightarrow_\beta \lambda PQe\phi.\neg(\exists(\lambda x.Px(x :: e)(\lambda e'.\neg(Q\ x\ e'\ \mathbf{stop})))) \wedge \phi e\end{aligned}$$

Pronoun

Syntactically, a pronoun belongs to the NP category. Its semantic type ought to be $(\iota \to o) \to o$. However, in standard logical semantics such as MG, no explicit entry for pronoun is provided. It is simply treated as a variable bound by the quantifier from the antecedent in standard logical semantics. In the vocabulary of TTDL, we have introduced the choice operator sel, which picks up an individual from the left context. So the dynamic entry for pronoun, such as *he*, can be given as follows:

$$\overline{[\![he]\!]} = \lambda Pe\phi.P(\mathsf{sel}\ e)e\phi$$

P is of type $\iota \to \Omega$, hence $\overline{[\![he]\!]}$ is of type $(\iota \to \Omega) \to \Omega$.

As shown in this section, every standard lexical entry can achieve its dynamic translation in TTDL through the systematic steps described in definition 4.4.4.

A.2 The Dynamic Translation of DN-TTDL

Proper Name

We take *John* as an example for proper name.

1. The standard entry for *John*:

$$[\![John]\!] = \lambda P.P\ \mathbf{john}$$

   It takes a property as input, and yields a proposition, its type is $(\iota \to o) \to o$.

2. According to definition 6.2.3:

$$\overline{\overline{[\![John]\!]}} = \overline{\overline{\lambda P.P\ \mathbf{john}}} = \lambda P.\overline{\overline{P\ \mathbf{john}}} = \lambda P.\overline{\overline{P}}\ \overline{\overline{\mathbf{john}}} = \lambda P.P\ \overline{\overline{\mathbf{john}}}$$

3. The individual constant **john** is of type ι, hence according to definition 6.2.2:

$$\overline{\overline{\mathbf{john}}} = \mathbb{D}^{dn}_{\iota}(\lambda e.\mathbf{john}) = \mathbf{john}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\overline{\overline{[\![John]\!]}} = \lambda P.P\ \overline{\overline{\mathbf{john}}} = \lambda P.P\ \mathbf{john}$$

   P is of type $\iota \to \Omega^{dn}$, **john** is of type ι, hence $\overline{\overline{[\![John]\!]}}$ is of type $(\iota \to \Omega^{dn}) \to \Omega^{dn}$.

Common Noun

We take *man* as an example for common noun.

1. The standard entry for *man*:

$$[\![man]\!] = \lambda x.\mathbf{man}\ x$$

   It takes an individual as input, and yields a proposition, its type is $\iota \to o$.

2. According to definition 6.2.3:

$$\overline{\overline{[\![man]\!]}} = \overline{\overline{\lambda x.\mathbf{man}\ x}} = \lambda x.\overline{\overline{\mathbf{man}\ x}} = \lambda x.\overline{\overline{\mathbf{man}}}\ x$$

3. The predicate constant **man** is of type $\iota \to o$, hence according to definition 6.2.2:

$$\begin{aligned}
\overline{\overline{\mathbf{man}}} &= \mathbb{D}^{dn}_{\iota\to o}(\lambda e.\mathbf{man}) \\
&= \lambda x.\mathbb{D}^{dn}_{o}(\lambda e.(\lambda e'.\mathbf{man})e(\mathbb{S}^{dn}_{\iota} xe)) \\
&\to_{\beta} \lambda x.\mathbb{D}^{dn}_{o}(\lambda e.\mathbf{man}(\mathbb{S}^{dn}_{\iota} xe)) \\
&= \lambda x.\mathbb{D}^{dn}_{o}(\lambda e.\mathbf{man}\ x) \\
&= \lambda x.\langle \mathbb{D}_{o}(\lambda e.\mathbf{man}\ x), \neg^{d}_{TTDL}(\lambda e.\mathbf{man}\ x)\rangle \\
&\twoheadrightarrow_{\beta} \lambda x.\langle \lambda e\phi.\mathbf{man}\ x \wedge \phi e, \lambda e\phi.\neg(\mathbf{man}\ x) \wedge \phi e\rangle
\end{aligned}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned}\overline{\overline{\llbracket man \rrbracket}} &= \lambda x.\overline{\overline{\mathbf{man}}}\ x \\ &= \lambda x.(\lambda x'.\langle \lambda e\phi.\mathbf{man}\ x' \wedge \phi e, \lambda e\phi.\neg(\mathbf{man}\ x') \wedge \phi e\rangle)x \\ &\to_\beta \lambda x.\langle \lambda e\phi.\mathbf{man}\ x \wedge \phi e, \lambda e\phi.\neg(\mathbf{man}\ x) \wedge \phi e\rangle\end{aligned}$$

x is of type ι, hence $\overline{\overline{\llbracket man \rrbracket}}$ is of type $\iota \to \Omega^{dn}$.

Intransitive Verb

We take *walk* as an example for intransitive verb.

1. The standard entry for *walk*:

$$\llbracket walk \rrbracket = \lambda S.S(\lambda x.\mathbf{walk}\ x)$$

   It takes a NP as input, and yields a proposition, its type is $((\iota \to o) \to o) \to o$.

2. According to definition 6.2.3:

$$\begin{aligned}\overline{\overline{\llbracket walk \rrbracket}} &= \overline{\overline{\lambda S.S(\lambda x.\mathbf{walk}\ x)}} \\ &= \lambda S.S\overline{\overline{(\lambda x.\mathbf{walk}\ x)}}\end{aligned}$$

3. Similar to $\overline{\overline{\lambda x.\mathbf{man}\ x}}$, we can infer

$$\overline{\overline{\lambda x.\mathbf{walk}\ x}} = \lambda x.\langle \lambda e\phi.\mathbf{walk}\ x \wedge \phi e, \lambda e\phi.\neg(\mathbf{walk}\ x) \wedge \phi e\rangle$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned}\overline{\overline{\llbracket walk \rrbracket}} &= \lambda S.S\overline{\overline{(\lambda x.\mathbf{walk}\ x)}} \\ &= \lambda S.S(\lambda x.\langle \lambda e\phi.\mathbf{walk}\ x \wedge \phi e, \lambda e\phi.\neg(\mathbf{walk}\ x) \wedge \phi e\rangle)\end{aligned}$$

   S is of the type of a dynamized NP, namely $(\iota \to \Omega^{dn}) \to \Omega^{dn}$, x is of type ι, hence $\overline{\overline{\llbracket walk \rrbracket}}$ is of type $((\iota \to \Omega^{dn}) \to \Omega^{dn}) \to \Omega^{dn}$.

Determiners

Existential Quantifier

We first look at the existential quantifier *a*.

1. The standard entry for *a*:

$$\llbracket a \rrbracket = \lambda PQ.\exists(\lambda x.Px \wedge Qx)$$

   It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 6.2.3:

$$\begin{aligned}
\overline{\overline{[\![a]\!]}} &= \overline{\overline{\lambda PQ.\exists(\lambda x.Px \wedge Qx)}} \\
&= \lambda PQ.\overline{\overline{\exists(\lambda x.Px \wedge Qx)}} \\
&= \lambda PQ.\overline{\overline{\exists}}(\lambda x.Px \mathbin{\overline{\overline{\wedge}}} Qx) \\
&= \lambda PQ.\exists^{d}_{DN\text{-}TTDL}(\lambda x.Px \wedge^{d}_{DN\text{-}TTDL} Qx)
\end{aligned}$$

3. According to formula 6.27 and 6.28:

$$\begin{aligned}
\overline{\overline{[\![a]\!]}} &= \lambda PQ.\exists^{d}_{DN\text{-}TTDL}(\lambda x.Px \wedge^{d}_{DN\text{-}TTDL} Qx) \\
&= \lambda PQ.(\lambda P'.\langle \exists^{d}_{TTDL}(\lambda x.\pi_1(P'x)), \\
&\qquad\qquad \neg^{d}_{TTDL}(\exists^{d}_{TTDL}(\lambda x.\pi_1(P'x)))\rangle) \\
&\qquad (\lambda x.(\lambda AB.\langle(\pi_1 A) \wedge^{d}_{TTDL} (\pi_1 B), \\
&\qquad\qquad \neg^{d}_{TTDL}((\pi_1 A) \wedge^{d}_{TTDL} (\pi_1 B))\rangle)(Px)(Qx)) \\
&\twoheadrightarrow_\beta \lambda PQ.\langle \lambda e\phi.\exists(\lambda x.\pi_1(Px)(x :: e)(\lambda e'.\pi_1(Qx)e'\phi)), \\
&\qquad\quad \lambda e\phi.\neg(\exists(\lambda x.\pi_1(Px)(x :: e)(\lambda e'.\pi_1(Qx)\ e'\ \textbf{stop}))) \wedge \phi e\rangle
\end{aligned}$$

P and Q are of type $\iota \to \Omega^{dn}$, hence $\overline{\overline{[\![a]\!]}}$ is of type $(\iota \to \Omega^{dn}) \to (\iota \to \Omega^{dn}) \to \Omega^{dn}$.

Universal Quantifier

We turn to the universal quantifier *every*.

1. The standard entry for *every*:

$$\begin{aligned}
[\![every]\!] &= \lambda PQ.\forall(\lambda x.(Px \to Qx)) \\
&= \lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))
\end{aligned}$$

It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 6.2.3:

$$\begin{aligned}
\overline{\overline{[\![every]\!]}} &= \overline{\overline{\lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}} \\
&= \lambda PQ.\overline{\overline{\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}} \\
&= \lambda PQ.\overline{\overline{\neg}}(\overline{\overline{\exists}}(\lambda x.Px \overline{\overline{\wedge}} \overline{\overline{\neg}}(Qx))) \\
&= \lambda PQ.\neg^{d}_{DN\text{-}TTDL}(\exists^{d}_{DN\text{-}TTDL}(\lambda x.Px \wedge^{d}_{DN\text{-}TTDL} \neg^{d}_{DN\text{-}TTDL}(Qx)))
\end{aligned}$$

3. According to formula 6.27, 6.23 and 6.28:

$$\begin{aligned}\overline{\overline{[\![every]\!]}} &= \lambda PQ.\neg^d_{DN\text{-}TTDL}(\exists^d_{DN\text{-}TTDL}(\lambda x.Px \wedge^d_{DN\text{-}TTDL} \neg^d_{DN\text{-}TTDL}(Qx)))\\ &= \lambda PQ.(\lambda A.\mathbf{swap}\ A)((\lambda P.\langle \exists^d_{TTDL}(\lambda x.\pi_1(Px)), \neg^d_{TTDL}(\exists^d_{TTDL}(\lambda x.\pi_1(Px)))\rangle)\\ &\qquad (\lambda x.(\lambda AB.\langle(\pi_1 A) \wedge^d_{TTDL} (\pi_1 B), \neg^d_{TTDL}((\pi_1 A) \wedge^d_{TTDL} (\pi_1 B))\rangle)\\ &\qquad\quad (Px)((\lambda A.\mathbf{swap}\ A)(Qx))))\\ &\twoheadrightarrow_\beta \lambda PQ.\langle \lambda e\phi.\neg(\exists(\lambda x.\pi_1(Px)(x :: e)(\lambda e'.\neg(\pi_2(Qx)\ e'\ \mathbf{stop})))) \wedge \phi e,\\ &\qquad \lambda e\phi.\exists(\lambda x.\pi_1(Px)(x :: e)(\lambda e'.\neg(\pi_2(Qx)e\phi)))\rangle\end{aligned}$$

Pronoun

Syntactically, a pronoun belongs to the NP category. Its semantic type ought to be $(\iota \to o) \to o$. However, in standard logical semantics such as MG, no explicit entry for pronoun is provided. It is simply treated as a variable bound by the quantifier from the antecedent in standard logical semantics. In the vocabulary of DN-TTDL, we have introduced the choice operator sel, which picks up an individual from the left context. So the dynamic entry for pronoun, such as *he*, can be given as follows:

$$\overline{\overline{[\![he]\!]}} = \lambda Pe\phi.\langle P(\mathsf{sel}\ e)e\phi, \neg(P(\mathsf{sel}\ e)\ e\ \mathbf{stop}) \wedge \phi e\rangle$$

P is of type $\iota \to \Omega^{dn}$, hence $\overline{\overline{[\![he]\!]}}$ is of type $(\iota \to \Omega^{dn}) \to \Omega^{dn}$.

As shown in this section, every standard lexical entry can achieve its dynamic translation in DN-TTDL through the systematic steps described in definition 6.2.3.

A.3 The Dynamic Translation of M-TTDL

Proper Name

We take *John* as an example for proper name.

1. The standard entry for *John*:

$$[\![John]\!] = \lambda P.P\ \mathbf{john}$$

   It takes a property as input, and yields a proposition, its type is $(\iota \to o) \to o$.

2. According to definition 7.3.5:

$$\overline{[\![John]\!]}^{\mathsf{m}} = \overline{\lambda P.P\ \mathbf{john}}^{\mathsf{m}} = \lambda P.\overline{P\ \mathbf{john}}^{\mathsf{m}} = \lambda P.\overline{P}^{\mathsf{m}}\ \overline{\mathbf{john}}^{\mathsf{m}} = \lambda P.P\ \overline{\mathbf{john}}^{\mathsf{m}}$$

3. The individual constant **john** is of type ι, hence according to definition 7.3.4:

$$\overline{\mathbf{john}}^{\mathsf{m}} = \mathbb{D}^m_\iota(\lambda e.\mathbf{john}) = \mathbf{john}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\overline{[\![John]\!]}^{\mathsf{m}} = \lambda P.P\ \overline{\mathbf{john}}^{\mathsf{m}} = \lambda P.P\ \mathbf{john}$$

P is of type $\iota \to \Omega_i$, **john** is of type ι, hence $\overline{[\![John]\!]}^{\mathsf{m}}$ is of type $(\iota \to \Omega_i) \to \Omega_i$.

Common Noun

We take *man* as an example for common noun.

1. The standard entry for *man*:

$$[\![man]\!] = \lambda x.\mathbf{man}\ x$$

It takes an individual as input, and yields a proposition, its type is $\iota \to o$.

2. According to definition 7.3.5:

$$\overline{[\![man]\!]}^{\mathsf{m}} = \overline{\lambda x.\mathbf{man}\ x}^{\mathsf{m}} = \lambda x.\overline{\mathbf{man}\ x}^{\mathsf{m}} = \lambda x.\overline{\mathbf{man}}^{\mathsf{m}}\ x$$

3. The predicate constant **man** is of type $\iota \to o$, hence according to definition 7.3.4:

$$\begin{aligned}
\overline{\mathbf{man}}^{\mathsf{m}} &= \mathbb{D}^m_{\iota \to o_i}(\lambda e.\mathbf{man}) \\
&= \lambda x.\mathbb{D}^m_{o_i}(\lambda e.(\lambda e'.\mathbf{man})e(\mathbb{S}^{dn}_\iota xe)) \\
&\to_\beta \lambda x.\mathbb{D}^m_{o_i}(\lambda e.\mathbf{man}(\mathbb{S}^{dn}_\iota xe)) \\
&= \lambda x.\mathbb{D}^m_{o_i}(\lambda e.\mathbf{man}\ x) \\
&= \lambda xe\phi i.((\lambda e'.\mathbf{man}\ x)ei \wedge \phi(\mathbf{up_context}\ e\ i\ ((\lambda e'.\mathbf{man}\ x)e))i) \\
&\twoheadrightarrow_\beta \lambda xe\phi i.(\mathbf{man}\ x\ i \wedge \phi(\mathbf{up_context}\ e\ i\ (\mathbf{man}\ x))i)
\end{aligned}$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned}
\overline{[\![man]\!]}^{\mathsf{m}} &= \lambda x.\overline{\mathbf{man}}^{\mathsf{m}}\ x \\
&= \lambda x.(\lambda x'e\phi i.(\mathbf{man}\ x'\ i \wedge \phi(\mathbf{up_context}\ e\ i\ (\mathbf{man}\ x'))i))x \\
&\to_\beta \lambda xe\phi i.(\mathbf{man}\ x\ i \wedge \phi(\mathbf{up_context}\ e\ i\ (\mathbf{man}\ x))i)
\end{aligned}$$

x is of type ι, hence $\overline{[\![man]\!]}^{\mathsf{m}}$ is of type $\iota \to \Omega_i$.

Intransitive Verb

We take *walk* as an example for intransitive verb.

1. The standard entry for *walk*:

$$[\![walk]\!] = \lambda S.S(\lambda x.\mathbf{walk}\ x)$$

It takes a NP as input, and yields a proposition, its type is $((\iota \to o) \to o) \to o$.

2. According to definition 7.3.5:

$$\begin{aligned}\overline{[\![walk]\!]}^{\mathsf{m}} &= \overline{\lambda S.S(\lambda x.\mathbf{walk}\ x)}^{\mathsf{m}}\\ &= \lambda S.S\overline{(\lambda x.\mathbf{walk}\ x)}^{\mathsf{m}}\end{aligned}$$

3. Similar to $\overline{\lambda x.\mathbf{man}\ x}^{\mathsf{m}}$, we can infer

$$\overline{\lambda x.\mathbf{walk}\ x}^{\mathsf{m}} = \lambda xe\phi i.(\mathbf{walk}\ x\ i \wedge \phi(\mathbf{up\_context}\ e\ i\ (\mathbf{walk}\ x))i)$$

4. As a result, based on the result of the previous step, we can obtain:

$$\begin{aligned}\overline{[\![walk]\!]}^{\mathsf{m}} &= \lambda S.S\overline{(\lambda x.\mathbf{walk}\ x)}^{\mathsf{m}}\\ &= \lambda S.S(\lambda xe\phi i.(\mathbf{walk}\ x\ i \wedge \phi(\mathbf{up\_context}\ e\ i\ (\mathbf{walk}\ x))i))\end{aligned}$$

   S is of the type of a dynamized NP, namely $(\iota \to \Omega_i) \to \Omega_i$, x is of type ι, hence $\overline{[\![walk]\!]}^{\mathsf{m}}$ is of type $((\iota \to \Omega_i) \to \Omega_i) \to \Omega_i$.

Determiners

Existential Quantifier

We first look at the existential quantifier a.

1. The standard entry for a:

$$[\![a]\!] = \lambda PQ.\exists(\lambda x.Px \wedge Qx)$$

   It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 7.3.5:

$$\begin{aligned}\overline{[\![a]\!]}^{\mathsf{m}} &= \overline{\lambda PQ.\exists(\lambda x.Px \wedge Qx)}^{\mathsf{m}}\\ &= \lambda PQ.\overline{\exists(\lambda x.Px \wedge Qx)}^{\mathsf{m}}\\ &= \lambda PQ.\overline{{}^{\iota}\exists}^{\mathsf{m}}(\lambda x.Px\ \overline{\wedge}^{\mathsf{m}}\ Qx)\\ &= \lambda PQ.{}^{\iota}\exists^{d}_{M\text{-}TTDL}(\lambda x.Px \wedge^{d}_{M\text{-}TTDL} Qx)\end{aligned}$$

3. According to formula 7.38 and 7.40:

$$\begin{aligned}\overline{[\![a]\!]}^{\mathsf{m}} &= \lambda PQ.{}^{\iota}\exists^{d}_{M\text{-}TTDL}(\lambda x.Px \wedge^{d}_{M\text{-}TTDL} Qx)\\ &= \lambda PQ.(\lambda P'e\phi.({}^{\iota}\exists_i(\lambda x.P'xe\phi)))(\lambda x.(\lambda ABe\phi.Ae(\lambda e'.Be'\phi))(Px)(Qx))\\ &\twoheadrightarrow_{\beta} \lambda PQe\phi.{}^{\iota}\exists_i(\lambda x.Pxe(\lambda e'.Qxe'\phi))\\ &= \lambda PQe\phi i.{}^{\iota}\exists(\lambda x.Pxe(\lambda e'.Qxe'\phi)i)\end{aligned}$$

   P and Q are of type $\iota \to \Omega_i$, hence $\overline{[\![a]\!]}^{\mathsf{m}}$ is of type $(\iota \to \Omega_i) \to (\iota \to \Omega_i) \to \Omega_i$.

Universal Quantifier

We turn to the universal quantifier *every*.

1. The standard entry for *every*:

$$\begin{aligned}[\![every]\!] &= \lambda PQ.\forall(\lambda x.(Px \to Qx)) \\ &= \lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))\end{aligned}$$

   It takes two properties and returns a proposition, its type is $(\iota \to o) \to (\iota \to o) \to o$.

2. According to definition 7.3.5:

$$\begin{aligned}\overline{[\![every]\!]}^{\mathsf{m}} &= \overline{\lambda PQ.\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}^{\mathsf{m}} \\ &= \lambda PQ.\overline{\neg(\exists(\lambda x.Px \wedge \neg(Qx)))}^{\mathsf{m}} \\ &= \lambda PQ.\overline{\neg}^{\mathsf{m}}(\overline{{}^{\iota}\exists}^{\mathsf{m}}(\lambda x.Px\overline{\wedge}^{\mathsf{m}}\overline{\neg}^{\mathsf{m}}(Qx))) \\ &= \lambda PQ.\neg^{d}_{M\text{-}TTDL}({}^{\iota}\exists^{d}_{M\text{-}TTDL}(\lambda x.Px \wedge^{d}_{M\text{-}TTDL} \neg^{d}_{M\text{-}TTDL}(Qx)))\end{aligned}$$

3. According to formula 7.38, 7.39 and 7.40:

$$\begin{aligned}\overline{[\![every]\!]}^{\mathsf{m}} &= \lambda PQ.\neg^{d}_{M\text{-}TTDL}({}^{\iota}\exists^{d}_{M\text{-}TTDL}(\lambda x.Px \wedge^{d}_{M\text{-}TTDL} \neg^{d}_{M\text{-}TTDL}(Qx))) \\ &= \lambda PQ.(\lambda Ae\phi.\neg_i(A\; e\; \mathbf{stop}_i) \wedge_i \phi e) \\ &\qquad ((\lambda Pe\phi.({}^{\iota}\exists_i(\lambda x.Pxe\phi))) \\ &\qquad\quad (\lambda x.(\lambda ABe\phi.Ae(\lambda e'.Be'\phi))(Px)((\lambda Ae\phi.\neg_i(A\; e\; \mathbf{stop}_i) \wedge_i \phi e)(Qx)))) \\ &\twoheadrightarrow_\beta \lambda PQe\phi.\neg_i({}^{\iota}\exists_i(\lambda x.(Pxe(\lambda e'.\neg_i(Q\; x\; e'\; \mathbf{stop}_i))))) \wedge_i \phi e\end{aligned}$$

Pronoun

Syntactically, a pronoun belongs to the NP category. Its semantic type ought to be $(\iota \to o) \to o$. However, in standard logical semantics such as MG, no explicit entry for pronoun is provided. It is simply treated as a variable bound by the quantifier from the antecedent in standard logical semantics. In the vocabulary of M-TTDL, we have introduced the choice operator sel. Different from the one in previous frameworks, the sel in M-TTDL is of type $(\iota \to o) \to o \to \iota$, based on an input property, it retrieves an individual from the background proposition. So the dynamic entry for pronoun, such as *he* and *it*, can be given as follows:

$$\overline{[\![he]\!]}^{\mathsf{m}} = \lambda Pe\phi i.P(\mathsf{sel}\;(\lambda x.\mathbf{human}\; x\; i \wedge \mathbf{male}\; x\; i)\;(\mathbf{bkgd}\; e\; i\; i))e\phi i$$

$$\overline{[\![it]\!]}^{\mathsf{m}} = \lambda Pe\phi i.P(\mathsf{sel}\;(\lambda x.\neg(\mathbf{human}\; x\; i))\;(\mathbf{bkgd}\; e\; i\; i))e\phi i$$

P is of type $\iota \to \Omega_i$, hence both $\overline{[\![he]\!]}^{\mathsf{m}}$ and $\overline{[\![it]\!]}^{\mathsf{m}}$ are of type $(\iota \to \Omega_i) \to \Omega_i$.

As shown in this section, every standard lexical entry can achieve its dynamic translation in M-TTDL through the systematic steps described in definition 7.3.5.

Appendix B

Linguistic Examples

In this appendix, we provide the collection of all linguistic examples that have been used in the thesis. Examples will be listed chapter by chapter.

Chapter 1: Introduction

(1) Colorless green ideas sleep furiously. Chomsky (1957)

(2) a. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. A law firm in which Mr Sarkozy owns shares was also searched, reports say. (episode from BBC News Europe on 3 July 2012)
b. Police have carried out searches of the home and offices of former French President Nicolas Sarkozy as part of a campaign financing probe. Tens of thousands have turned out in the streets of the Spanish capital Madrid to welcome the national football team after their victory at Euro 2012. (mixed episode from BBC News Europe on 3 July 2012)

(3) a. John walks in. He smiles.
b. Bill walks in. He smiles.

(4) a. $\underline{\text{Russell}}_i$ admired $\underline{\text{him}}_{*i/j}$.
b. $\underline{\text{Russell}}_i$ admired $\underline{\text{himself}}_{i/*j}$. Huang (2006)

(5) a. $\underline{\text{John}}_i$ loves $\underline{\text{his}}_i$ mother.
b. $\underline{\text{Every man}}_i$ loves $\underline{\text{his}}_i$ mother. Evans (1980)

(6) $\underline{\text{A man}}_i$ walks in the park. $\underline{\text{He}}_i$ whistles.

(7) Every farmer who owns $\underline{\text{a donkey}}_i$ beats $\underline{\text{it}}_i$.

(8) Bill doesn't have $\underline{\text{a car}}_i$. *$\underline{\text{It}}_i$ is black. Karttunen (1969)

(9) You must write $\underline{\text{a letter}}_i$ to your parents. *They are expecting $\underline{\text{the letter}}_i$. Karttunen (1969)

(10) a. John did not fail to find $\underline{\text{an answer}}_i$. $\underline{\text{The answer}}_i$ was even right.
b. John did not remember not to bring $\underline{\text{an umbrella}}_i$, although we had no room for $\underline{\text{it}}_i$. Karttunen (1969)

(11) Either there's $\underline{\text{no bathroom}}_i$ in the house, or $\underline{\text{it}}_i$'s in a funny place. Roberts (1989) (motivated by Barbara Partee)

(12) If John bought a book$_i$, he'll be home reading it$_i$ by now. It$_i$'ll be a murder mystery. Roberts (1989)

(13) A thief$_i$ might break into the house. He$_i$ would take the silver. Roberts (1989)

Chapter 2: Linguistic Preliminaries

(14) Max$_i$ claims he$_i$ wasn't told about it. Huddleston et al. (2002)

(15) The idea was preposterous$_i$, but no one dared say so$_i$. Huddleston et al. (2002)

(16) Wim Kok is the prime minister of the Netherlands. Krahmer and Piwek (2000)

(17) The people who work for him love Al. Conroy et al. (2009)

(18) I was at a wedding$_i$ last week.
a. The bride$_i$ was pregnant.
b. The mock turtle soup$_i$ was a dream. Geurts (2009)

(19) Every man$_i$ thinks that he$_i$ deserves a raise. Carlson (2002)

(20) No one$_i$ wanted to admit that he$_i$ might be wrong. Partee (2008)

(21) John needed some cash so he went to a bank. Krahmer and Piwek (2000)

(22) a. When he$_i$ saw the damage, the headmaster$_i$ called in the police.
b. The repeated attacks on him$_i$ had made Max$_i$ quite paranoid. Huddleston et al. (2002)

(23) a. He$_i$'s the biggest slob I know. He$_i$'s really stupid. He$_i$'s so cruel. He$_i$'s my boyfriend Nick$_i$. Wikipedia (2014)
b. It$_i$'s a complete mystery to me: Why did he turn down such a marvellous offer$_i$? Huddleston et al. (2002)

(24) He went to Spain last week. Huddleston et al. (2002)

(25) a. He's up early.
b. I'm glad he's left. Evans (1980)

(26) a. The President of the U.S. visited France last week.
b. The moon orbits around the earth.

(27) I asked for a green shirt$_i$, but he gave me a white one$_i$. Huddleston et al. (2002)

(28) These boxes$_i$ are more suitable than the others$_i$. Huddleston et al. (2002)

(29) a. Ann$_i$ blamed herself$_i$ for the accident. (Reflexive)
b. They$_i$ are required to consult with each other$_i$/one another$_i$. (Reciprocal)
c. Everyone$_i$ had cast his$_i$ vote. (Possessive)
d. I raised some money$_i$ by hocking the good clothes I had left, but when that$_i$ was gone I didn't have a cent. (Demonstrative)
e. She wrote personally to those$_i$ whose$_i$ proposals had been accepted. Huddleston et al. (2002) (Relative)

(30) John insulted the ambassador$_i$. It$_i$ happened at noon. Gundel et al. (2005)

(31) a. It is fortunate that Nadia will never read this thesis.
b. It is half past two. Hirst (1981)

(32) Mary saw a movie$_i$ last week. The movie$_i$ was not very interesting. Abbott (2006)

(33) John$_i$ was playing. The tall boy$_i$ was happy. Elworthy (1992)

(34) Ross$_i$ used his Bankcard so much, the poor guy$_i$ had to declare bankruptcy. Hirst (1981)

(35) a. She agreed to help$_i$, but she did so$_i$ reluctantly.
b. If we are going to live together$_i$, we may as well do it$_i$ properly.
c. There are times when I'd just like to go down to the library and get some books$_i$, but often you can't do that$_i$ on the spur of the moment. Huddleston et al. (2002)

(36) a. Daryel thinks$_i$ like I do$_i$.
b. When Ross orders sweet and sour fried short soup$_i$, Nadia does$_i$ too. Hirst (1981)

(37) a. Some careless$_i$ driver backed into our car. Such$_i$ people make me mad. Partee (2008)
b. I was looking for a purple$_i$ wombat, but I couldn't find such$_i$ a wombat. Hirst (1981)

(38) a. In the mid-sixties$_i$, free love was rampant across campus. It was then$_i$ that Sue turned to Scientology.
b. In the mid-sixties$_i$, free love was rampant across campus. At that time$_i$, however, bisexuality had not come into vogue. Hirst (1981)

(39) a. John's brother$_i$ is an anti-war campaigner, and Bill's $\emptyset_i$ is an anti-globalization activist. Huang (2006) (Nominal Ellipsis)
b. *A*: Have you finished your assignment$_i$ yet?

(40) a. The President (of the United States)$_i$ walked off the plane. He$_i$ waved to the crowd.
b. The President$_i$ is elected every four years. He$_i$ has been from a southern state ten times. Carlson (2002)

(41) The man who gave his paycheck$_i$ to his wife was wiser than the man who gave it$_i$ to his mistress. Karttunen (1969)

(42) Kelly is seeking a unicorn$_i$ and Millie is seeking one$_i$ too. Luperfoy (1991)

(43) Ross likes his hair$_i$ short, but Daryel likes it$_i$ long. Hirst (1981)

(44) Leonard$_i$ is a famous conductor. He$_i$ writes operas. Carlson (2002)

(45) Few professors$_i$ came to the party. They$_i$ had a good time. King (2013)

(46) a. I met two people$_i$ yesterday. The woman$_i$ told me a story. (set membership)
b. I looked into the room$_i$. The ceiling$_i$ was very high. (necessary part)
c. I walked into the room$_i$. The chandeliers$_i$ sparkled brightly. (inducible part) Krahmer and Piwek (2000)

(47) Every man thinks that every man deserves a raise.

(48) No one wanted to admit that no one might be wrong.

(49) John$_i$ loves his$_i$ wife. Partee (2008)

(50) John loves his wife and so does Bill. Partee (2008)

(51) Zelda$_i$ bores herself$_{i/*j}$. Büring (2005)

(52) Bob$_i$ was nominated by him$_{*i/j}$. Carlson (2002)

(53) She$_{*i/j}$ hoped that Mary$_i$ would win the contest. Carlson (2002)

(54) a. They$_i$ saw each other$_i$'s pictures.
b. They$_i$ saw their$_i$ pictures. Huang (1983)

(55) a. They$_i$ saw pictures of each other$_i$.
b. They$_i$ saw pictures of them$_i$. Huang (1983)

(56) *He$_i$ exploits the secretary that each of the tenors$_i$ hired. Büring (2005)

Chapter 3: Mathematical Preliminaries

(57) a. John loves Mary.
b. Mary loves John.
c. John loves Mary and Mary does not love John.

(58) a. Casper is bigger than John.
b. John is bigger than Peter.
c. Casper is bigger than Peter. Gamut (1991a)

(59) Every man loves Mary.

(60) He loves Mary.

(61) a. Sandy might be home.
b. Sandy must be home. von Fintel (2006)

(62) It is not possible for pigs to fly. Holton (2004)

Chapter 4: Dynamic Frameworks

(63) a. I dropped ten marbles and found all of them, except for one. It is probably under the sofa.
b. I dropped ten marbles and found only nine of them. ?It is probably under the sofa. Heim (1982)

(64) a. A delegate arrived. She registered.
b. It is not the case that every delegate failed to arrive. *She registered. Kamp et al. (2011)

(65) a. A man who walks in the park whistles
b. A man$_i$ walks in the park and he$_i$ whistles.

(66) a. *Every dog$_i$ came in. It$_i$ lay down under the table.
b. *No dog$_i$ came in. It$_i$ lay down under the table. Heim (1982)

(67) A man$_i$ walks in the park. He$_i$ whistles. He$_i$ smokes.

(68) Omnis homo habens asinum videt illum.
(Every man owning a donkey sees it.)

(69) Every farmer who owns a donkey$_i$ beats it$_i$.

(70) *If every man$_i$ meets a nice woman$_j$, he$_i$ smiles at her$_j$. Van Eijck and Kamp (1997)

(71) *If John owns no donkey$_i$, he wants it$_i$. Chierchia (1995)

(72) *He$_i$ whistles. A man$_i$ walks in the park. Geurts (1999)

Chapter 5: Discourse Referent and Exceptions

(73) Bill doesn't have a car$_i$.

a. *It$_i$ is black.
b. *The car$_i$ is black.
c. *Bill's car$_i$ is black. Karttunen (1969)

(74) a. It is not the case that a man$_i$ walks in the park. *He$_i$ whistles.
b. No man$_i$ walks in the park. *He$_i$ whistles. Groenendijk and Stokhof (1991)

(75) a. ?Jones owns a car$_i$ or he hides it$_i$. Kamp and Reyle (1993)
b. *John has a new car$_i$ or Mary has it$_i$. Chierchia (1995)
c. *Either Jones owns a bicycle$_i$, or it$_i$'s broken. Simons (1996)

(76) a. Either John is home or he went out to get a coke$_i$. *It$_i$ is sugar free. Chierchia (1995)
b. Jane either borrowed a car$_i$ or rented a truck$_j$ to get to Boston. *It$_{i/j}$ broke down on the way. Simons (1996)

(77) a. If a farmer$_i$ owns a pedigree donkey$_j$, he$_i$ is rich. *It$_j$ lives on caviar.
b. Every farmer owns a donkey$_i$. *It$_i$ brays in distress. Elworthy (1992)

(78) a. Every man$_i$ walks in the park. *He$_i$ whistles.
b. Every farmer$_i$ who owns a donkey$_j$ beats it$_j$. *He$_i$ hates it$_j$. Groenendijk and Stokhof (1991)

(79) Bill didn't find a misprint$_i$. Can you find it$_i$? Karttunen (1969)

(80) Bill didn't see a misprint.

a. "There is a misprint which Bill didn't see."
b. "Bill saw no misprints." Karttunen (1969)

(81) John wants to catch a fish.

a. "There is a particular fish which John wants to catch."
b. "What John wants to catch is a fish." Karttunen (1969)

(82) Mary may want to marry a Swede.

a. "There is some Swede whom Mary may want to marry."
b. "It may be the case that there is some Swede whom Mary wants to marry."
c. "It may be the case that Mary wants her future husband to be a Swede." Karttunen (1969)

(83) Bill intends to visit a museum every day.

a. "There is a certain museum that Bill intends to visit every day."
b. "Bill intends that there be some museum that he visits every day."
c. "Bill intends to do a museum visit every day." Karttunen (1969)

(84) a. John wants to catch a fish$_i$. You can see the fish$_i$ from here.

b. Mary may want to marry a Swede$_i$. She introduced him$_i$ to her mother yesterday. Karttunen (1969)

(85) a. You must write a letter$_i$ to your parents. *They are expecting the letter$_i$.
b. Bill can make a kite$_i$. *The kite$_i$ has a long string. Karttunen (1969)

(86) a. John wants to catch a fish$_i$. *Do you see the fish$_i$ from here?
b. Mary expects to have a baby$_i$. *The baby$_i$'s name is Sue.
c. I doubt that Mary has a car$_i$. *Bill has seen it$_i$. Karttunen (1969)

(87) a. You must write a letter$_i$ to your parents and mail the letter$_i$ right away. *They are expecting the letter$_i$.
b. John wants to catch a fish$_i$ and eat it$_i$ for supper. *Do you see the fish$_i$ over there? Karttunen (1969)

(88) I don't believe that Mary had a baby$_i$ and named her$_i$ Sue. *The baby$_i$ has mumps. Karttunen (1969)

(89) a. Bill is not a linguist. Karttunen (1969)
b. A lion is a mighty hunter. Karttunen (1969)
c. A donkey is an animal. Le Pore and Garson (1983)
d. A blue-eyed bear is (always) intelligent. Heim (2011)

(90) a. John managed to find an apartment$_i$. The apartment$_i$ has a balcony.
b. Bill ventured to ask a question$_i$. The lecturer answered it$_i$. Karttunen (1969)

(91) a. John didn't manage to find an apartment$_i$. *The apartment$_i$ has a balcony.
b. Bill didn't dare to ask a question$_i$. *The lecturer answered it$_i$. Karttunen (1969)

(92) a. John managed to solve the problem.
b. John solved the problem. Karttunen (1971)

(93) a. John didn't manage to solve the problem.
b. John didn't solve the problem. Karttunen (1971)

(94) a. John forgot to write a term paper$_i$. *He cannot show it$_i$ to the teacher.
b. John fails to find an answer$_i$. *It$_i$ was wrong. Karttunen (1969)

(95) John knew that Mary had a car$_i$, but he had never seen it$_i$. Karttunen (1969)

(96) Bill didn't realize that he had a dime$_i$. It$_i$ was in his pocket. Karttunen (1969)

(97) a. John brought an umbrella.
b. John didn't bring an umbrella.
c. It is not true that John didn't bring an umbrella.

(98) a. It is not true that John didn't bring an umbrella$_i$. It$_i$ was purple and it$_i$ stood in the hallway. Krahmer and Muskens (1995)
b. It is not true that there was no lion$_i$ in the cage. I saw it$_i$ sleeping and heard it$_i$ snoring. Kaup and Lüdtke (2008)

(99) a. Some of the students$_i$ passed the examination. They$_i$ must have studied hard.
b. Not all the students$_i$ failed the examination. ?They$_i$ must have studied hard. Hintikka (2002)

(100) Either Jones does not own a car$_i$ or he hides it$_i$. Kamp and Reyle (1993)

(101) a. Either Jones does not own a car$_i$ or else he hides it$_i$.

b. Either Jones does not own a car$_i$ or otherwise he hides it$_i$.
c. Either Jones does not own a car or Jones owns a car$_i$ and he hides it$_i$. Kamp and Reyle (1993)

(102) Either Jones does not own a car or if Jones owns a car$_i$ then he hides it$_i$.

(103) If Jones owns a car$_i$ then he hides it$_i$.

(104) A thief$_i$ might break into the house. *He$_i$ takes the silver.

(105) If John bought a book$_i$, he'll be home reading it$_i$ by now. *It$_i$'s a murder mystery. Roberts (1989)

Chapter 6: Double Negation

(106) John brought an umbrella$_i$. It$_i$ was purple and it$_i$ stood in the hallway.

(107) It is not the case that either there's no bathroom$_i$ in the house, or it$_i$'s in a funny place. *It$_i$ is well-furnished.

(108) It is not the case that if a farmer$_i$ owns a donkey$_j$, he$_i$ beats it$_j$. *He$_i$ hates it$_j$.

(109) It is not the case that every farmer$_i$ who owns a donkey$_j$ beats it$_j$. *He$_i$ hates it$_j$.

(110) Bill has a car$_i$. It$_i$ is black. Karttunen (1969)

(111) It is not the case that Bill doesn't own a car$_i$. It$_i$ is black.

(112) *Either there's a bathroom$_i$ in the house, or it$_i$'s in a funny place.

(113) a. A student$_i$ passed the examination. He$_i$ studied hard.
b. Not every student$_i$ failed the examination. *He$_i$ studied hard.

Chapter 7: Modality

(114) a. All Maori children must learn the names of their ancestors.
b. The ancestors of the Maoris must have arrived from Tahiti. Kratzer (1977)

(115) a. In view of what their tribal duties are, the Maori children must learn the names of their ancestors.
b. In view of what is known, the ancestors of the Maoris must have arrived from Tahiti. Kratzer (1977)

(116) a. According to his dating coach, John must dance at parties.
b. Since John hangs out with Linda at parties, he must dance at parties. Starr (2012)

(117) a. It has to be raining. [after observing people coming inside with wet umbrellas; epistemic modality]
b. Visitors have to leave by six pm. [hospital regulations; deontic]
c. You have to go to bed in ten minutes. [stern father; bouletic]
d. I have to sneeze. [given the current state of one's nose; circumstantial]
e. To get home in time, you have to take a taxi. [telelological] von Fintel (2006)

(118) John must donate to charity, and he might do so. Starr (2012)

(119) a. Jockl must have been the murderer (in view of what we know).
b. Jockl is the murderer.
c. Jockl might have been the murderer (in view of what we know). Kratzer (1991)

(120) a. It is barely possible to climb Mount Everest without oxygen.
b. It is easily possible to climb Mount Toby.
c. They are more likely to climb the West Ridge than the Southeast Face.
d. It would be more desirable to climb the West Ridge by the Direct Route. Kratzer (1991)

(121) A wolf$_i$ might walk in. It$_i$ would growl. Asher and Pogodalla (2011a)

(122) A wolf$_i$ might walk in. It$_i$ would growl. A second wolf$_j$ might then walk in, but it$_j$ wouldn't growl. Asher and Pogodalla (2011a)

(123) John might buy a house$_i$. He earns enough to get a mortgage. He could rent it$_i$ out for the Festival. Kibble (1994)

(124) A wolf$_i$ might walk in. We would be safe because John$_j$ has a gun$_k$. He$_j$ would use it$_k$ to shoot it$_i$. Stone (1999)

(125) A wolf$_i$ walks in. It$_i$ might growl.

(126) a. A wolf$_i$ might walk in. *It$_i$ growls.
b. A wolf$_i$ would walk in. *It$_i$ growls.

(127) A wolf$_i$ might walk in. John has a gun$_j$. John would use it$_j$ to shoot it$_i$.

Chapter 8: Conclusion

(128) a. John had a great evening last night.
b. He had a great meal.
c. He ate salmon$_i$.
d. He devoured lots of cheese.
e. He then won a dancing competition.
f. *It$_i$ was a beautiful pink. Asher and Vieu (2005)

(129) a. John took Mary to Acapulco. They had a lousy time.
b. Last month John took Mary to Acapulco. Fred and Suzie were already there. The next morning they set off on their sailing trip. Kamp and Reyle (1993)

(130) a. Every student$_i$ wrote a paper. They$_i$ also read a book.
b. Every student wrote a paper$_i$. They$_i$ weren't very good. Nouwen (2003b)

(131) John won't buy a car$_i$ because he wouldn't have room for it$_i$ in his garage. Partee (1973)

(132) John won't buy a car because if he did buy a car$_i$, he wouldn't have room for it$_i$ in his garage. Partee (1973)

(133) a. I didn't submit a paper$_i$. They wouldn't have published it$_i$. Kibble (1994)
b. John didn't buy a mystery novel$_i$. He would be reading it$_i$ by now. Krifka (2001)
c. Mary didn't buy a microwave$_i$. She would never use it$_i$. Frank (1997)

d. Fred didn't draw a picture$_i$. He would have made a mess of it$_i$. Frank (1997)

(134) a. I didn't submit a paper. If I had submitted a paper$_i$, they wouldn't have published it$_i$. Kibble (1994)

b. John didn't buy a mystery novel. If he had bought a mystery novel$_i$, he would be reading it$_i$ by now. Krifka (2001)

c. Mary didn't buy a microwave because if she had bought a microwave$_i$ she would never have used it$_i$. Frank (1997)

d. Fred didn't draw a picture because if he had drawn a picture$_i$ he would have made a mess of it$_i$. Frank (1997)

(135) a. You must write a letter$_i$ to your parents. It$_i$ has to be sent by airmail. The letter must get there by tomorrow. Karttunen (1969)

b. Mary wants to marry a rich man$_i$. He$_i$ must be a banker. Karttunen (1969)

c. Harvey courts a girl$_i$ at every convention. She$_i$ always comes to the banquet with him. The girl$_i$ is usually very pretty. Karttunen (1969)

d. A train$_i$ leaves every hour for Boston. It$_i$ always stops in New Haven. Sells (1985)

e. Every chess set comes with a spare pawn$_i$. It$_i$ is taped to the top of the box. Sells (1985)

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Résumé Mini

Cette thèse prend ses racines dans la tradition sémantique montagovienne et dynamique standard. L'objet est les conditions dans lesquelles un syntagme nominal peut agir comme antécédent d'une expression anaphorique. Le travail porte sur l'accessibilité des référents de discours dans un système formel de la sémantique dynamique. Le cadre choisi est celui proposé par De Groote, type théorique Dynamic Logic (TTDL) car il fait appel à des outils mathématiques et logiques standards, qui permettent de conserver le principe de compositionnalité. Nous étendons la couverture de la gestion de l'accessibilité des référents dans TTDL à deux cas naturellement problématique pour les théories sémantiques dynamiques classiques, en particulier, l'anaphore sous la double négation et les modalités. Une adaptation est définie pour chaque cas et enfin, l'intégration des différentes solutions est proposée, ce qui montre la souplesse de TTDL.

Mini Abstract

This thesis has its roots in the standard Montagovian and dynamic semantic tradition. The subject is conditions under which a noun phrase may act as antecedent of a particular anaphoric expression. The work thesis deals with the accessibility of discourse referents using a formal system of dynamic semantics. The framework used is the one proposed by De Groote, Type Theoretic Dynamic Logic (TTDL) because it follows the Montagovian tradition and only makes use of standard mathematical and logical tools which allows to maintain compositionnality. We extend the coverage of TTDL to cases which are naturally problematic for classical dynamic semantic theories. In particularly, this thesis aims to extend TTDL's coverage of the accessibility of referents to two exceptions of classical dynamic theories, namely anaphora under double negation and modality. An adaptation is defined for each case and finally, an integration of various solutions is proposed, which shows the flexibility of TTDL.